



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

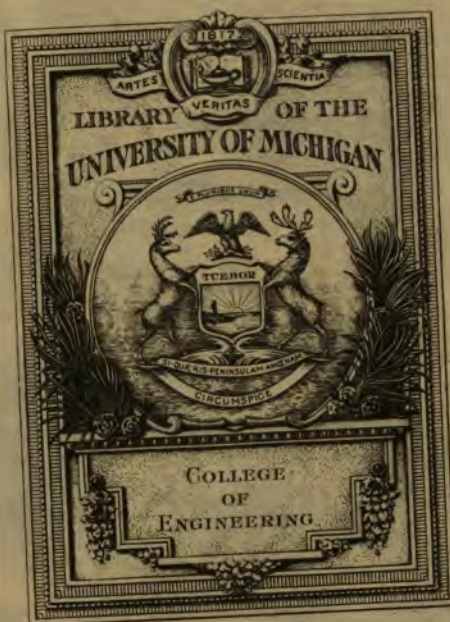
- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

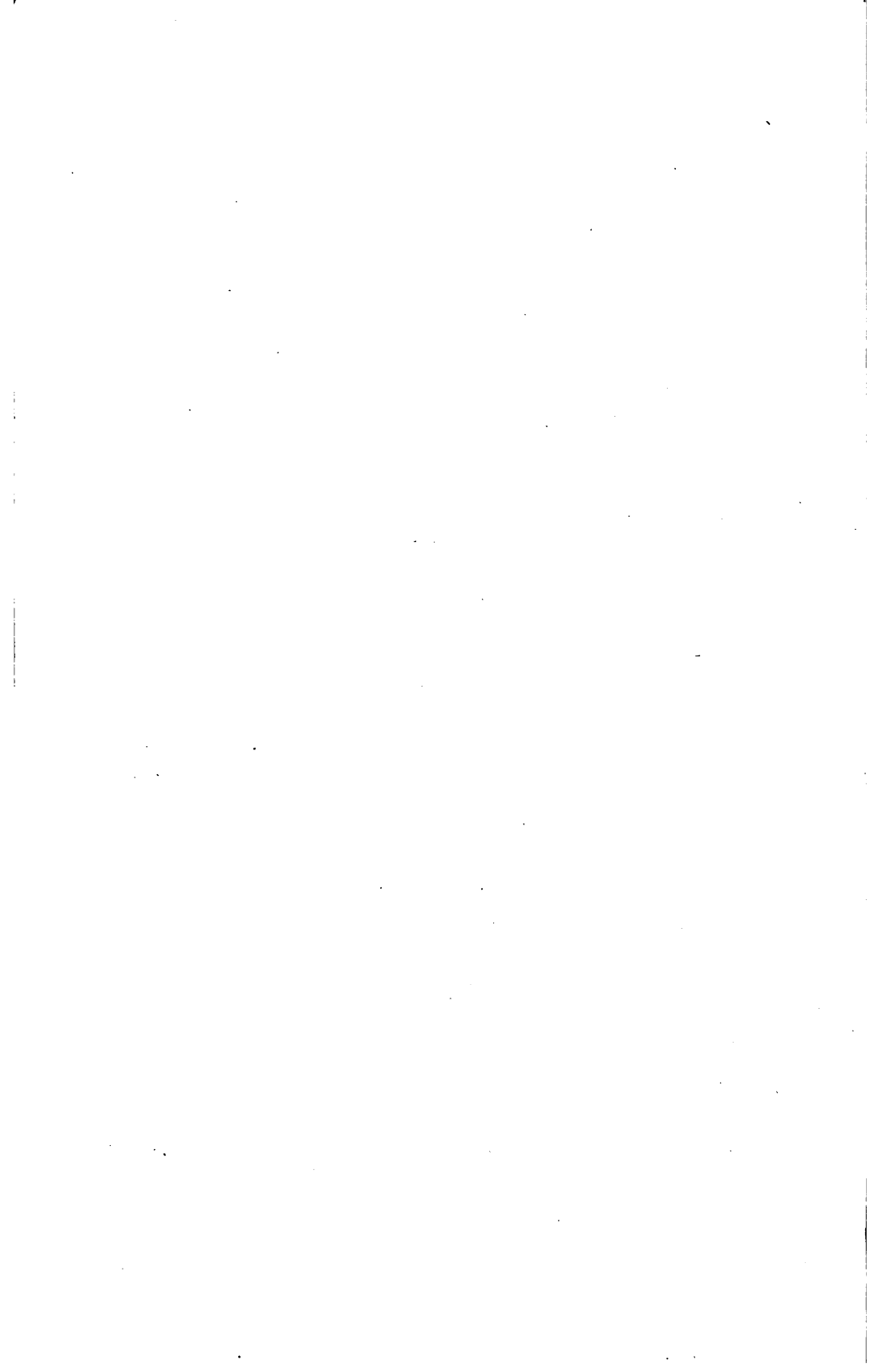
About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

B 465292

Graphical Analysis
OF
Roof Trusses





ENGINEER

LIBRARY

TG

270

.68

1900

WORKS OF PROF. C. E. GREENE

PUBLISHED BY

JOHN WILEY & SONS.

Graphics for Engineers, Architects, and Builders.

A Manual for Designers, and a Text-book for Scientific Schools.

Trusses and Arches. Analyzed and Discussed by Graphical Methods. In THREE PARTS.

PART I. Roof Trusses. Diagrams for Steady Load, Snow, and Wind. 8vo, cloth, \$1.25.

PART II. Bridge Trusses. Single, Continuous, and Draw Spans; Single and Multiple Systems; Straight and Inclined Chords. 8vo, cloth, \$2.50.

PART III. Arches in Wood, Iron, and Stone. For Roofs, Bridges, and Wall Openings; Arched Ribs and Braced Arches; Stresses from Wind and Change of Temperature. 8vo, cloth, \$2.50.

Structural Mechanics: The Action of Materials Under Stress. A work on the Strength and Resistance of Materials and the Elements of Structural Design. Ann Arbor, Mich., 1897. Printed for the author. 8vo, 300 pp., 100 illustrations. Price \$3.00.

Graphics for Engineers, Architects, and Builders:
A MANUAL FOR DESIGNERS, AND A TEXT-BOOK FOR TECHNICAL SCHOOLS

TRUSSES AND ARCHES

ANALYZED AND DISCUSSED BY GRAPHICAL METHODS

BY
CHARLES E. GREENE, A.M., C.E.,
PROFESSOR OF CIVIL ENGINEERING, UNIVERSITY OF MICHIGAN; CONSULTING ENGINEER.

IN THREE PARTS.

I.

ROOF-TRUSSES: DIAGRAMS FOR STEADY LOAD, SNOW, AND WIND.

II.

BRIDGE-TRUSSES: SINGLE, CONTINUOUS, AND DRAW SPANS; SINGLE AND MULTIPLE SYSTEMS; STRAIGHT AND INCLINED CHORDS.

III.

ARCHES, IN WOOD, IRON, AND STONE, FOR ROOFS, BRIDGES, AND WALL-OPENINGS; ARCHED RIBS AND BRACED ARCHES; STRESSES FROM WIND AND CHANGE OF TEMPERATURE; STIFFENED SUSPENSION BRIDGES.

PART I.—ROOF-TRUSSES.

THREE FOLDING PLATES.

REVISED EDITION.

FOURTH THOUSAND.

NEW YORK
JOHN WILEY & SONS
LONDON
CHAPMAN & HALL, LTD.
1900.

COPYRIGHT. 1890.
By CHARLES E. GREENE.

Braunworth, Munn & Barber
Printers and Binders
Brooklyn, N. Y.

Engrin. Lib.
925-1930-
Pt. 1-3
ad. 1. copy.

PREFACE TO PART I.

THE use of Graphical Analysis for the solution of problems in construction has become of late years very wide-spread. The representation to the eye of the forces which exist in the several parts of a frame possesses many advantages over their determination by calculation. The accuracy of the figures is readily tested by numerous checks. Any designer who fairly tries the method will be pleased with the simplicity and directness of the analysis, even for frames of apparently complex forms. Those persons who prefer arithmetical computation will find a diagram a useful check on their calculations. Being founded on principles absolutely correct, these diagrams give results depending for their accuracy on the exactness with which the lines have been drawn, and on the scale by which they are to be measured. With ordinary care the different forces may be obtained much more accurately than the several parts of the frame can be proportioned.

It is advisable to draw the figure of the frame to quite a large scale, as the lines of the stress diagram are drawn parallel to the several pieces of the frame. If it is objected by any that a slight deviation from the exact directions will materially change the lengths of some of the lines, and therefore give erroneous results, it may be suggested that just so much change in the form of the frame will produce this change in the forces; one is therefore warned where due allowance for

such deformation should be made by the proper distribution of material. The comparison of different types of truss for the same locality can be made with ease, and the changes produced in all of the forces in any frame by a modification of a few of its pieces can be readily shown. By applying each new principle to a new form of truss, quite a variety of patterns have been treated without an undue multiplication of figures.

The method of notation used was introduced by Mr. Bow, in his "Economics of Construction." The diagrams, as here developed, are credited in England to Prof. Clerk-Maxwell, and the method is known by his name. The arrangement of the subjects, the application of the method, and the minor details have been carefully studied by the author. A very limited knowledge of Mechanics will enable the reader to understand the method of treatment here carried out.

NOTE TO REVISED EDITION.

THE reception of this Part at the hands of teachers and designers, since its first appearance as a reprint of a series of articles in "*Engineering News*," has been so hearty and sustained, that it has been thought best to put ROOF-TRUSSES in a uniform dress and agreement with BRIDGE-TRUSSES and ARCHES. The opportunity has been seized to arrange the material in a more systematic order, introduce some additional problems, and improve, as it is thought, in some matters of detail.

Quite a modification has been made in the way of regarding trusses which exert horizontal thrust, and Chapter VIII., Special Solutions, is new. The solution by *reversal of a diagonal* has been used in the author's class-room for several years. The concluding example of this chapter will afford a good test of the reader's mastery of the preceding principles.

C. E. G.

ANN ARBOR, MICH., March 11, 1890.

TABLE OF CONTENTS.

CHAPTER I.

	PAGES
General Principles. Triangle of Forces ; Notation ; Illustrations, .	1-6

CHAPTER II.

Trusses with Straight Rafters ; Vertical Forces. Triangular, King-post, and Fink Trusses,	7-15
---	------

CHAPTER III.

Trusses for Flat Roofs. Queen-post, Warren, and Howe Trusses, .	16-21
---	-------

CHAPTER IV.

Wind Pressure on Pitched or Gable Roofs. Formula for Wind-pressure ; Examples with Roller Bearings ; Wind on Alternate Sides, 22-32	
---	--

CHAPTER V.

Wind Pressure on Curb (or Mansard) and Curved Roofs. Examples with and without Rollers,	33-43
---	-------

CHAPTER VI.

Trusses with Horizontal Thrust. Scissor and Hammer-beam Trusses, 44-49	
--	--

CHAPTER VII.

Forces not Applied at Joints,	50-52
---	-------

CHAPTER VIII.

Special Solutions. Reversal of Diagonal ; Trial and Error ; Example, 53-58	
--	--

TABLE OF CONTENTS.

ix

CHAPTER IX.

	PAGES
Bending Moment and Moment of Resistance. Equilibrium Polygon ;	
Graphical Solution for Moment of Resistance,	59-71

CHAPTER X.

Load and Details. Weight of Materials ; Allowable Stresses ; Ties,	
Struts, Beams, Details,	72-77

ROOF-TRUSSES.

CHAPTER I.

GENERAL PRINCIPLES.

1. **Aim of the Book.**—It is proposed, in this volume, to explain and illustrate a simple method for finding the stresses in all of the pieces of such roof or other trusses, under the action of a steady load, as permit of an exact analysis; to show how the wind or any oblique force alters the amount of the stresses arising from the weight; to add a device for solving some systems of trussing which otherwise appear insoluble by the above method; and to conclude with such an explanation of bending moments and moments of resistance as will make this part reasonably complete for roof designing.

2. **Triangle of Forces.**—Taking it for granted that, if two forces, acting at a common point, are represented in length and direction by the two adjacent sides of a parallelogram ca and cn , Fig. 2, their resultant will be equal to the diagonal cb of the figure, drawn from the same point,—it follows that a force equal to this resultant, and acting in the opposite direction, will balance the first two forces. Hence, considering one-half of the parallelogram, we have the well-known proposition that, if three forces in equilibrium act at a single point, and a triangle be drawn with sides parallel to the three forces, these sides will be proportional in length, by a definite scale, to these forces. The forces will also be found to act in order

round the triangle, and must necessarily lie in one plane. If the magnitude of one force is known, the other two can be readily determined.

For example:—Let a known weight be suspended from the points 1 and 2, Fig. 1, by the cords 1-3, 3-2, and 3-4. Draw cb vertically to represent the weight by any convenient scale of pounds to the inch. This line will then be parallel to, and will equal the tension in 3-4. Draw ca parallel to 1-3, and ba parallel to 3-2. Then will the sides of the triangle cba represent the forces which act on the point 3, and they will be found to follow one another round the triangle, as shown by the arrows.

3. **Notation.**—A notation will now be introduced which will be found very convenient when applied to trusses and diagrams. In the frame diagram write a capital letter in every *space* which is cut off from the rest of the figure by lines, real or imaginary, along which forces act. See Fig. 2 and following figures. Thus D represents the space within the triangular frame, A the space limited by the external forces acting at 1 and 2, B the space between the line to 2 and the line which carries the weight. Then let that piece of the frame or that force which lies between any two letters be called by those letters; thus, the upper bar of the triangle is AD, the right hand bar is BD, the cord to the point 1 is AC, that to the weight, or the weight itself, is CB, etc. In the diagrams drawn to determine the magnitude and kind of the several forces acting upon or in the frames the corresponding small letters will be used; thus cb will be the vertical line representing the force in CB, ba the tension of the cord BA, and ac the pull on 1.

4. **External Forces.**—Returning to Fig. 1, let us suppose that a rigid, triangular frame is made fast to those cords, so that, as shown by Fig. 2, the cords are attached to the vertices of the triangle, while their directions are undisturbed. It is evident that the same stresses still exist in those cords, if the frame has no weight, and that the portion of the cords

within the triangle may be cut away without destroying the equilibrium of this combination. Hence we see that the equilibrium of this frame is assured, if the directions of these cords, or forces external to the frame, meet, if prolonged, at a common point.

The external forces CB , BA and AC , taken in the order CBA , or passing around the exterior of the triangle in a direction contrary to the movement of the hands of a watch, give the triangle of forces cba , in which cb acting in a known direction, i.e. downwards, determines the direction of ba and ac in relation to their points of application to the frame, since for equilibrium, by § 2, they must follow one another in order round the stress triangle.

5. Stresses in the Frame.—Consider the left-hand apex of the triangle. This point is in equilibrium under the action of three forces, viz., those in AC , CD , and DA , which we read around the point *in the same order as before*; we found the direction and magnitude of AC in the previous section, and the inclinations of the other two are known. The three forces at this joint must therefore be equal to the three sides of a stress triangle, as before.

Begin with AC , the fully known force, and pass from a to c , because that is the direction of the action of the force AC on the joint under consideration. Next, from c , draw cd parallel to CD , prolonging it until a line from its extremity d , parallel to the piece DA , will strike or close on a . The stress cd is found in CD , and the stress da exists in DA . The direction in which we passed around acd , that is, from c to d , and then to a , shows that CD and DA both exert tension on the joint where they meet.

Next take the lowest joint. Remembering again to take the three forces in equilibrium here in the order in which the external forces were taken, and commencing with the first known one, we go, in the stress diagram, from d to c ; because, since we have just found that cd represents the pull of CD on the left-hand apex of the frame, dc must be the equal and op-

posite pull of DC on the lowest joint. Next comes cb , along which we pass *down*, the direction in which the weight acts; and finally we draw from b , bd parallel to the piece BD . This last line will close on the point d , if the construction has been carefully made, and the direction in which we pass over it, from b to d , shows that the piece BD exerts tension on the lowest joint. If the reader will now run over the triangle dba , which must belong to the right-hand joint, he will see that the directions just given are again complied with.

The reader can invert Fig. 2; then the weight will press down upon the upper apex of the triangle, and he will find, upon drawing the stress diagram, that the three external forces are thrusts, and that compression exists in each piece of the frame.

6. Second Illustration: External Forces.—In order to make these first principles more plain let us take another case. Suppose a triangular frame, Fig. 3, to rest against a wall by one angle, to have a weight of known amount suspended from the outer corner, and to be sustained by a cord attached to the third angle and secured to a point 2. Since this frame is at rest under the action of three external forces which are not parallel, their lines of action must, by § 2, meet at one common point; and since the known directions of two of these forces, AC and CB , will meet at 4, if prolonged, the force exerted on the frame by the wall at 1 must have the direction of the line 1-4. The magnitude and kind of the two unknown external forces therefore will be found by the following construction, observing the rules of interpretation already laid down:—

Draw ac , vertically down, equal to the known weight and force AC ; next, from c , a line parallel to the cord and force CB , and prolong it until, from its extremity b , a line may be drawn parallel to BA , to strike a . As we went from c to b , and from b to a , CB must pull on, and BA must thrust against, the frame.

7. Stresses in the Frame.—Take whichever joint is most

convenient, for instance the one where the weight is attached ; pass down ac for the external force and then, observing the order in which the triangle of external forces was drawn, draw cd parallel to CD and da parallel to DA . Since cd , in the triangle acd (made up of forces ac , cd , and da), must represent a force acting upwards, CD exerts tension on this joint ; and, similarly, da (not ad) shows that DA thrusts against the same joint.

Take next the joint at 1. Here the reaction, as before ascertained, is ba ; next comes ad , the thrust of the piece AD against this joint ; and lastly db , drawn parallel to DB , to close on b the point of beginning, shows that DB also thrusts with this amount at 1.

✓ 8. **Third Illustration.**—Once more, suppose that the triangular frame, Fig. 4, has a weight attached to its lowest angle and that the two other points are supported by inclined posts. The forces 1-4 and 2-4 must intersect 3-4 at the same point. Draw ab vertically downwards, and equal to the given weight ; draw bc parallel to 2-4 or BC and ca parallel to 1-4 or CA . Hence bc and ca are thrusts. For the lowest joint, after passing down ab for the weight, draw bd parallel to BD and da parallel to DA , thus finding that BD and DA both pull on the joint AB , and hence are tension members. As in former cases, find dc , which proves to be compression.

9. **General Application.**—Since, in Mechanics, the polygon of forces follows naturally from the triangle of forces, being simply a combination of several triangles, the same rules will apply when we have to deal with several external forces or a number of pieces meeting at one joint. 1°. Draw the polygon of external forces for the whole frame, taking them in order round the truss, either to the left or right, as may seem convenient. 2°. Take any joint where not more than two stresses in the pieces are unknown, and draw the polygon of forces for it. Treat the pieces and external forces which meet at the joint in that order, to the left or right, in which the external forces were taken, and begin, if possible,

with the first known force, so that the two unknown forces will be the last two sides of that particular polygon. 3°. The direction in which any line is passed over, in going round the polygon as above directed, shows whether the stress in the piece to which that line was drawn parallel acts towards or from the joint to which the polygon belongs, and hence is compression or tension. The reader must understand this principle in order to correctly interpret his diagrams.

10. **Reciprocal Figures.**—Prof. Clerk-Maxwell called the frame and stress diagrams *reciprocal* figures; for, referring to the figures already drawn, we see that the forces which meet at one point in the frame diagram give us a triangle or closed polygon in the stress diagram, and the pieces which make the triangular frame have their stresses represented by the lines which meet at one point in the stress diagram. The same reciprocity will exist in more complex figures, and it is one of the checks which we have upon the correctness of our diagrams.

The convenience of the notation explained in § 3 depends upon the above property.

CHAPTER II.

TRUSSES WITH STRAIGHT RAFTERS; VERTICAL FORCES.

11. Triangular Truss; Inclined Reactions.—Suppose that the roof represented in Fig. 5 has a certain load per foot over each rafter, and let the whole weight be denoted by W . It is evident that one-half of the load on the rafter CF will be supported by the joint B and one-half by the upper joint; the same will be true for the rafter DF ; therefore the joint B will carry $\frac{1}{2}W$, the upper joint $\frac{1}{2}W$, and the joint at E $\frac{1}{2}W$. The additional stress produced in CF by the bending action of the load which it carries is not considered at this time, but must be noticed and allowed for separately. (See Chap. IX.) Taking the external forces in order from right to left over the roof, lay off ed , or $\frac{1}{2}W$, vertically, to represent the weight ED acting downward at the joint E , next dc equal to $\frac{1}{2}W$, for the weight DC , and lastly cb for the weight at B . Call eb the *load line*.

Let the two reactions or supporting forces for the present be considered as a little inclined from the vertical, as shown by the arrows BA and AE . Since the truss is symmetrical and symmetrically loaded, the resultant of the load must pass through the apex of the roof, and, as the two supporting forces must meet this resultant at one point, the two reactions must be equally inclined. Then, to complete the polygon of external forces:—as we have drawn ed , dc , and cb in order, passing over the frame to the left,—draw next ba , up from the extremity b of the load line, and parallel to the upward reaction BA ; and lastly a line ae , parallel to the other reaction AE , to close on e , the point of beginning.

12. Triangular Truss: Stresses.—While in this truss we might find the stresses at any joint, let us begin at B . Here

we have equilibrium under the action of four forces, of which the two external ones are known. Taking the latter in the same order as above, and beginning at c (§ 9, 2°), pass over cb downwards and ba upwards; then draw af parallel to AF , in such a direction that fc , drawn from f parallel to FC , will strike c , the point of beginning. Because we passed from a to f , AF will pull on the joint B , and as we then passed from f to c , FC will exert a thrust on B . (It is usual to draw af from a and fc from c till they meet at f ; but to determine the *kind* of stress, one must pass over the lines in the directions noted.)

Passing next to the apex of the roof, and again taking the forces in the same order, pass down the line dc for the external force, thence up to f for the thrust cf , and finally draw fd parallel to FD , thus determining the thrust of that rafter against the top joint. If this line does not close on d , the drawing has not been made with care. As all the stresses are now found we need not examine the remaining joint. It may again be noted that we pass over a stress line in one direction when we analyze the stresses at the joint at one end of the piece to which the line is parallel, and in the reverse direction when we consider the joint at the other end of the same piece.

13. Effect of Inclined Reactions.—If the supporting forces had been more inclined from the vertical, the point a , of their meeting in the stress diagram, would have been nearer f , thus diminishing the tension in AF , but not affecting the compression in the rafters. The inclination might be so much increased that a would fall on f , when the piece AF would have no stress, the thrust of the rafters being balanced without it. If a fell to the right of f , af would be a thrust.

14. Triangular Truss: Vertical Reactions.—If the two reactions are vertical, as will be the case when the roof truss is simply placed upon the wall, BA and AE , Fig. 6, will each be $\frac{1}{2}W$, and the point a will therefore be found at the middle of eb . The polygon of external forces has closed up and be-

come a straight line, but in the analysis it must still be used. Thus we pass down $ed + dc + cb$ for the weights at the joints and back over $ba + ae$ for the reactions. The rest of the diagram follows from § 12.

The diagrams which the reader draws may be inked in black and red, one denoting compression, the other tension, or the two kinds of stress may be indicated by the signs $+$ and $-$.

15. King-post Truss.—In the truss of Fig. 7 the rafters are supported at points midway between their extremities. Each point of junction of two or more pieces is considered a joint around which the pieces would be free to turn were they not restrained by their connections with other points. Whatever stiffness the joint may possess from friction between its parts, or from the continuity of a piece, such as a rafter, through the joint, is not taken into account, and may add somewhat to the strength of the truss.

In this example, therefore, half of the uniform load on CL will be carried at B, and be represented by the arrow BC; the other half together with half of the uniform load on DK will make the force CD, and so on, three of the joints carrying each one-quarter of the whole load, and the two extreme ones one-eighth each.

On a vertical line lay off $gf = \frac{1}{8} W$, $fe = ed = dc = \frac{1}{4} W$ and $cb = \frac{1}{8} W$; then $ba = ag = \frac{1}{2} W$, the two supporting forces. In the order shown by the arrow, for the joint B we have cb external load, ba reaction; then draw al , tension, § 9, 3°, parallel to AL and lc , compression, parallel to LC. At the joint CD the unknown forces now are those in LK and KD. Begin with the load dc , following with cl , the stress just found in CL; then draw lk , compression, parallel to LK, and kd , compression, parallel to KD, to close on d . Passing next to the joint DE, ed is the load, dk the thrust of DK on this joint, ki the tension in KI,* and ie , to close on e , is the compression in IE. Take next the joint in the middle of the

* It will be seen that KI is a tension member or tie, and not a post as would be inferred from the name given to this truss by old builders.

lower tie; we know ik , kl , and la ; the next stress lies in AH ; as we have just arrived at a from l , we must pass back horizontally until a line from h parallel to HI will close on i , the point from which we started. The remaining line hf is easily determined by taking either the joint $E F$ or the one at G .

It will be noticed that, since the truss is symmetrically made and loaded, the stress diagram is symmetrical; ki must be bisected by al ; dk and ei must intersect on al . Attention to such points ensures the accuracy of the drawing.

A truss, Fig. 8, is now submitted, which the reader is advised to analyze for himself, as a test whether the principles thus far explained are clearly understood.

16. Wooden Truss with Frequent Joints.—The truss represented by Fig. 9, a simple extension of Fig. 7, is one well adapted for construction in timber, the verticals alone being made of iron. It can be used for roofs of large span. In any actual case, before beginning to draw the diagram, assume an approximate value for the weight of the truss, add so much of the weight of the purlins, small rafters, boards and slates, or other covering, as is supported by one truss, and divide this total weight by the number of equal parts, such as DI or EL , in the two rafters. We thus obtain the weight which is supposed to act at each joint where two pieces of the rafter meet. The weight at each abutment joint will be half as much. If the rafter is not supported at equidistant points, divide the total load by the combined length of both rafters, to obtain the load per foot of rafter, and then multiply the load per foot by the distance from the middle of one piece of the rafter to the middle of the next, to obtain the load on the joint which connects them. Numerical values will be introduced in later chapters.

Draw the vertical load line equal to the total weight, and beginning with bc as the load on B from one-half of CH , space off the weights cd , de , etc., in succession, closing at p with a half load as at b . The point of division a , at the middle of pb , marks off the two supporting forces pa and ab ,

which close the polygon of external forces. Beginning now at B, draw, as heretofore directed, § 9, *abcha* for this joint. The order of these letters gives the directions of the forces on the joint B. Then for the joint CD we have *hcdih*; for HK we have *ahika*; for DE we have *kidelk*, etc. Observe that, by taking the joints in this order, first the one on the rafter, and then the one below it on the tie, we have in each case only two unknown forces, out of, at some joints, five forces. We repeat, also, the remark that it is expedient, when possible, first to pass over all the known forces at any joint, taking them in the order observed with the external forces when laying off the load line. The rest of the diagram presents no difficulty.

After the stress in NO is obtained, the diagram will begin to repeat itself inversely, the stress in OG being equal to that in FN. It is therefore unnecessary to draw more than one-half of this figure, except for a check on the accuracy of the drawing by the intersections which are seen on inspection of this diagram. Noting the stresses found in the several polygons, we see that all the inclined pieces are in compression, while the horizontal and vertical members are in tension.

17. Superfluous Pieces.—Sometimes a vertical rod is introduced in the first and last triangles, where dotted lines are drawn. It is evident that this rod will be of no service if all the load is assumed to be concentrated on the joints of the rafters, and this fact can be determined from the stress diagram as well. Thus, taking the joint below H, Fig. 9, we have three forces in equilibrium; begin at *a* in the stress diagram and pass to *h* along the line already found for AH; then we are required to draw a vertical line from *h* and, from its extremity, a horizontal line to close on the point *a* from which we started; the vertical line therefore can have no length. All that this vertical rod can do is to keep the horizontal tie from sagging, by sustaining whatever small weight is found at its foot.

Therefore, whenever there are at a joint but three pieces or

lines along which forces can act, and two of these pieces lie in one straight line, it follows from the above that the third piece must be without stress, and that the first two pieces or lines will have the same stress. Thus, L K of Fig. 7 and H I of Fig. 9 would have no compression if the external load C D were removed. This fact will often prove of service in analysis.

18. Problem.—Draw the stress diagram for the truss illustrated by Fig. 10, which is supported on a shoulder at the wall and by an overhead tie running from the right end. It will be convenient to imagine that tie replaced by the inclined reaction shown by the arrow at the right, as thus the reaction is kept on the right of the load at that joint. The reaction at the wall will cut the tie where the resultant of the load cuts it; if the load is uniform over the rafter, that intersection is at the middle of the tie.

Next, try this problem with the two inclined diagonals reversed, so as to slant up to the right. Notice the upper left-hand joint. Compare the two cases, as to difference in magnitude and kind of stress.

19. Joints where three Forces are Unknown.—It appears impracticable to determine the stresses at any joint where more than two forces are unknown. In Fig. 9, we could not start with the joint C D or at D E; for we should know only the external force or load, and have three unknown stresses to find; therefore our quadrilateral, of which one side is known, might have the other sides of various lengths, but still parallel to the original pieces of the frame. When the joints were taken in the order observed this difficulty was not met with.

When, in some cases, we find three or more apparently unknown forces at a joint we may have some knowledge of the proportion which exists between one or more of them and a known force, and can thus determine the proper length of the line in the stress diagram. An example of such a case will be given in Fig. 11. In Chapter VIII. will be found a treatment

that is applicable to certain trusses which otherwise offer difficulties in solution.

20. Polonceau or Fink Truss.—Fig. 11 shows a truss which is often built in iron. The loads at the several joints of the rafters are found by the method prescribed in § 16. It will be unnecessary to dwell upon the manner of finding the stresses at the joints B, C D, and H K, for which the stresses will be ch , ha , ak , ki , hi and id . But when we attempt to analyze the joint D E, we find that, with the external load, we have six forces in equilibrium, of which those along E M, M L, and L K are unknown. If we try the joint L A we find four forces, three of which are at present unknown. We are therefore obliged to seek some other way of determining one of the stresses.

It will be seen, upon inspection, that the joint E F is like the joint C D; and it will appear reasonable that N M should have an equal stress with I H. We may then expect that there must be as much and the same kind of stress exerted by M L to keep the foot of the strut N M from moving laterally as is found necessary in K I to restrain the foot of I H.

Returning then to the joint D E, and beginning with ki , pass next over id , then de , then draw em , parallel to E M, to such a point m , that (having drawn ml until its extremity l comes in the middle of what will be the space between em and fn , or until ml equals in length ik), the line lk shall close on k whence we started. The ties and struts can be readily selected by the direction of movement over these lines in reference to the joint D E. The remaining joints when taken in the usual order of succession offer no difficulty, and the other half of the diagram need not be added, unless one desires a check on the results.

This truss will be treated again in § 74.

The polygon which we have just traced, $kide m l k$, affords a good illustration of the rule that the forces which meet at a joint make a closed polygon in the stress diagram. The symmetry of the triangles hik and mnl , and their resemblance to

klo, are worth noting, and will assist one in drawing diagrams for trusses of this type.

21. Cambering the Lower Tie.—Sometimes it is thought desirable to raise the tie *AO*, either to give more height below the truss or to improve its appearance. The effect on the stresses of such an alteration is very readily traced, and one then can judge how much change it is expedient to make. Let it be proposed to raise the portion *AO* of the tie to the position indicated by the dotted line, and thus to introduce such changes in the other members that they shall coincide with the other dotted lines in Fig. 11, while the load remains unchanged.

The line *ch* for joint *B* now becomes *ch'*, being prolonged until *h'a* can be drawn parallel to *HA* in its new position. Next come *h'i'* and *i'd*; then we easily draw *i'k'*, *k'l'*, *l'm'*, *m'n'*, etc. The struts *HI*, *KL*, and *MN* are the only pieces in this half of the truss unaffected by the change; the amount of increase, and the serious increase, of the other stresses for any considerable elevation of the lower member can be readily seen.

22. Load on all Joints.—If one prefers to consider that a portion of the weight of the truss, or that a floor, ceiling or other load is supported at the lower joints, the load may be distributed as in Fig. 12. Here the joints *QR* and *RS* carry their share of the weight of the pieces which touch these joints, as well as such other load as may properly be put there. Each supporting force, if the load is symmetrical, will still be one-half the total load, but the two will no longer divide the load line equally, nor can the load line be at once measured off as equal to the total weight.

Begin, if convenient, with the extremity *H* of the truss, and lay off *hi*, *ik*, *kl*, etc., downwards, ending with *op*. Passing on, around the truss, lay off next the reaction *pq* upwards, equal to one-half the *total* weight, then *qr* and *rs* downwards, and finally *sh* upwards, for the other supporting force, to close on *h*. The polygon of external forces, therefore, doubles back

on itself as it were, and hp is still the load on the exterior of the roof. The diagram can now be drawn, by taking three joints on the rafter in succession before trying the joint QR ; when taking that joint remember that there is a load upon it. The loads on the horizontal tie cause the stresses in its three parts to be drawn as three separate lines, instead of being superimposed as in the figures before given.

A diagram may now be drawn for Fig. 13. The upper part of the roof, dotted in the figure, throws its load, through the small rafters, on the upper joints of the truss.

23. Stresses by Calculation.—It is evident, from inspection of the preceding diagrams, that the stresses may be calculated by means of the known inclinations of the parts of the trusses. The degree of accuracy with which they can be scaled equals, however, if it does not exceed the approximation which designing and actual construction make to the theoretical structure.

24. Distribution of Load on the Joints.—In Unwin's "Iron Bridges and Roofs" the rafter is treated as a beam continuous over three or more supports, and the distribution of the load on the several joints is there determined by that hypothesis. That such an analysis may be true, it is necessary that all the points at which the rafter is supported shall remain in definite positions, usually a straight line. As slight deformations of the truss and unequal loading of the joints will prevent the realization of that assumption, a division of the load at any point of a rafter or other piece so that the joints at its two ends shall be loaded in the inverse ratio of the two segments into which the point divides the piece will best represent the case. Uniform loads will be distributed easily by § 16. A different distribution of the load, however, if one prefers it, will only require a corresponding division of the load line. (See Part II., Bridge Trusses, Chaps. VIII. and IX.)

CHAPTER III.

TRUSSES FOR FLAT ROOFS.

25. Trapezoidal Truss; Equal Loads.—A consideration of the trapezoidal, or queen-post, truss, represented by Fig. 14, will bring out two or three points which will be of use in the analysis of other trusses. In this case, let us suppose the load to be on the lower part, or bottom chord, of the truss. In order to separate the supporting forces from the small weights on the ends of the truss, and to permit them to come consecutively with the other weights in the load line, let us draw the supporting forces above the tie, instead of below as before. The rectangle formed by the two vertical and two horizontal pieces might become distorted; we will therefore introduce the brace HI , represented by the full line. The rectangle is thus divided into two triangles and movement prevented. The dotted line shows a piece which might have been introduced in place of the other.

If the truss is symmetrically loaded, or $CD = DE$, we shall get the first stress diagram. The stress in each vertical is here seen to be the load at its foot. The stress in the piece HI proves to be zero. If the load had been on the upper joints, no stress would have been found in the verticals also. (See § 17.) It is evident that a trapezoidal truss, when symmetrically loaded, requires no interior bracing. This fact might readily be seen if we considered the form assumed by a cord, suspended from two points on a level, and carrying two equal weights symmetrically placed.

26. Trapezoidal Truss; Unequal Loads.—The second stress diagram will be drawn when the weight CD is less than DE . Let us suppose that bc and ef are of the same

magnitude as in the first diagram, and let the span of the truss, or distance between supports, which we shall denote by l , be divided by the joints into three equal parts. The first step is to find the supporting forces. If each external force be multiplied by the perpendicular distance of its line of action from any one assumed point, which distance may be called its leverage, and all the products added together, those which tend to produce rotation about this point in one direction being called plus, and those tending the other way minus, it is necessary for equilibrium that the sum of these products shall be zero; otherwise the rotation can take place. A convenient point to which to measure the distances will be one of the points of support, for instance the right-hand one. Then we shall have

$$A F \cdot l - F E \cdot l - E D \cdot \frac{2}{3} l - D C \cdot \frac{1}{3} l - C B \cdot 0 + B A \cdot 0 = 0,$$

or

$$A F \cdot l = F E \cdot l + E D \cdot \frac{2}{3} l + D C \cdot \frac{1}{3} l;$$

therefore

$$A F = F E + \frac{2}{3} E D + \frac{1}{3} D C.$$

If $E D$ be taken as $3 D C$,

$$A F = F E + \frac{1}{3} E D.$$

It will be seen that the object in taking the point or axis at B is to eliminate $B A$, and have only one unknown quantity, $A F$. This method of determination is called *taking moments*, and is at once the simplest and most generally applicable. Lay off the above reaction at $f a$; $a b$ will be the reaction at the right support. One cause of a diagram's failure to *close*, when drawn by a beginner, is carelessness in placing the reactions on the load line in the wrong order.

The point a being now located, we can proceed to draw the second diagram. The construction requires no explanation; but we will call attention to the fact that a compressive stress here exists in $H I$. If, in place of the diagonal represented by the full line, the one shown by the dotted line is now supplied, the reader can without difficulty trace out for himself

the change in the diagram, which is denoted by the dotted lines and the letters marked by accents. The stress in this diagonal will be seen to be tensile. Changing the diagonal reverses its stress.

It is also worthy of notice that the only pieces affected by the substitution of one diagonal for the other are those which form the quadrilateral enclosing the diagonals. This fact will be of service later.

27. Use of Two Diagonals.—If, at another time, this excess of load might fall on CD in place of DE, the stress on either diagonal would be reversed: that is, if it sloped down to the right it would be a tie; if to the left, a strut. As a tension diagonal is likely to be a slender iron rod, which is of no practical value to resist a thrust, while the compression member, unless made fast at its extremities, will not transmit tension, a weight or force which may be shifted from one joint to another may require the designer to introduce two diagonals in the same rectangle or trapezium, or else to so proportion and fasten one diagonal as to withstand either kind of stress.

Where both diagonals occur the diagram can still be drawn. Determine which kind of stress, tension or compression, the two shall be designed to resist, and then, when drawing the diagram, upon arriving at a particular panel or quadrilateral, try to proceed as if only one of the diagonals existed. If a contrary kind of stress to the one desired is found to be needed, erase the lines for this panel only, and take the other diagonal. In the treatment for wind pressure, this method becomes serviceable, since the wind may blow on either side of the roof.

This truss can be used for a bridge of short span.

28. Trusses for Halls.—It is sometimes the case that, in covering a large building, it is desired to have the interior clear from columns or partitions, while a roof of very slight pitch is all that is needed. As it is not expedient to have a truss of much depth, since the space occupied by it is not generally available for other purposes, one of several types of

parallel-chord bridge trusses may be employed, for instance the "Warren Girder," of Fig. 15, which is an assemblage of isosceles triangles. In a public hall, galleries may be suspended from the roof, and the weight of a heavy panelled or otherwise ornamented ceiling may be added to what the truss is ordinarily expected to carry. The depth may be less than here drawn, but, for clearness of figure, we have not made the truss shallow.

If the roof pitches both ways from the middle of the span, the top chord may conform to the slope, making the truss deeper at the middle than at the ends; but a light frame may be placed above, as shown by the dotted lines, and supported at each joint of the top chord. The straight-chord truss is more easily framed. If the roof pitches slightly transversely to the trusses, it will be convenient to make them all of the same depth and put on some upper works to give the proper slope. The ends of the truss could readily be adapted to a mansard roof.

29. Warren Girder.—In Fig. 15, each top joint is supposed to be loaded with the weight of its share of roof, in which case the joint LM or PQ will have three-quarters of the weight on NO or OP, if the roof is carried out to the eaves as marked on the left; or practically the same as NO, if the roof follows the line IL. The bottom joints are supposed to carry the weight of the ceiling, and in addition the tension of a suspending rod to a gallery on each side. The load line will be equal to the weight on the upper part of the truss, and the polygon of external forces will overlap, as in Fig. 12, previously explained, § 22. We go from *k* to *r*, for the loads on the exterior in sequence, then up to *s* for the left-hand reaction, then down to *w* for the loads on the interior, and finally close on *k* with the right-hand reaction.

Upon drawing the diagram it will be seen that the stress is compression in the top chord and tension in the bottom chord; that the stresses in the chords increase from the supports to the middle; that the stresses in the braces decrease from the

ends of the truss to the middle, and that alternate ones are in compression and in tension, those which slant up from the abutment towards the centre being compressed, and those which incline in the other direction being in tension. The tie-braces are, therefore, A B, C D, F G, and H I. A decrease of depth in the truss will increase the stresses in the chords.

30. **Howe Truss; Determination of Diagonals.**—A truss with parallel chords may be employed, in which the braces are alternately vertical and inclined. The designer will choose whether the verticals shall be ties and the diagonals struts, in which case the type is called the "Howe Truss," Fig. 16, or the verticals struts and the diagonals ties, when it is known as the "Pratt Truss." There is an advantage in having the struts as short as possible, but, if one desires to use but little iron, the Howe is a good form.

To decide which diagonal of the rectangle shall be occupied by the piece:—Start from the wall as a fixed point; it is evident that, to keep the load C D from sinking, C Q must be a strut. If we wish to put a tie in this panel, it must lie in the other diagonal, shown by the dotted line. C D now being held in place, P O as a strut will uphold D E. We thus may work out from each wall until we have passed as much load as equals the amount supported, or the reaction, at that wall. If the last load passed exactly completes the amount required to equal the reaction, no diagonal will be required in the next panel. We might draw diagonals, one in each panel, sloping in either direction as we pleased, and then construct the stress diagram. If we found a stress in any diagonal opposite to the stress we desired, § 27, we could then erase that diagonal and substitute the other, erasing also so much of the diagram as referred to the pieces in that panel. Were the chords not parallel, this method might be necessary (see Fig. 20), but in the present case it is better to draw the load line first, find the dividing point a , Fig. 16, for the two reactions, see what load it cuts, and then incline the diagonals from each wall either up or down, as preferred, towards that loaded joint.

31. Howe Truss; Diagram.—In the present example CD is supposed to be four times DE, etc. A tower on that end of the truss or some suspended load will account for the difference. Recalling the manner in which the supporting forces were found when the load was unsymmetrical, § 26, use a panel as a unit of distance, call a panel length p and the ordinary weight on a joint w . Then we shall have, taking moments about H,

$$w \cdot p (1 + 2 + 3) + 4w \cdot 4p + \frac{1}{2}w \cdot 5p = R \cdot 5p, \text{ or } R = 4.9w,$$

the reaction at B, or ab . The two supporting forces will then be ha and ab . Draw the stress diagram as usual; the diagonals will all come in compression as intended, and the verticals will be ties. There will plainly be no stress in the dotted vertical ON. The stress in the chords is inversely proportional to the depth of the truss, and economy of material in the chords will be served by making the depth as much as possible, within reasonable limits. In bridge trusses this depth is seldom less than from one-sixth to one-eighth of the span.

32. Moving Load.—If the joint DE also might become heavily loaded, we could draw another diagram for that case, and, as the joints in succession had their loads increased, we might make as many diagrams. From a collection of diagrams for all positions of a moving load, we could select the maximum stress for each piece. A truss designed to resist such stresses would answer for a bridge. We should find that the greatest stresses in the chords occurred in all panels when the bridge was heavily loaded throughout, and that the greatest stress in a diagonal was found when the bridge was heavily loaded from this piece to one end only, that end generally being the more distant one. As we have more expeditious methods of analyzing a bridge truss, this one is not used. The graphical treatment of bridge trusses is found in Part II. of this work.

CHAPTER IV.

WIND PRESSURE ON PITCHED ROOFS.

33. **Action of Wind.**—The forces hitherto considered have been vertical; the wind pressure on a roof is inclined. It was once usual to deal with the pressure of the wind as a vertical load, added to the weight of the roof, snow, etc., and the stresses were obtained for the aggregate pressure. This treatment manifestly cannot be correct. The wind may be taken without error as blowing in a horizontal direction; it exerts its greatest pressure when blowing in a direction at right angles to the side of a building; it consequently acts upon but one side of the roof, loads the truss unsymmetrically, and sometimes causes stresses of an opposite kind, in parts of the frame, from those due to the steady load. Braces which are inactive under the latter weight may therefore be necessary to resist the force of the wind.

It will not be right to design the roof to sustain the whole force of the wind, considered as horizontal; nor will it be correct to decompose this horizontal force into two rectangular components, one perpendicular to the roof, and the other along its surface, and then take the perpendicular or normal component as the one to be considered; for the pressure of the wind arises from the impact of particles of air moving with a certain velocity, and these particles are not arrested, but only deviated from their former direction upon striking the roof. Yet the analysis applicable to a jet of water striking an inclined surface cannot be used here, for water escapes laterally against the air, a comparatively unresisting medium, while the wind particles, if we may so term them, deflected by the roof, are turned off against a stream of similar air, also in motion, which retards their lateral progress and thus causes

them to press more strongly against the roof. We are obliged, therefore, to have recourse to experiments for our data, and from them to deduce a formula.

34. Formula for Wind Pressure.—It appears that, for a given pressure exerted by a horizontal wind current on any square foot of a vertical plane, the pressure against a plane inclined to its direction is perpendicular to the inclined surface, and is greater than the normal component of the given horizontal pressure. Unwin quotes Hutton's experiments as showing that, if P equal the horizontal force of the wind on a square foot of a vertical plane, the perpendicular or normal pressure on a square foot of a roof surface inclined at an angle i to the horizon may be expressed by the empirical formula

$$P \sin i^{1.84} \cos i - 1.$$

If, then, the maximum force of the wind be taken as 40 pounds on the square foot, representing a velocity of from 80 to 90 miles per hour, the normal pressure per square foot on surfaces inclined at different angles to the horizon will be :

Angle of Roof.	Normal Pressure.	Angle of Roof.	Normal Pressure.
5°	5.2 lbs.	35°	30.1 lbs.
10	9.6	40	33.4
15	14.0	45	36.1
20	18.3	50	38.1
25	22.5	55	39.6
30	26.4	60	40.0

For steeper pitches the pressure may be taken as 40 pounds.

Any component in the plane of the roof, from the friction of the air as it passes up along the surface, or from pressure against the butts of the shingles or slates, is too slight to be of any consequence.

The above maximum is a sufficient amount to be provided for, although wind gauges have been known to register somewhat higher pressures at rare intervals.

35. Example: Steady Load.—The truss of Fig. 17 is supposed to be under the action of wind pressure from the

left. If the truss is 67 feet span, and the height is 15 feet, the angle of inclination will be $24^{\circ} 7'$, and the normal wind pressure, interpolated from the table, will be 21.8 pounds per square foot. The rafter will be 36.7 feet long. If the trusses are 10 feet apart, the normal wind pressure on one side will be

$$36.7 \times 10 \times 21.8 = 8000 \text{ lbs.}$$

For steady load of slates, boards, rafters, purlins, and truss, let us assume 11 pounds per square foot of roof, or

$$36.7 \times 10 \times 2 \times 11 = 8074 \text{ lbs., total vertical load.}$$

The truss is here drawn to a scale of 30 feet to an inch, and both diagrams are drawn to a scale of 6000 pounds to an inch. In actual practice these figures should be much larger, the diagrams showing perhaps 1000 pounds or 800 pounds to an inch.

We will, in the present case, treat the two kinds of external force separately. The diagram on the right for steady load needs no description. Each supporting force will be 4037 pounds, and the weights at the joints of the rafters will be, 673 pounds for the end ones, and 1346 pounds for each of the others. The above weights are laid off on a vertical load line and the diagram then drawn. The stresses in the various pieces for half of the truss are given in the table to follow, the sign + denoting compression, and the sign —, tension.

36. Wind Diagram; Reactions.—The normal pressure of 8000 pounds distributed uniformly over the whole of the left side of the roof, and on that alone, will have its resultant, shown by the dotted arrow, at the middle of that rafter. To find the supporting force on the right we may take moments about the left-hand wall, remembering to multiply each force by the lever arm drawn perpendicular to its direction: or

$$AP \times HT = 8000 \times HK,$$

or

$$AP \times 61.15 = 8000 \times 18.35;$$

whence $AP = 2400$ pounds, and $AH = 5600$ pounds.

But since these arms, HT and HK , are proportional to the span and the left part of the horizontal tie cut off by the resultant, an easier way to get the supporting pressures due to an inclined force is to prolong this force until it cuts the horizontal line joining the two abutments, when the two reactions will be inversely proportional to the two segments into which the horizontal line is thus divided, the larger force being on the side of the shorter segment, or, for ordinary pitches, on the side on which the wind blows.

The pressures on the joints will be 2667 pounds each on IK and KL , and 1333 pounds each on HI and LM , as denoted by the arrows. Draw mh by scale, equal to 8000 pounds, so inclined as to be in the direction of the given forces, that is, perpendicular to the roof; divide the reactions of the supports by means of the point a , and lay off the joint forces in their proper order, ml , lk , ki and ih . Before going further be sure that the external forces and the reactions follow one another in their proper order, down and up the load line; for, through heedlessness, the reactions are sometimes interchanged.

37. Wind Diagram; Stresses.—Proceed with the construction of the diagram by the usual rules, remembering that wind alone is being treated. After the joint KL has given $lkcdel$, the joint EA gives $edafe$. Taking next the apex LM , and passing along ml , le and ef , we find that there will be no line parallel to FG , since gm , parallel to GM , will exactly close on m , the point of beginning. As no stress passes through FG , the remainder of the bracing on this side can experience no stress, and therefore the compression gm affects the whole of the right-hand rafter while the tension af is found in the remainder of the horizontal tie. The stress triangle for the point P will therefore be $mgam$. That the above result is true will be seen if we notice that the piece QR , having no wind pressure at its upper end, can, by § 17, have no stress. Then it follows that RS is now free from stress, and next SG and lastly GF , all by § 17. Further:

imagine all of the braces in the right half to be removed ; it is evident that the right rafter is a sufficient support to the joint LM, conveying to the wall the stress gm which compresses its upper end, while the tie AF keeps the truss from spreading. If the lower tie or the rafter was not straight, some of the braces would come into action, as will be seen later.

38. **Remarks.**—At another time the wind may blow on the right side. Then the braces on the right will be strained as those on the left now are, and those on the left will be unstrained. The wind stresses are placed in the third column of the table. As in this truss they are all of the same kind, in the respective pieces, as those from the steady load, they are added to give the total or maximum stresses. The force gm , being smaller than, while it is of the same kind as le , is of no consequence ; for, with wind on the right, MG would have to resist a stress equal to le .

A combination of the two components of the supporting forces at each end, as shown in the figure, by either the parallelogram or triangle of force, will give the direction and amount of each reaction from the combined load. Wind on the other side will exactly reverse the amounts and bring them on the opposite side of the vertical line.

TABLE OF STRESSES FOR FIG. 17.

Piece.	Steady Load.	Wind.	Total.
Tie {	AB — 7520 lbs.	10,440 lbs.	17,960 lbs.
	AD — 6020	7,160	13,180
	AF — 4520	3,900	8,420
Braces {	EF — 1830	3,990	5,820
	CD — 1500	3,280	4,780
	BC + 1230	2,670	3,900
	DE + 1840	4,000	5,840
Rafter {	IB + 8240	9,530	17,770
	KC + 7690	9,530	17,220
	LE + 5760	6,550	12,310

If the truss is simply placed upon the wall-plates, and either of the supporting forces makes a greater angle with the

vertical than the angle of repose between the two surfaces, the truss should be bolted down to the wall ; otherwise there will be a tendency to slide, diminishing the tension in the tie, perhaps causing compression in that member, and changing the action of other parts of the truss. This matter will be treated of further.

If the weight of snow is also to be provided for, it may readily be done by taking the proper fraction of the stresses from the steady load and adding them to the above table.

39. **Truss with Roller Bearing; Dimensions and Load.**

—We propose, in the example illustrated by Fig. 18, to consider the truss as supported on a rocker or rollers at the end T, where the small circle is drawn, to allow for the expansion and contraction of an iron frame from changes of temperature. It is therefore plain that the reaction at T must always be practically vertical. The truss is supposed to be 79 feet 8 inches in span, and 23 feet in height, which gives an angle of 30° with the horizon, and makes the length of rafter 46 feet. It would be proper usually to support the rafter at more numerous points; but our diagram would not then be so clear, with its small scale, from multiplicity of lines, and one can readily extend the method to a truss of more pieces.

This frame supports 8 feet of roof, and the steady load per square foot of roof is taken, including everything, as 14 pounds. The total vertical load will then be

$$14 \times 46 \times 2 \times 8 = 10,304 \text{ lbs.,}$$

or 1717 lbs. on each joint except the extreme ones.

We find, from the table of § 34, that the normal pressure of the wind, for a horizontal force of 40 pounds on the square foot, may be taken as 26.4 pounds per square foot of a roof surface inclined at an angle of 30°. The total wind pressure, normal to the roof, will therefore be

$$26.4 \times 46 \times 8 = 9715 \text{ lbs.,}$$

or 3238 lbs. and 1619 lbs. on the middle and end joints

respectively of one rafter. The truss is drawn to a scale of 40 feet to an inch, and the diagrams to that of 8000 pounds to an inch.

40. Diagram for Steady Load.—The diagram for steady load, having a vertical load line, is the one above the truss, and a little more than one-half is shown. The only piece at all troublesome is GF . On arriving in our analysis at the apex of the roof, or at the middle joint of the lower member, we find three pieces whose stresses are undetermined: but as we have reached the middle of the truss, we know that the diagram will be symmetrical, and therefore that gf will be bisected by al . In the case of an unsymmetrical load we can recommence at the other point of support and close on the apex. The stresses caused by this load are given in the first column of figures in the table in § 44, compression being marked +, and tension —. If it is thought necessary to provide for snow, in addition to the stresses yet to be found for wind, make another column in the table, of amounts properly proportioned to those just found.

41. Wind on the Left; Reactions.—Upon turning our attention to the other diagrams, we shall find that the rollers at T cause something more than a reversal of diagram,—often a considerable variation of stress, when the wind is on different sides of the roof. Taking the wind as blowing from the left, we draw the diagram marked $W. L.$ The line qm , 9715 lbs., § 39, is divided and lettered as shown for the four loads at the joints where arrows are drawn. The resultant of the wind pressure, at the middle point of the rafter, when prolonged by the dotted arrow, will divide the horizontal line or span in the proportion in which the load line should be divided to give the two parallel reactions, if there were no rollers at T . This proportion, for a pitch of 30° , is 2 to 1; it locates the point a' , and gives $ma' = 6477$ lbs., and $a'g = 3238$ lbs.

But the reaction at T must be vertical, and consequently only the vertical component of $a'g$ can be found at T , while

the horizontal component of $a'q$ must come, through the lower member, from the resistance of the other wall. Therefore draw $a'a$ horizontally and we shall get aq as the vertical reaction at T, while ma , to close this triangle of external forces, must give the direction and amount of the reaction at M.

42. Verification.—It may, at first sight, strike the reader that this analysis will not be correct; for, if only the vertical component is resisted at T, and if we decompose the resultant of the wind pressure at O, where it strikes the roof, into two components, we get results as follows:

$$\begin{array}{l} \text{Vert. comp. of 9715 lbs., for angle } 30^\circ = 8414 \text{ lbs.} \\ \text{Hor. " " " " " " = 4858 lbs.} \end{array}$$

The vertical from the middle point of the rafter will divide the span at $\frac{1}{4}$ M T. Therefore, amount of vertical component carried at T = 2103 lbs., and the remainder is supported at M, with all of the horizontal component. But take next into account the moment, or the tendency of the horizontal component at O to cause the truss to overturn. It naturally decreases the pressure at M and increases that at T, or, in other words, the couple formed by the horizontal component at O and the equal horizontal reaction at M with an arm of half the height of the truss must be balanced by an opposite couple, composed of a tension at M and an equal compression at T, with a leverage of the span. Making the computation of this tension, or compression T, we have

$$\begin{array}{l} 4858 \times 11.5 = T \times 79\frac{1}{2}, \text{ or } T = 702 \text{ lbs.} \\ 2103 + 702 = 2805 = \frac{1}{4} \text{ of } 8414 \text{ lbs.} \end{array}$$

as obtained by the first process.

Still another way to find the supporting forces is to prolong the resultant until it intersects the vertical through T, then to draw a line from M to the point of intersection, and finally to draw ma and qa parallel to the lines from M and T. This method depends for its truth on the fact that the three external

forces which keep the truss in equilibrium, not being parallel must meet in one point.

43. Diagram for Wind on Left.—Having completed the triangle of external forces, and laid off the pressures on the joints, we can readily draw the diagram. It will be found, as in Fig. 17, § 37, that braces on the right experience no stress, the lines gf and eg closing the polygon which relates to the joint P Q. If the lower tie were cambered to the joint D C, we should find a stress from wind in E F and C D, but not in B C or C E, as explained in § 37.

Upon combining with the inclined reaction ma the steady load reaction also marked ma , the direction of the resultant supporting force at M will be found; and it may be so much inclined to the vertical that provision against sliding on the wall-plate at M should be made. The stresses given by this diagram for wind on the left are found in the table to follow, in the column marked W. L. It will be seen that all of them agree in *kind* with those for steady load.

44. Diagram for Wind on Right.—This diagram is marked W. R. The supporting force at T, while still vertical,

TABLE OF STRESSES FOR FIG. 18.

Piece.	Steady Load.	W. L.	W. R.
Rafters {	BS	+ 8570 lbs.	5600 lbs.
	CR	+ 6850	5600
	EQ	+ 5700	5600
	IP	+ 5700	5880
	KO	+ 6850	5600
	LN	+ 8570	5600
Tie {	LA	— 7440	11400
	HA	— 5450	7050
	DA	— 5450	4850
	BA	— 7440	4850
Braces {	BC	+ 1720	0
	CE	+ 1520	0
	EF	— 1000	0
	FG	— 2300	2500
	GI	— 1000	2150
	IK	+ 1520	3300
	KL	+ 1720	3800
			0

is greater in amount than before. If diagram W. L. has been already constructed, the reaction at T can be taken as that portion of the vertical component of the wind pressure not included in aq of that figure; that is, $aq + ta =$ vertical component of qm or pt . If this should be the first diagram drawn, find the supporting forces in one of the three ways given above. The reaction at M is rightly denoted by ap , for, when the wind is on the right, there is no external force to divide the space from M to P.

The point a is moved considerably from its place in diagram W. L., and this change affects the amounts of stress in the horizontal member, but not in those pieces which bear similar relations to the two sides of the truss; in other words, IP and EQ interchange stresses, etc. In some forms of truss, however, we find more material changes. In the present example it happens that the vertical fg strikes the point a , so that ip , the stress in the rafter, coincides with ap , the reaction at M; the wind on the right consequently causes no stress in LA and HA. The stresses from this diagram are found in the last column of the table.

45. Remarks.—There is no need to tabulate the stress in KH, if that in IG is given, nor gh , if ki is given. Notice that the joint KG or CF gives a parallelogram in each diagram, the stress in KI passing to GH without change, so that the diagonals which cross may be considered and built as independent pieces. It will be seen on inspection of the table that the combination of steady load with wind on the left gives maximum stresses in IP, KO, LN, LA, HA, DA, GI, IK, and KL, while the remainder, with the exception of FG, have maximum stresses for wind on the right. FG is strained alike in both cases.

These wind diagrams may be drawn on either side of the line of wind force, as in the case of steady load, by changing the order in which the supporting forces are taken, going round the truss and joints in the opposite direction. Although there exist two four-sided spaces C and K, the

structure is sufficiently braced against distortion; for these spaces are surrounded by triangles on all sides but one.

It may perhaps not be amiss to suggest again how to determine the kind of stress in any member without retracing the whole polygon for any joint. Notice, from the load line, whether the forces were taken in right-hand or left-hand rotation. Read the letters of a piece in that order with reference to the joint at one end of it; then read the stress in the diagram in that same order, and it will show the direction of the stress in the piece, either to or from that joint. Thus diagram W. L. is written in left-hand rotation; K L is then the reading for that brace at its *lower* end, and *kl* reads downward or is thrust. If we read L K, it must apply to its upper end, and *lk* acts upwards or thrusts against the joint near N.

Wind diagrams for the truss of Fig. 21 can now be drawn. The apex of the roof can be treated first, and the stresses, obtained in the dotted lines, can then be transferred to the ends of the upper horizontal member. The truss proper goes no higher.

CHAPTER V.

WIND PRESSURE ON CURB (OR MANSARD) AND CURVED ROOFS.

46. Truss for Curb Roof; Steady Load Diagram.—To have a definite problem we will assume that the truss of Fig. 19, drawn to scale of 20 feet to an inch, is 50 feet in span, that the height to ridge is 20 feet, to hips $14\frac{1}{2}$ feet, and that CD is 14 feet. The sides KB and GE are practically $16\frac{3}{4}$ feet long, at an angle of 60° with the horizon, so that their horizontal projection is $8\frac{1}{2}$ feet. The upper rafters are $17\frac{1}{2}$ feet long, and therefore make an angle with the horizon of $18^\circ 19'$. The trusses are assumed to be 8 feet apart, and are loaded at the joints only. The rafters in a larger truss would commonly be supported at intermediate points; but more lines would make our diagrams less plain.

The steady load is taken at 12 pounds per square foot of roof surface, or

$$(2 \times 16\frac{3}{4} + 2 \times 17\frac{1}{2}) 12 \times 8 = 6560 \text{ lbs., total load.}$$

The joint L will carry one-half the load on KB, or 800 pounds; the joint IK will carry one-half the load on KB and one-half of that on IC, or $800 + 840 = 1640$ pounds; IH = $840 + 840 = 1680$ pounds, etc. These weights are laid off, in the diagram marked S. L., from *l* to *f* by a scale of 4000 pounds to an inch, and the diagram is drawn. It shows that the rafters are in compression, marked +, and all the braces in tension, marked —.

47. Snow Diagram.—In treating this truss for snow load, it is considered that KB and EG are too steep for any weight of snow to accumulate there, as whatever fell on them would

soon slide off. Therefore a weight of 12 pounds per *horizontal* square foot, for the upper rafters only, is taken for the maximum snow load, and, as the horizontal projection of IC + DH is $33\frac{1}{2}$ feet, that load will be

$$12 \times 33\frac{1}{2} \times 8 = 3200 \text{ lbs.},$$

laid off from *k* to *g*, in the diagram marked S. The end portions, *ki* and *hg*, are each 800 pounds, and *ih* is 1600 pounds. The division into two equal reactions at the points of support gives *a*. This diagram much resembles the other, but there is one point worth noticing; the lines of stress, *ic* and *hd*, cross in the first diagram, but do not in the second; while the reverse is the case with *ed* and *bc*. The result is that the stress of CD is reversed by the maximum snow load, and, as this stress is greater in amount than the one for the weight of roof and truss, CD will be a compression member whenever such a load of snow falls on the roof; and will be in tension when that load is removed. The stresses from these two diagrams are marked on the truss above each piece on its left with the usual signs. This strain sheet is more convenient than the table of § 44.

48. Wind from the Left; No Roller.—When the rafters do not slope directly from the ridge to the eaves, but are broken into two or more planes of descent, we shall have wind pressures of different directions and intensities on the two portions, IC and KB. From the table of wind pressures, § 34, we see that the intensity of pressure on KB will be 40 pounds, and on IC 16.9 pounds, normally, per square foot of roof. The total pressure on KB therefore will be $40 \times 16\frac{1}{2} \times 8 = 5333$ pounds, of which one-half will be supported at the joint L, and the other half at the joint J, as indicated by the two arrows perpendicular to KB. The pressure on IC will be $16.9 \times 17\frac{1}{2} \times 8 = 2366$ pounds, or 1183 pounds on each joint.

If the truss has no rollers under it, the diagram marked W. L., I. is obtained. On a scale of 4000 pounds to an inch,

$hi = ij = 1183$ pounds; $jk = kl = 2667$ pounds. For ij and jk may be substituted ik , if desired, the resultant of these two components at J.

To find the supporting forces:—Prolong the resultants of the wind pressure from the middle point of each rafter to intersect the span L F. The resultant K will be resisted at L and F by two reactions parallel to it, and *inversely* proportional to the two segments into which this resultant divides L F, as shown in § 36. The same will be true for the resultant I. By scale, or from the known angles, it will be found that resultant K cuts L F at $16\frac{2}{3}$ feet, or one-third the span, from L, and that resultant I cuts it at 22.4 feet from the same end. Dividing jl at $\frac{1}{3}$ its length, we have la' for one component of the reaction at L and $a'j$ for one component of the reaction at F. If we divide hj at $\frac{22.4}{50}$ of its length, ja'' will be a component of the supporting force at L, and $a''h$ at F. By drawing the parallelogram $a'ja''a$ we shall bring the component reactions for each wall together, and shall have, for the supporting force at L, or L A, la' and $a'a$, or their resultant la ; and for that at F, aa'' and $a''h$, which combined give ah , properly called A H in the truss, since the letters from F to H are not in use at present. Take care to lay off the component reactions on the proper ends of the wind-pressure lines.

The polygon of external forces, when there is no roller under the truss, is therefore hi, ik, kl, la , and ah . The completion of the diagram, by drawing lines parallel to the several pieces, will be easy without further explanation. That the point e should apparently fall on ik is accidental. The signs affixed to the lines will enable one to see readily that the stresses in B C and E A are now reversed, the pressure I K obliging us to use a strut to keep that joint in place. The resultant, however, from the combined stresses in E A is still tension. The amounts given by diagram W. L, I. have not been placed on the truss, as we prefer to treat it from another

point of view. Had they been used, it would be unnecessary to draw a diagram for wind on the right, for the different members of the truss would exchange stresses symmetrically; that is, AB would have the stress of EA, and EA that of AB; DH of CI, etc., CD remaining the same.

49. Wind from the Left; Roller at Left.—If rollers are placed at L, to permit of movement resulting from change of temperature, the supporting forces will be modified, LA becoming vertical. The diagram marked W. L., II. shows the effect of this change. So far as drawing the lines of wind pressure $hijkl$, the polygon of external forces will be obtained in the same manner as before. We may then draw the parallelogram and locate the point here marked a' ; then draw $a'a$ horizontally, and we shall get la , the vertical reaction at L, equal to the vertical component of la of the figure just preceding.

In case the former diagram has not been drawn, a readier way to determine la will be as follows:—Draw hl , plainly the resultant of hj and jl ; then, having prolonged the dotted arrows at I and K until they meet, draw a line, parallel to hl , through their intersection. This line will give the position of the resultant of the wind pressures, and lh is now to be divided in the inverse ratio of the two segments into which the resultant divides the span LF. The point of division will fall at a'' , from which draw horizontally $a''a$, and the reaction la is thus determined. This method will not answer for finding the supporting forces if they are both inclined, as it will make LA and AH parallel to one another. The reaction at L being la , the one at F is ah , requiring the resistance at F of the entire horizontal component of the wind pressure.

A comparison of the two W. L. diagrams will show that the stress in every piece is changed very decidedly in amount, and that in a number of pieces the stresses are reversed by rollers at L. These latter stresses are marked on the truss, at the right of each piece.

50. **Wind from the Right.**—When the wind blows from the right, the diagram marked W. R. will be obtained. The lines $ihgf$, representing the wind pressures, will correspond in value with $hikl$ of the preceding figure, and, since the other diagram has been constructed, the vertical reaction at L will now be obtained by drawing the horizontal line $a'a$, from either the angle of the parallelogram or the proper point of division of the resultant if , so as to give ai , the smaller part of the vertical component of the wind pressure; that, is la from W. L., II., plus ai from W. R., equals the vertical projection of the polygon of external forces.

51. **Results.**—When this diagram is completed by the customary rules, a comparison of it with the one preceding will make clear the effect of wind on different sides. The stress in the rafters is much greater when the wind blows on the side farther from the rollers, but it is always compressive. The forces in the braces are all reversed.

The weight of the roof and truss may be the only external force, or snow may be added; and, in either case, the wind may also blow on one side or the other. Selecting then those stresses which may exist together, we find the maximum tension and compression marked below each piece. The rafters are always compressed, and A B is always in tension. The other pieces must be designed to resist both kinds of stress, although the compression in D E is quite insignificant.

✓ 52. **Curved Roof Truss: Example.**—If the truss has a curved exterior outline, the pressure of the wind will make a different angle with the horizon for every point. But there will be no sensible error if the pressure on each piece is assumed to be normal to the curve at its middle point, or, what is practically the same thing, perpendicular to the straight line joining its two extremities. Thus, in the truss of Fig. 20, the wind pressure on C T is taken as perpendicular to a straight line from B to the next joint in the rafter.

The span of this truss, drawn on a scale of 30 feet to an inch, is 60 feet; height at middle of rafters 15 feet, at middle

of main tie 6 feet. The curves are arcs of circles, the radii of the upper and lower members being respectively $37\frac{1}{2}$ feet and 78 feet. The rafters are spaced off at intervals of $11\frac{1}{2}$ feet each way from the middle, and the tie is divided into $10\frac{1}{4}$ feet lengths. The end portions will differ slightly from these measures. The trusses are to be 10 feet apart. From the data, radius $37\frac{1}{2}$ feet, and half-chord or sine $5\frac{1}{4}$ feet, it is easy to calculate that the chord of the first piece of rafter from the middle will make an angle with the horizon of $8^{\circ} 49\frac{1}{4}'$. The second piece will be inclined three times as much, or $26^{\circ} 28'$, and the last five times as much, or $44^{\circ} 6'$. The intensity of normal wind pressure will then be, when interpolated in the table, § 34, 8.6 pounds per square foot for the upper length, 23.7 pounds for the next length, and 35.6 pounds for the lowest piece. Multiplying these intensities by $11\frac{1}{2} \times 10$, we get 989 pounds, 2725 pounds, and 4094 pounds, respectively, represented by the small arrows, as if concentrated at the middle points of E, D, and C. The steady load is taken at a small figure, 2300 pounds per piece of rafter, to allow the disturbing effect of the wind to be more marked.

The diagonals in this truss are light iron rods, not adapted to resist compression, and therefore, if a compressive stress would occur in a particular diagonal, in case it were alone in a panel, we substitute the other diagonal, which will then be in tension. In lettering the figure, that *tie* which is required for a particular distribution of load is supposed to be present, and the other diagonal is not taken account of. Thus, in the panel through which the dotted arrow is drawn, if the brace which goes from the top of O P to the bottom of Q R is under stress, it will be called P Q, while the rafter will be Q E and the bottom tie P A. If the other diagonal is strained, the rafter will be called P E and the main tie Q A.

53. Steady-Load Diagram.—The diagram for weight of roof and truss is drawn on a scale of 8000 pounds to an inch. The vertical load line is *ib*, and the polygon for the point of support B is *cbatc*. On passing to the next joint in the top

or bottom member we find three pieces whose stresses are unknown. Both diagonals RS cannot be in action as ties at once; therefore suppress one, for instance that which runs to the upper end of ST. We then shall have only two unknown stresses at the upper joint, and can draw ts' and $s'd$. The lower joint will then give $s't$, ta , ar' , and $r's'$. But $r's'$ will be a compressive stress, as we read from r' to s' , and this diagonal is not the desired one. Taking the other, and trying the lower joint first, we have $tast$, and the upper joint then gives $dctsr d$, where sr is tension. Notice that change of diagonal affects the stresses in no pieces beyond those which bound the quadrilateral or panel in which the diagonal is changed. Analogy will rightly lead us to take the other diagonals which slope the same way, that is, down towards the middle. It is therefore easy, after the first attempt, to decide which diagonal to reject and which to retain.

54. Remarks.—If dr had been slightly more inclined, so as to strike s , no diagonal RS would have been required for this distribution of load. It will be seen that the stresses, all tensile, in the bracing are very small as compared with those in the main members, a fact due to the approximation of the rafter outline to the equilibrium curve or polygon for a load distributed as in this case. See § 88. If the outline of a truss coincides with the equilibrium polygon pertaining to a certain distribution of load, no interior bracing will theoretically be needed for such distribution; but if the distribution or direction of the external forces is at any time changed, bracing will be called into action. Further discussion of this subject comes in Parts II. and III.

The length of hk , etc., as compared with HK , etc., shows the necessity of drawing the truss skeleton on a large scale, to secure parallelism of the respective lines in each figure. As a slight change in the inclinations of the rafter and lower tie lines will change the magnitude of the stresses in those pieces quite materially, we are warned by the appearance of the diagram to provide, by an increase in size of these pieces,

against such a change in the truss as would be caused by slight errors in construction or by deflection under the load. Stress diagrams are particularly serviceable in this way.

55. Wind and Steady Load.—We might analyze the effect of the wind separately upon the truss, but, as there is a likelihood that the wind will reverse the stress in some of the diagonals which experience tension from the steady load, and that we shall be obliged, therefore, to substitute the other diagonals in such panels, it seems better to draw the diagram for the wind and the weight of the roof in conjunction. Therefore the two diagrams marked W. R. and W. L. are drawn for the maximum force of wind on either side, combined with the weight of the roof, etc. The external load line bi of one case is the exact reverse of ib of the other. An explanation of the construction of W. R. will suffice for both.

When the wind blows from the right, there is only the steady load on the left half of the truss. Beginning therefore with the joint at I, lay off vertically $hi = 1150$ pounds, or one-half the load on HK; next $gh = 2300$ pounds, load at GH, and so on to FE, as in the steady-load diagram already discussed. At FE we find, in addition to 2300 pounds vertical pressure, an inclined force perpendicular to the tangent at E, or to the chord of the piece, and equal to one-half of 989 pounds, the wind pressure before computed for E. We thus get the inclined line as far as e in the diagram. The joint DE gives de , manifestly made up of the other half of 989 pounds, of the vertical 2300 pounds as usual, and finally of one-half of 2725 pounds from the next length of rafter, and perpendicular to it. The forces for the remaining joints CD and BC will be plotted in the same manner, and we therefore see that, commencing at B, as is proper for this load line, we lay off the vertical and inclined forces in regular succession from one side of the truss to the other. If one draws a straight line from c to d , it will be the resultant of the combined external forces at CD.

56. Reactions and Diagrams.—Connect b with i by the dotted line, which will be the resultant of all these forces. As the resultant of the dead weight, symmetrically distributed, acts in the line of the vertical OP , and hence through the centre of curvature of the rafters, and as the wind pressures all point to the same centre of the circle, the resultant, parallel to bi , must pass through the same point. Therefore draw the dotted arrow through the centre from which the rafter was struck, and parallel to bi . This arrow cuts the span BI , by measurement, at $25\frac{1}{4}$ feet from B , or $34\frac{1}{4}$ feet from I . The resultant bi scales 20,620 pounds. If the supporting force at B were parallel to this resultant, it would be found by taking moments about I , when we should have

$$B \times 60 = 20,620 \times 34\frac{1}{4}; \quad \text{or} \quad B = 11,942 \text{ lbs.}$$

Lay off this force from b to a' . If rollers are placed at B , that reaction will be vertical, and the horizontal component of $a'b$ must be resisted at I . Let fall ba vertically, determining the point a by drawing $a'a$ horizontally, and connect i with a . The two supporting forces will be ia and ab .

In the $W. L.$ diagram the point a' comes nearer to b than to i ,—that is, the quantity just obtained now applies to the point of support I ,—and a falls very near to, but just outside of f , in the prolongation of the vertical line.

If there are no rollers under the truss, find the supporting forces for each oblique pressure separately, as in § 48. The same course must be pursued when the curve of the rafters is not circular, as the forces will not then meet at a common centre. Having thus completed, in either case, the polygon of external forces, the remainder of the construction will be made as in any example. After the first trial to ascertain the proper diagonal, it appears that, in each case, the diagonals all slant one way; so that, for wind on one side, one set of diagonals is in tension, and for wind on the other, all of the other set are strained.

57. Change of Diagonal.—The effect on the five pieces of a panel, top, bottom, two sides and the diagonal, of drawing the diagram so as to give compression in a diagonal, is shown anew in the W. L. figure for the panel P Q. Instead of op and qr , we get op' and $q'r$, considerably increased in amount but the same in kind; for ep and aq are substituted eq' and ap' , unchanged in kind, but having practically what is taken from one added to the other; while the diagonal stress is, as we said, reversed, but very nearly the same in amount.

It might be practicable to deduce some rule for determining beforehand the diagonal which would have the desired kind of stress, but the tentative process seems easy. We find it convenient to draw the lines parallel to the rafter and main tie first, as ep and ap' , then to sketch roughly two lines for the suspending piece and diagonal, see whether that diagonal comes in tension, and finally draw the right ones carefully.

58. Resultant Stresses.—It is not necessary to put the signs + and — on these lines, for it may be seen that all the rafter is compressed, the whole lower member extended, and all of the diagonals are in tension, as well as all the suspending pieces except O P and Q R, which are compressed a trifle when the maximum wind comes from the right. Such pieces are easily selected, if one notices that op and qr in the W. R. diagram are drawn in a direction opposite to the prevailing one.

The stresses are given in the following table. The lengths of rafter are denoted by a single letter. The pieces of the main tie, having the letter A in common, have also the letters which stand before the stresses in the proper columns. The inclination of the diagonal is shown by the sign prefixed to the stress. The effect of the wind on the roller side is to materially reduce the stress in a large portion of the main tie. The light bracing required is a marked feature of this type of truss, and the predominance of tensile members favors the use of iron bars. The two compressions, marked +, are too insignificant to require an increase of section.

TABLE OF STRESSES FOR FIG. 20.

	S. L.	W. R.	W. L.	Max.
				+
Rafters.....	C	12,600	18,900	18,200
	D	11,400	17,500	15,600
	E	10,800	15,000	16,200
	F	10,800	18,300	17,900
	G	11,400	12,700	20,100
	H	12,600	13,100	21,800
				-
Main Tie.....A	K	9,600	K 5,500	K 19,500
	L	9,500	L 5,500	M 18,900
	N	10,400	N 7,200	O 16,000
	Q	10,400	P 9,000	Q 14,200
	S	9,500	R 10,900	S 12,300
	T	9,600	T 12,800	T 12,300
				-
Diagonals...	LM	\ 900	\ 1,800	/ 1,800
	NO	" 400	" 2,100	" 2,400
	PQ	/ 400	" 2,400	" 2,200
	RS	" 900	" 2,200	" 2,100
				-
Suspenders.	KL	1,200	700	1,200
	MN	1,000	200	900
	OP	900	+ 100	700
	QR	1,000	+ 50	1,000
	ST	1,200	400	1,600

If the designer proposes to proportion the pieces with regard to minimum as well as maximum stresses, he can readily select the former from the table.

If a fall of snow is supposed to be uniformly distributed over the roof, the increased action of the several pieces can be easily obtained by proportion from column S. L. But, if it is thought that the inclination of the portions near C and H is too great to permit of snow accumulating there, a diagram for snow should be drawn. The horizontal projection of a piece of the rafter is properly taken when reckoning a snow load.

We think the reader will have no difficulty in drawing diagrams for a truss of similar outline, but with only a system of simple triangular bracing.

CHAPTER VI.

TRUSSES WITH HORIZONTAL THRUST.

59. Scissor Truss.—When it is desired to strengthen the rafters in a roof of moderate span by supporting them at their middle points, a simple means, often employed, is to spike on a piece from the lower end of one rafter to the middle of the other, as shown in Fig. 22. The two pieces may or may not be fastened together where they cross. At the first glance we should say that, to draw the diagram, we must lay off the load line ke , divide it as usual, and then, beginning at the joint E, draw $a'b'$ and $b'f$, parallel to AB and BF. Next, for the joint FG, we should get the lines $b'c'$ and $c'g$. For the apex we should have three lines, viz., hg , gc' , and a line from c' parallel to CH to strike h . There is evidently something wrong here. If we start from the other point of support K, we obtain the remainder of the diagram in dotted lines, and find that we have two points marked c' , some distance apart, which ought to come together; we also have two conspiring forces, gc' and hc' , whose vertical components ought to balance hg .

Abandoning this diagram for the present, let us start at the apex of the roof, where we may feel sure that there are but two unknown forces. Taking the load hg at that point, draw the full lines gc and ch . Next for the joint GF, starting with cg , pass down gf and draw fb and bc . The joint HI will similarly give the figure $ihcdi$. Lastly, the joint AC will add ba and ad to the stresses dc and cb . To close the polygon for the joint E we must now supply to $abfe$ the line ea , which must be the inclined reaction at E, required to keep this truss

from sliding outwards on the wall-plates, on the supposition that the points of meeting of two or more pieces are true joints (ones about which the parts are free to turn). As ea may be decomposed into ea' and $a'a$, the force $a'a$ is called the horizontal thrust of the truss, which may be resisted by the wall or by a tie-rod from E to K. The pieces of this truss are all in compression.

60. Horizontal Thrust or an Additional Member Necessary.—That the truss is not in equilibrium without this inclined or horizontal reaction at the walls is seen, if we suppose that E and K are not prevented from sliding laterally; the joint AC will drop, the joints FG and HI will approach one another, and the angle at the apex will become sharper. This change will take place unless the above or some other restraining force is applied. The trouble arises from the four-sided space C, which is here free to change its form. A member added in either diagonal of this space will cure the evil. One from the apex to the joint CA will plainly act as a tie, and will be found to supply the missing line $c'c'$ in the dotted diagram first drawn. From this diagram we see that the stresses in most of the pieces will then be greater than when the resistance comes from the wall. A strut between the joints FG and HI will also make the truss secure; the reader can try such a diagram, and see what pieces have their stresses reversed by the change. Either of the above modifications puts the truss into the class having vertical reactions.

61. Remarks.—As these trusses are usually made, reliance against change of form, where little or no horizontal thrust is supplied by the walls, is placed upon the stiffness of the rafters, which are of one piece from ridge to eaves, and on that of the two braces; but a failure to get a good horizontal resistance from the walls has sometimes resulted in an unsightly sagging or springing of rafters and braces. The bending moments on these pieces are due to the horizontal

thrust. Bending moments on a rafter or other piece will be considered later.

It is worthy of notice that cd equals ba , or that the thrust is constant throughout the brace. Two members crossing as at A must naturally give a parallelogram in the stress diagram; the component of the load at H I which starts down the brace will pass to E without being affected by crossing the other brace; yet, to resist the tendency to sag spoken of above, and for the reason that the braces are better able to resist thrust by mutually staying one another, it is advisable to spike them together at their intersection.

62. Hammer-beam Truss; Curved Members.—Another example where the horizontal thrust of the truss against the walls must be ascertained is shown in Fig. 23. This frame is called a hammer-beam truss, and is a handsome type often employed, in this country and abroad, for the support of church roofs, the bracing being visible from below, and the spaces containing more or less ornamental work. When the church has a clear-story, the windows come between the trusses at B, the truss is supported on columns, and the roof of the side aisle takes up the horizontal thrust. If there are no side roofs, the main walls are properly strengthened by buttresses.

It will be well to note in advance that a curved piece in a truss, so far as the transmission of the force from one joint to another is concerned, acts as if it lay in the straight line between the two joints. The curved members in the present example are the quadrants of a circle. They may have any other desired curve, depending somewhat upon the pitch of the roof. If, now, we consider the point of support B P of the truss, and remember that the curved brace A O transmits the force between its two extremities as if it were straight, it will be evident that the thrust of the inclined piece, if any thrust exists in it, must have a horizontal component which cannot be neutralized by a vertical supporting force alone. Therefore, in addition to the reaction of half the weight of the roof

and truss, there must be supplied by the wall, assisted perhaps by a buttress or a side roof, a certain horizontal thrust.

63. Amount of the Horizontal Thrust.—To determine the value of this thrust:—Let W equal the weight of truss and load. We have nine loaded joints, and there is, therefore, $\frac{1}{8}W$ at each joint except the two extreme ones. The portion 213 may be considered a small truss, like Fig. 7, superimposed on the lower or main truss 462375, and thus bringing additional loads on the points 2 and 3. If then we regard the main truss as a trapezoidal truss, and consider that the pieces LA and QA are unnecessary because the load is the same on the two halves of the frame, the trapezoidal truss will be 4235, the brace 4-2 being made up of an assemblage of pieces. LA and QA will be required when wind acts upon the roof. Considering the trapezoidal truss 4235 alone, the joint 2 will carry a load equal to that on DM , EK , and FI , or $\frac{3}{8}W$, the joint 3 will carry the same amount, while 4 will support $\frac{1}{8}W$ from CN , and 5 the remainder. If then we lay off on a vertical line $\frac{3}{8}W$, for the load on 2, and draw lines parallel to 2-4 and 2-3 from its extremities, the line parallel to 2-3 will be the stress in the same, and will also, since the load is vertical, be the horizontal thrust of the foot of the compound brace 2-4. This force is marked H in the dotted triangle drawn below the truss. A reference to § 25, Fig. 14, may aid one in understanding the above.

64. Stress Diagram.—We now have the data for the stress diagram, of which one-half is shown. For the point 4, or BP , we have the upward supporting force $bp = \frac{1}{2}W$, next $pa = H$, the horizontal thrust just determined of the wall, etc., against the joint, ao parallel to the line of action of AO , and finally ob , the pressure of the post OB on 4. The resultant of bp and pa , or ba , may of course be used for the reaction of the wall. Taking next the joint 6, we have cb the load, bo the thrust of BO , and we then draw on and nc . The joint CD gives $dcnmd$. The joint MA already has the lines mn , no and oa ; since the line which is to close on m must be

parallel to LM, and a is already vertically over m , al can have no length, and there is no stress in AL, as before assumed. Upon taking the joint DE we find also that no stress exists in LK. The reader must not think this fact at variance with the value H which was said to exist in 2-3 when we considered the trapezoid alone; the triangular truss 123 will plainly cause a tension in 2-3, and, with this distribution of load, such tension will exactly neutralize the compression caused in the same piece by 4-2. If one will consider the truss as loaded at 6, 2, 1, 3, and 7 only, thus doing away with NM, KI, IG, etc., he will find that a diagram will then give some compression in KL.

Another method of treatment will be applied to this truss later, § 75.

65. Different Horizontal Thrusts Consistent with Equilibrium.—In studying Fig. 22 we saw that the stresses in GC and CH were determined by the load GH, and that the space C would become distorted unless a horizontal thrust of a definite amount, here $a'a$, was supplied by the walls. In Fig. 23 also the same things are true; the trapezoidal truss 4235 requires a certain horizontal thrust at the points 4 and 5 to balance its load; a greater or less thrust will cause the truss to rise or fall, so long as LA and QA are neglected, for in that case motion can freely occur at joints 2 and 3. If, however, these pieces are under stress, a greater or less horizontal thrust may be applied, the truss will still be in equilibrium, and the diagram will close. Indeed a vertical reaction is a supposable one, in which case OA must be without stress. The same statement applies to Fig. 22, if one of the diagonals of the space C is put in. As all roof-trusses of small depth in their middle section, as compared with their total rise, have a tendency to spread under a load, and hence to thrust against their supports, their diagrams should be drawn for a moderate amount of thrust at least, if it is desired to have them maintain their shape; and the supports should be able to offer this resistance, or a tie should be carried across

below. Otherwise, in addition to the sagging, a large increase of stress is likely to be found in some of the parts as a result of a vertical reaction. The determination of the horizontal thrust in a braced frame of this kind is not very simple, but may be worked out by a method given in Part III, "Arches," Chap. XII.

66. **Proof.**—That such trusses are in equilibrium under a greater or less amount of horizontal thrust, or even when the reactions are vertical, provided the pieces are able to withstand the resulting stresses, is illustrated by Fig. 24. Here the load CB is taken as twice DC . The vertical reactions ba and ad are calculated by the method of § 26. The diagram with unaccented letters is then drawn and closed as usual. Next, any horizontal thrust aa' at the points of support is assumed and the diagram with accented letters is drawn. This diagram also closes. The reduction of all of the stresses except that in fg is most marked. We see from these cases that only when the truss admits of deformation by the distortion of some interior space such as C of Fig. 22, or R of Fig. 27, is the horizontal thrust determinate by the method of these chapters; and that moderately inclined reactions or the tension of a horizontal tie between points of support are favorable to a reduction of the stresses.

Arched ribs of a nearly constant depth, not infrequently employed in railroad stations and public halls, will be treated in Part III.

CHAPTER VII.

FORCES NOT APPLIED AT JOINTS.

67. First Diagram.—In the trusses heretofore treated the loads have been concentrated at those points only which were directly supported. It sometimes happens that the cross-beams or purlins, which connect the trusses and convey the weight from the secondary rafters to the main rafters, rest upon the latter at points between the joints. Let us, in Fig. 25, assume that a load rests upon the middle of each of the upper rafters. If we neglect the bending action of the load EG upon the rafter and proceed as usual, we consider that one-half of the load EG will be supported at each of the joints CE and GK, and similarly for the load KM. Therefore, having laid off the weights and the two equal reactions of the walls on the load-line of the first diagram, we may increase the loads on the joints CE, GK, and MO by the new points of division, and complete this diagram, taking first B, then the next joint on the inside, and then the outside one. It will be noticed that all of the pieces except the rafters are ties.

68. Supplying Imaginary Forces.—This diagram gives but one stress along the whole of the upper rafter; but it is plain that the vertical force EG must have a component along the rafter and cause a different stress to exist in ET from what exists in GT. If, however, we suppose a joint to be at EG, the transverse component of EG will cause it to yield, as there is no brace beneath to hold it in place. To secure equilibrium here we may supply an imaginary force EF, shown by the dotted line, equal and directly opposed to this.

transverse component. This imaginary force will take the place of a perpendicular strut, will steady the joint, and will leave the longitudinal component to affect the rafter. But the transverse component of FG actually gives a pressure at the joints CE and GK , while the imaginary force EF , just added, will lift the ends of this rafter by the same amount; therefore we must restore the pressure, and the equilibrium of the rafter FT as a whole, by adding imaginary forces, each one-half of EF , at CD and GH . This added system of forces cannot interfere with the stresses in any other pieces, for they balance by themselves. Treat the similar load KM in the same way.

69. Second Diagram.—In the second diagram the two supporting forces, pa and ab , are each equal to one-half the total load. Lay off bc as before; draw the dotted line cd , equal and parallel to the first imaginary force CD ; then de vertical, as before; then ef , equal to, and in the direction of EF ; then fg , and so on, arriving finally at p , as usual.

The construction of the rest of the diagram presents no difficulty; the joints are taken in the same order as before, and, when we have more than one external force on a joint, we take them in succession, in the order first observed for the external forces. When we reach the upper rafters, we find that g falls on the line et ; et is greater and gt is less than the line for the same piece in the first diagram.

70. Comparison of Results.—Thus it appears that the first diagram gives the stress which would exist in the whole length of the rafter ETG , if the load FG were actually at its extremities; but, being at its middle point, one-half of the longitudinal component of FG goes to diminish the compression otherwise existing in GT , and the other half to increase the compression in ET . A comparison of the two diagrams will also show the truth of the former statement, that the system of imaginary forces does not affect any of the truss outside of the particular pieces to which it may be applied. It is still necessary to provide for the bending action of the

transverse portion of FG , or a force equal and opposite to EF upon the rafter, considered as a beam extending from hip to apex, a joint of course not being made at EG . This subject will be treated in Chapter IX.

71. Remarks.—If the action of the wind upon this truss is considered, it will be seen at once that no special treatment is needed; for the wind pressure is normal, and the addition of the opposite force EF at once balances the force on this joint, and transfers it to the ends D and H as the first analysis did. The bending action on the rafter must, however, be provided for.

The treatment of loads or forces not directly resisted, as above, is given by Mr. Bow in his "Economics of Construction," and may be applied to frames where one or more of the internal spaces are not triangles, but quadrilaterals. If such spaces are not surrounded by triangular spaces on at least all sides but one, the truss is liable to distortion, unless the resistance of some of the pieces to bending or the stiffness of the *theoretical* joints is called into play. A use of this treatment at many points in the same diagram will, however, be apt to make confusion.

Another application of imaginary forces, where a bending moment exists, will be made at the close of the next chapter.

CHAPTER VIII.

SPECIAL SOLUTIONS.

72. Reversal of Diagonal.—Difficulty is sometimes experienced in completing the diagram for a truss because, after passing a certain point, no joint can be found where but two stresses are unknown; while yet, judging from the arrangement of the pieces, the stresses ought apparently to be determinate. Such a case was found in Fig. 11, and was solved in § 20 by what might be called the law of symmetry. A method of more general application to these cases is what may be styled *Reversal of a Diagonal*.

It has been pointed out already that, if any quadrangular figure in a truss is crossed by one diagonal, the other diagonal of the quadrangle may be substituted for the former without affecting the stresses in any pieces except those which make up the quadrangle. See §§ 26 and 53. It will be found that such a change often reduces the stress in one or more pieces of the quadrangle to zero, and thus makes the truss solvable graphically. It will be well, if the reader fails to distinguish readily the altered truss from the original one, to temporarily erase from a pencil sketch the pieces thus rendered superfluous, or to draw the truss anew with the proper changes as has been done in Figs. 26 and 27. The modified truss will then be easily analyzed, and, when the old members are restored, enough stresses will be known to make the final solution practicable.

73. Example.—This method will first be applied to the roof-truss, Fig. 26, of a railroad station at Worcester, Mass. The span of this roof is 125 feet; entire height, wall to apex,

45 feet; camber of main tie 8 feet; rafter divided into six equal panels; trusses 50 feet apart.

Under steady load the tie bars ST , TU , UW , WS , which cross the centre line of the truss, will be without stress, as in Fig. 14, § 25. Indeed, as these two centre ties are independent of one another, but one can be in action at a time, as, for instance, SW and TU when the wind is on the left side. If we begin our diagram from B with $cbakc$, we meet with no difficulty until we have passed the joint EF , for which we drew $fenopf$. At either of the next joints are three unknown stresses. As all stresses are determined up to the piece PQ , change the diagonal QR in the adjoining quadrilateral from the position of the full line to the dotted one. Then the joint FG , as seen in the sketch below, will give us $gfpq'g$. As the full-lined diagonal has been removed, the joint RW has disappeared; for, if three supposed forces are in equilibrium at one point, § 17, and two of them act in one line, the third force must be zero, and RS therefore can have no stress. The stress in SW will also be zero unless it resists wind on the left, and the stress in ST is then zero. In either case we can draw $hgq'r'h$ for the upper joint, and then find aw and wr' , if it exists, at the lower joint. The dotted peak is not in the main truss, but in the jack-rafters which transfer their load to GH and HI ; if one prefers, he may put a load at the peak and draw the triangle of forces for that point.

After using the above expedient on the other half of the truss also, if the load is unsymmetrical, we replace the *reversed diagonal* and find the true stresses in the pieces affected by the change,—the diagonal and the four sides of the containing quadrilateral. Hence we may draw $poawqp$ for the lower joint or $hgrsh$ for the upper joint, and finally $gfpqrg$ for the left-hand joint of the quadrilateral.

74. Polonceau Truss.—The left half of Fig. 29 is the same as Fig. 11. It will be remembered that we were stopped at the piece DE of Fig. 29 by having three unknown stresses at

either end. Change the full line EF to the dotted one. The stress in FG at once becomes zero, as did RS in Fig. 26. We may now find the stresses in DE and EL at the joint KL; in dotted EF and GM at joint LM, and in AH and HF at the lower joint. Then the diagonal may be replaced and the stresses in DE, EF, FG, EH, and FL rectified. The right half of Fig. 29 may be similarly solved by reversing the diagonal PQ, which change makes the stress in OP zero.

75. Hammer-Beam Truss, by Reversal of Diagonals.

—The hammer-beam truss of Fig. 27 differs from that of Fig. 23 by the omission of the vertical in the space R. As pointed out in § 66, this omission renders the horizontal thrust of this truss definite. In attempting to draw a diagram, however, we cannot apparently begin at the wall until we know the horizontal thrust, and, if we begin at FG, we soon meet with joints where three unknown forces are found. The method of the preceding sections will first be applied to the right half. Draw *gfr* for the upper joint, *hgrsh* for joint GH, and *feqrfe* for EF. As joints HI and RA are now insoluble, draw dotted TW for the full-lined diagonal TW, and do the same with XY. The truss will thus be changed to the form of the sketch below. For, since TA and YA act in the same straight line (shown dotted on left half of truss), the stress in WX is now zero, and TA and YA have the same stress. Further, at joint KL there remain KY, YL, and the exterior force or load KL, which latter acts in the vertical line YL; hence the stress in KY is now zero, and YL carries KL only. We can therefore draw *ihst'i* for joint HI, *kit'w'k* for joint IK, *w't'srq . . . aw'* for joint AR, and *lkw'al* for the abutment. The reaction *al*, being thus determined, can be used to draw the diagram, as in Fig. 23. The diagram for the left half of the truss is given in full lines, and it may be seen that AP and AT are now useful.

76. **Method of Trial and Error.**—Where the unknown stress in but one piece appears to stand in the way of a

solution, the diagram may sometimes be drawn with comparative ease by trial. Thus, in the left half of Fig. 27, we may assume the value of the horizontal thrust or of the stress in PQ and proceed with the diagram. Upon its failing to close, we can change the assumed quantity and try again. Thus, beginning at the apex, draw $gfrg$, $feqrf$, and $hgrsh$; then assume qp' and its equal st' . The middle joint will give $t'srqp'a't'$; the joint DE , $p'qed'o'p'$, etc.; and finally the horizontal line from n' will fail to meet a line parallel to AM on the load line, to give mb in the post. It is evident, upon a slight inspection, that qp' is too long. The reader will find that he can soon bring the diagram to a closure by diminishing qp' .

By the use of such approximations one of necessity loses that check on the accuracy of the diagram, of having it close with reasonable exactness.

Fig. 30, in case one or the other of the dotted diagonals is used, will serve as an example for the practice of the preceding suggestions. Which diagonal tie, if either, will be needed for wind, and which for steady load?

77. **Example.**—We will close this branch of the subject with an example which will introduce one or two new points in addition to a combination of principles heretofore illustrated separately. The example shows the capabilities of this method in handling complex problems. The structure drawn in Fig. 28 is to be treated as a whole in its resistance to wind pressure.

The steady-load diagram would present no difficulty. The truss is carried upon columns which are hinged at their lower ends B and P , each being connected by a pin to its pedestal. The brace at R is therefore necessary to prevent overturning. The proportions of the frame are as follows: Distance between columns, 76 ft.; $AC = 15$ ft.; $QR = 7$ ft.; camber of lower tie, 3 ft.; $1-A = 19$ ft.; height of space $1 = 16$ ft.; of $Y = 7$ ft.; extreme height, ground to peak, 48 ft.

Distance between trusses, 12 ft. Scale 40 ft. = 1 in. Scale of diagram, 8000 lbs. = 1 inch. No wind on C.

Wind pressure on main roof, 12,000 lbs. = bj ; therefore fg , gh , etc., = 3000 lbs.; wind pressure on KX = 3360 lbs. = $j-10$; on LY = 3500 lbs. = $10-m$. The dotted arrows are resultants of wind pressure on the sloping surfaces. By moments about P, or by proportion of segments of span BP, as in § 48, we find

$$\begin{array}{rcl} \text{that } 8368 \text{ lbs. of } bj \text{ is carried at B, and } 8632 \text{ lbs. at P.} & & \\ \text{that } 940 \text{ " " } 10-m \text{ " " " } 2460 \text{ " " "} & & \\ \hline 9308 \text{ lbs.} = b-9 & \text{" " " " } & 6092 \text{ " " "} \end{array}$$

The horizontal force, $j-10$, at K, may be supposed to be resisted equally at each point of support, since the two posts will be alike. Hence $jk = 9-a' = \frac{1}{2}(j-10) = 1680$ lbs. is carried at B. The moment of this horizontal force K about B or P, tending to overturn the frame, or the couple formed by K and the equal reaction in the line PB, will cause an increased upward vertical force at P and an equal downward force or diminished pressure at B. Its value, § 42, will be

$$\frac{3360 \times 39\frac{1}{4}}{76} = 1760 \text{ pounds} = a'a. \text{ The reaction at B must}$$

balance the components, $b-9$, $9-a'$, and $a'-a$, and hence will be ab . The reaction at P will then be m (or p) a , which may be checked in detail, if desired.

The reaction ab , at B, will now be decomposed into its vertical and horizontal components ac and cb . The piece AC can resist ac as a strut or post, but must carry cb 5900 lbs. by acting like a beam. Were there a real joint at D the structure would fall. It is therefore necessary to make the post of one piece, or as one member from B to R. The magnitude of the horizontal force at F caused by the 5900 lbs. of horizontal force at B will be in the ratio of the two segments of the column (beam) or as 15 to 7, or 12,643 lbs. These two forces must be balanced at D by a force equal to their sum,

or 18,543 lbs. As in § 68, Fig. 25, this beam action of the post must be neutralized, before the diagram can be drawn, as these diagrams take no account of bending moments, for which see Chap. IX.

We therefore apply at BC the imaginary horizontal force $bc = 5900$ lbs., opposed to the direction of the reaction, and leaving only ac , the vertical component, which is balanced by the post; at CD we apply $cd = 18,543$ lbs.; and at EF , we add $ef = 12,643$ lbs. The sum of these three imaginary horizontal forces being zero, the stresses in the truss are not disturbed. The same steps must be taken at P , the horizontal forces mn , no , and op being obtained by the same process from the horizontal component po of the reaction pa .

The load line therefore finally becomes $bcdefghijklmnop$, the force DE being shifted laterally as shown, and ik being the resultant of ij and jk . The stress in DQ is readily obtained by drawing deg . Then the point D of the post gives the figure $acdgra$, determining the stresses in the upper part of the post and the brace RA . The remainder of the diagram presents no difficulty.

The column must be designed to resist the large bending moment to which it is liable, as well as the thrust qr . For bending moments, etc., see the next chapter, and also Part II. As this structure is supposed to be open below, the lower member should be adapted to resist such compression as may come upon it from the tendency of a gust of wind, entering beneath, to raise the roof.

CHAPTER IX.

BENDING MOMENT AND MOMENT OF RESISTANCE.

78. Load between Joints.—Having treated of the action of external forces upon a great variety of trusses, we propose now to investigate the graphical determination of the bending moments which arise from the load on certain pieces, and of the stresses due to the moments of resistance by which the bending moments must be met.

To recapitulate some statements of earlier chapters :—In case the transverse components of the load upon a portion of a rafter, or other piece of a truss, are not immediately resisted by the supporting power of some adjacent parts, or, in other words, unless the load on a structure is actually concentrated at the several joints, such transverse components will exert a bending action on the portion in question, and the additional stress thus caused in the piece may be too great to be safely neglected. Further, in case the piece makes any other than a right angle with the line of action of the load, or has an oblique force acting upon it, the stress along it, given by the diagram, will be less than the maximum, and will generally be the mean stress. Lastly, in case a piece is curved, a bending moment will be exerted upon it by the force acting along the straight line joining its two ends, this bending moment being a maximum at the point where the axis or centre line of the piece is farthest removed from the line drawn between its ends.

but **79. Example.**—To illustrate the former statements by a simple example :—Suppose the rafters AC and BC, Fig. 31, to be loaded uniformly over their whole extent. Let us assume, in the first place, that the tie AB is not used, but

that the thrust of the rafters is resisted by the walls which carry the roof. Consider the piece AC . Since the roof is symmetrically loaded, the thrust at C must be horizontal, and therefore the reaction which supports this end of AC will lie in the line CE . The centre of gravity of the load on AC is at D , its middle point, and the resultant of the load will, if prolonged upwards, intersect CE at E . Since the rafter is in equilibrium under the load and the reactions at C and A , the direction of the reaction of the wall at A must also pass through E (compare Figs. 3 and 4). Draw AE and prolong ED to G . Let EG be measured by such a scale as to represent the load on AC . The three forces meeting in the common point E will then be equal to the respective sides of the triangle $AE G$, drawn parallel to them; and, since AG equals EC , the reactions at A and C will be AE and CE .

We now decompose AE and CE into components along and transverse to the rafter, and have AF , direct compression on the rafter at A , and CF , direct compression at C . The compression on successive sections of the rafter increases from C to A by the successive longitudinal components of the load. The two components AL and CQ , which, combined with AF and CF , give the original forces AE and CE , are analogous to the supporting forces of a beam or truss, and through them we obtain the bending action of the load on this rafter. If, now, the rafters simply rest on the wall, being secured against spreading by the tie AB , the reaction AE will be replaced by the two components, AI , the upward supporting force of the wall, and AG , the stress exerted by the tie; these two forces give the same stress and bending moments on the rafter as before.

80. Comparison with Diagram.—Consider, next, the method by diagram. The load is now to be concentrated at the joints, and in place of EG , we shall have AN and CP , each one-half of the load on one rafter. Lay off 1-2 to represent the total load on the roof, make 1-3 equal to AN and 1-4 to AI , and draw 3-5 and 4-5 parallel to the rafter and tie.

A G will equal 4-5, and therefore the stress in the tie is given correctly; but, since $AI - AN = AK = 3-4$, 3-5 equals A D, and this is the stress given by the diagram as existing from A to C, a supposition which is true when the load is actually concentrated at the joints, but is not true for a distributed load. But A D, or 3-5, is equal to one-half of $AF + FC$, and is manifestly the value of the direct compression at the middle point D of the rafter; all of the load from A to D was, when we drew the diagram, considered to be concentrated at the joint A. To 3-5, or A D, we should add D F, to obtain the correct compression A F at the lower end; therefore a piece which supports a distributed load should have a compression, equal to the longitudinal component of so much of the load as is transferred to its lower end, added to its stress obtained from the stress diagram. The amount to be added, however, is generally insignificant as compared with the truss stress.

The load on the principal rafters of a roof-truss is usually concentrated at series of equidistant points, by means of the *purlins*, or short cross-beams which extend from one truss to another, and which are themselves weighted at a series of points by the pressure of the secondary rafters. These secondary rafters, when employed, carry the boards, etc., and thus have a uniformly distributed load. It is only in cases where purlins rest at other points than the so-called joints that bending action occurs in the principal rafters, or in very light trusses where the boards are nailed directly to the main rafters. We need to determine the maximum bending moments on such main rafters, on the purlins and secondary rafters, in order to intelligently provide sections sufficiently strong to resist them.

✕ 81. **Bending Moment.**—It will first be well to explain what *bending moment* and *moment of resistance* are. A horizontal beam A B, Fig. 32, supported at its two ends, when loaded with a series of weights, distributed in any manner, is in equilibrium under the action of vertical forces, the weights acting downwards and the two supporting forces acting up-

wards. These supporting forces are easily calculated by the principle of the lever, or by taking moments as explained in §§ 26 and 36. They will be found graphically presently. As the beam is at rest, there must be no tendency to rotate, and therefore, if we assume any point for an axis, the sum of the moments, that is of the products of each force by its distance from the axis, must equal zero. A moment which tends to produce rotation in one direction being called plus, one which acts in the other direction is called minus. If then we pass an imaginary vertical plane of section through any point in the beam, such as E, the sum of the moments on one side of the plane of section must balance or equal that on the other. The sum of these moments on one side or the other is called the *bending moment*: the reason for the name will soon be evident.

82. **Moment of Resistance.**—These bending moments on opposite sides of the section in question can balance one another only through the resistance of the material of the beam at the section where stresses between the particles are set in action to resist the tendency to bend. The beam becomes slightly convex, and the particles or fibres on the convex side are extended, while those on the concave side are compressed. Experiment shows that, for flexure within such moderate limits as occur in practice, the horizontal forces exerted between contiguous particles vary uniformly as we go from the top of the beam to the bottom, the compressive stress being most intense on the concave side, diminishing regularly to zero at some point or horizontal plane, called the *neutral axis*, then changing to tension and increasing as we approach the convex side. The two sets of stresses reacting against each other may be represented to the eye by the arrows in the vertical section marked E'.

Since all of the external forces are vertical, these internal stresses, being horizontal, must balance in themselves, or the total tension must equal the total compression, whence it follows that the neutral axis must pass through the centre of

gravity of the section. To make this fact clear, let one consider that the distance of the centre of gravity from any assumed axis or the position of the resultant of parallel forces is found by multiplying each force or weight by its distance from that axis and dividing by the sum of the forces. Now if we attempt to find the centre of gravity of a thin cross-section of this beam, and take our axis through the point where the centre of gravity happens to lie, the sum of the moments of the particles on each side will balance or be equal, and we can see that the distance of each particle from the axis will vary exactly as these given stresses; hence the neutral axis must lie in the centre of gravity of each cross-section.

As these stresses are caused by and resist the external bending moment on each side of the section, the moment in the interior of the beam, made up of the sum of the products of the stress on each particle multiplied by its distance from the neutral axis, or indeed from any axis, and known as the *moment of resistance*, must equal the bending moment at the given section. As the tensions and compressions on one side of the plane of section tend to produce rotation about the neutral axis in the same direction, their moments are added together.

83. **Formula for Bending Moment.**—The bending moment, then, in the beam *AB* of the figure, at any section *E*, will be, if P_1 is the supporting force on the right, W_1 , W_2 , etc., the weights,

$$P_1 \cdot BE - W_1 \cdot CE - W_2 \cdot DE;$$

or, in general, if L equal the arm of any weight, and Σ be the sign of summation,

$$M \text{ (the bending moment)} = P_1 \cdot BE - \Sigma W \cdot L,$$

it being remembered always to take only the weights between one end and the plane of section.

The moment of resistance, being numerically equal to the bending moment, is therefore equal to the above expression, and the maximum stress at any section can thence be

determined, or the required cross-section to conform to the proper working stress for the material. The weights on one side of the section may all be considered to be concentrated at their common centre of gravity, or point of application of their resultant, so far as the bending moment at that section is concerned; the load when continuous is always so taken.

If the reader will take a special case, and, having a beam of known length with weights in given positions, will first find the supporting forces, and then calculate the bending moment on either side of a plane of section, he will obtain the same result with opposite signs, showing that the two moments balance one another. The numerical result, being the product of two quantities, is read as so many foot-pounds or inch-pounds, according to the units employed. As the stress in any material is usually expressed in pounds on the square inch, the latter units are the better.

84. Equilibrium Polygon.—Let us suppose that the weights which, in Fig. 32, rest upon the beam are transferred to a cord at the several points c, d, f , and g , vertically below their former positions C, D, F , and G , the cord itself being attached to two fixed points a and b , at equal distances vertically from A and B . Let us further suppose that the amount of the weight at G alone is at present known. This cord can be treated as if it were a frame. Taking the joint g into consideration, draw $5-4$ vertically, equal to the weight, then $5-0$ parallel to ag and $4-0$ parallel to gf . The two lines just drawn must be the tensions in ag and gf . For the joint f , fg is now known; therefore $4-3$ parallel to the weight and $3-0$ parallel to fd will determine the other forces at f . The side $4-3$ must equal the weight at F , and must lie in the same straight line with $5-4$; for this triangle was constructed on the side $4-0$ previously found. Continuing the construction for the successive angles of the cord, we find that a vertical line $5-1$ will represent by its several portions the successive weights, and that the tensions in the different parts of the cord will be given by the lines parallel to these parts, drawn

from the points of division of the load line, and all converging to the common point 0. Draw 0-6 horizontally, and hence parallel to ab ; this line will be the horizontal component of the tension at any point of the cord, and is here denoted by H . The form assumed by the cord for a given distribution of weights is called the *Equilibrium Polygon*, as the system will be in equilibrium or at rest; and it is also called in mechanics a funicular polygon. Students of mechanics will recall the fact, so easily shown here, that the horizontal component H is a constant quantity at every point.

85. Reactions.—If now the cord, instead of being fastened to fixed points at a and b , is attached to the two ends of a rigid bar ab , and the whole system is then suspended from A and B by two short cords, its equilibrium will not be disturbed. The pull 5-0 at a will be decomposed into 0-6, compression in ba , and 6-5, tension along aA . Similarly at b , 0-1 will be decomposed into 1-6 along bB and 6-0 along ab . 6-0 balances 0-6, while 1-6 and 6-5 must be the supporting forces at b and a . As the supporting forces do not depend upon the form of the frame or truss, the reactions which carry the beam at B and A must be these same quantities.

86. Equilibrium Polygon, General Construction.—We may make the construction more general by drawing an equilibrium polygon from any point a' , vertically below A , and finding the outline of a cord which will sustain in equilibrium the given weights at the given horizontal distances from A . Lay off the weights in succession from 5 to 1; assume any point $0'$ arbitrarily and connect it with all the points of division of the load line. Begin at a' , and draw $a'g'$ parallel to 5-0', stopping at the vertical dropped from G ; then draw $g'f'$ parallel to 4-0', etc., and finally $c'b'$ parallel to 1-0'. That this will be the figure of a cord suspended from a' and b' follows from the preceding demonstration. Connect b' with a' ; a line, parallel to $b'a'$, from $0'$ must strike the same point 6 which the line from 0, parallel to ba , touched. The sup-

porting forces, if $b'a'$ exists, will be 1-6 and 6-5 as before; but $0'-6'$ will be the horizontal component H' for this cord.

87. The Equilibrium Polygon Gives Bending Moments.—If we turn again to the first cord, attached at a and b , the piece ab being dispensed with, the moment of all the forces on one side of any point, such as e , must be the bending moment there; but as the cord is perfectly flexible and at rest, this bending moment will equal zero. Using, instead of 1-0, its two components 1-6 = P , and 6-0 = H , multiplying each force by the perpendicular distance of its line of action from e , calling the combined moments of the weights on one side of e $\Sigma W \cdot L$ as before, and denoting the tendency to produce rotation in opposite ways by opposite signs, we shall have, for moments of forces on the right of, and around e ,

$$P_2 \cdot bk - \Sigma W \cdot L - H \cdot ek = 0,$$

or

$$H \cdot ek = P_2 \cdot bk - \Sigma W \cdot L.$$

But $P_2 \cdot bk = P_2 \cdot BE$, and $P_2 \cdot BE - \Sigma W \cdot L = M$, the bending moment at the section E of the beam, as shown in § 83; therefore

$$M = H \cdot ek.$$

By a similar analysis of the lower cord we have

$$P_2 \cdot ik' - \Sigma W \cdot L = (6-0') \cdot e'l = M.$$

From similarity of triangles $le'k'$ and $6'0'6$, we have

$$e'l : e'k' = 6'-0' : 6-0',$$

or

$$(6-0') \cdot e'l = (6'-0') \cdot e'k';$$

therefore

$$M = (6'-0') \cdot e'k' = H' \cdot e'k',$$

as in the other case. The solution is therefore general, and the bending moment at any section of the beam equals the product of H from the stress diagram 015 by the vertical ordinate, below the section, from the cord to the line connecting its two extremities.

88. Remarks.—The relative situations of a' and b' will depend upon the choice of the position of O' , and this point may be taken wherever convenient. H' is measured by the same scale used in plotting 5-1, while $e'k'$ must be measured by the scale to which AB is laid off. The two scales, one representing pounds, the other inches, need not be numerically the same; their product will be inch-pounds.

A single load on the beam will have for its equilibrium polygon two straight lines from a' and b' , meeting at a point vertically under the weight. A uniformly distributed load will give a parabola with the maximum ordinate at the middle of the span. This load may be treated as if concentrated at any convenient number of points along the beam, as we have done in getting the loads at the several divisions of a rafter, and the angles of the polygon will lie in the desired parabola. When the beam is inclined the transverse components alone of the load produce any bending, as explained for a uniform load in § 79. Wind pressure will act as a uniform normal or transverse load on the piece which directly resists it.

The equilibrium polygon has much more extended applications in Parts II. and III.

89. Moment of Resistance of Rectangular Cross-Section.—Next, to determine the moment of resistance for a particular form of cross-section:—Consider a beam of rectangular cross-section, represented by $ABCD$ of Fig. 33. The intensity of stress, as shown at E' , Fig. 32, varies uniformly each way from the neutral axis which, lying through the centre of gravity G of the cross-section, will be at EF , the middle of the depth. The stress on a square inch will be most intense on the fibres at the edge AB or CD , and less intense on any intermediate layer, such as IK , in the proportion of EI to EA . If then we draw from G the lines GA and GB , and imagine that the layer IK is replaced by $I'K'$, which has its breadth diminished in the same proportion, the total stress on $I'K'$, if of the intensity found at AB , will be equal to the total stress of less intensity actually existing on IK . The

former stress will also have the same leverage about E F as does the actual stress on I K. By the same reasoning for all layers of the cross-section, we obtain two triangular, shaded areas, A B G and G D C, which may be termed *equivalent areas of uniform stress* of intensity equal to the actual maximum; one of them, usually the upper one, when multiplied by this maximum intensity of stress, represents the total compression, and the other the total tension at the section. The moments of this tension and compression about the neutral axis will be most readily obtained by considering the stress, which is now uniformly distributed over the triangle, as concentrated at its centre of action, the centre of gravity G' of the triangle, distant two-thirds of its height from the apex G.

Let b represent the breadth and h the height of the cross-section in inches; the area of one triangle will be $\frac{1}{2}b \cdot \frac{1}{2}h$; and the lever arm about E F will be $\frac{2}{3} \cdot \frac{1}{2}h$. Let f represent the maximum stress on the square inch at A B. Since the tension and compression tend to produce rotation in the same direction, we add the moments of the two forces together and have

$$2\left(\frac{bh}{4} \cdot f \cdot \frac{1}{3}h\right) = \text{moment of resistance} = \frac{1}{6}fbh^2.$$

Putting this value equal to the bending moment M, we obtain

$$H' \cdot e'k' = \frac{1}{6}fbh^2.$$

If we select the maximum value of $e'k'$, introduce the safe working stress f for the extreme fibres, and assume either b or h , we can compute the other required dimension, and thus determine the beam when of uniform section throughout. If the cross-section is to vary, its moment of resistance at different points must at least be equal to the bending moments. As the stiffness of the beam depends principally upon h , the depth must not be made too small. If the beam has too little breadth the compressed edge will yield sideways.

90. Moment of Resistance of T Section.—It is easy to compute the size of a beam of rectangular cross-section by the

above formula, but for less regular sections the determination of the moment of resistance by this graphical method may prove of service. In applying it to a beam of the section shown in Fig. 34 we must begin by finding the centre of gravity of the section. By multiplying each rectangular area by the distance of its centre of gravity from either the top or the bottom, adding these products, and dividing by the whole area, we find the distance of the neutral axis from that edge. If $GI = b$, $AB = b'$, $GE = h$, and $CA = h'$, we have

$$\frac{bh \cdot \frac{1}{2}h + b'h'(h + \frac{1}{2}h')}{bh + b'h'} = \text{distance of neutral axis from GI.}$$

The construction of the shaded area APB needs no explanation, as it follows the previous example. The stress on the fibres at the edge GI will not be so great as at the edge AB , because they are not so far from the neutral axis. If the fibres at GI were removed to KL , so as to be equally remote with AB , they would be equally strained. Then to reduce the layer GI to one which, if it had the same intensity of stress with AB , would give the same total stress which now exists on GI , project GI to KL , draw KP and LP , and $G'I'$ will be the desired reduced length. The remainder of the shaded area for the lower rectangle follows the usual rule. In the same way, the fibres at CD will be projected at QR , and, by drawing QP and RP , we determine $C'D'$, and thus complete the shaded portion. These triangles, etc., can be readily scaled, or computed from the known proportions of the beam, their centres of gravity found and the moment of resistance calculated.

91. Moment of Resistance of an Irregular Section.—A good example of a section whose moment of resistance is not readily determined by computation alone is afforded by a deck-beam, Fig. 35, often employed in floors and roofs. It is here drawn to one-quarter scale, showing height of section 6 inches, breadth of flange AB $3\frac{1}{2}$ inches, thickness of web $\frac{3}{8}$ inch, weight per yard 44 lbs.; therefore the area of cross-section is about 4.4 square inches.

The readiest way to determine the moment of resistance of such a cross-section is as follows :—Transfer its outlines from the book of shapes or by such data as you have to a sheet of heavy paper, and make a tracing for construction purposes. Cut the section from the heavy paper, balance on a knife-edge and thus determine the neutral axis CD . Then on the tracing draw KL horizontally at the same distance from CD that ST is. AB will be projected at KL , and lines from K and L to P , the middle point of CD , or the centre of gravity of this section, will cut AB at A' and B' , making $A'B'$ the reduced length of AB , and now considered to have the same stress per square inch as exists at IG . In the same way the end M of MN will be projected at O , the point U at V , and the lines from O and V to P will cut the horizontal lines through M and U at new points in the desired curve. Thus enough points are soon obtained to locate the boundary of the shaded portion from B' to P . The part of the web with straight sides gives of course a triangle, found at once by drawing a line from W to P . The curve $A'P$ corresponds with $B'P$. For the lower portion, project EF on TS , draw lines to P , and get in a similar way enough points for this curve. Cut out the two shaded figures from the heavy paper, balance each one over a knife-edge and thus determine their respective centres of gravity Q and R . Calculate the area of one; the area of the other should exactly equal it, for the total tension equals the total compression. Calling this area A and the safe working stress on the square inch f , we shall then have for the moment of resistance

$$f \cdot A \cdot PQ + f \cdot A \cdot PR = f \cdot A \cdot QR.$$

In this example $A = 1.29$ sq. inches, $PQ = 2.12$ inches, and $PR = 2.66$ inches. If therefore for a static load $f = 12,000$ lbs., the moment of resistance equals

$$12,000 \times 1.29 \times 4.78 = 74,000 \text{ inch-pounds.}$$

92. Moment of Resistance of I Beam.—In simpler cases the required size of beam to sustain a given load is more read-

ily found by formula. If I beams are used, the web being thin, and the top and bottom flanges alike, an approximate formula may be used. If F represents the area in square inches of the cross-section of either flange, W the area of the web, h the depth from centre to centre of flanges or the entire depth minus thickness of one flange (that is, between centres of gravity approximately), and f the safe stress on the square inch, the moment of resistance is nearly equal to

$$f h (F + \frac{1}{3} W).$$

CHAPTER X.

LOAD AND DETAILS.

93. Lateral Bracing.—The principal trusses, if large, should be braced together in the planes of the rafters to prevent wind, in a direction perpendicular to the gable ends, from producing any lateral movement. The roof boards, if laid close, and well nailed, will stiffen trusses of moderate span. It is often customary also to fasten the trusses down to the walls, especially in those buildings where wind may get below the roof. In such cases it is proper to consider and provide for the tendency of the wind to reverse the stresses in a roof which has a light covering.

94. Weight of Materials.—The weight of the roof covering can be ascertained in advance. The bending moments on the jack-rafters and the purlins can then be found, their sizes computed and their weights added in. The weight of the truss must then be assumed from such data as may be at hand. After the diagrams have been drawn and the truss has been roughly designed, its weight should be calculated to see how well it agrees with the assumed weight. If this agreement is not sufficiently exact, the proper allowance is then to be made.

Trautwine says that, for spans not exceeding about 75 feet, and trusses 7 feet apart, of the type shown in Figs. 11 and 29, the total load per square foot, including the truss itself, purlins, etc., complete, may be taken as follows :

Roof covered with corrugated iron, unboarded, . . .	8 lbs.
Same if plastered below the rafters,	18 "
Roof covered with corrugated iron, on boards, . . .	11 "

Same if plastered below the rafters,	21 lbs.
Roof covered with slate, unboarded or on laths,	13 "
Same on boards 1½ inches thick,	16 "
Same if plastered below the rafters,	26 "
Roof covered with shingles on laths,	10 "

For spans from 75 feet to 150 feet it will suffice to add 4 lbs. to each of these totals.

The weight of an ordinary lathed and plastered ceiling is about 10 lbs. per square foot; and that of an ordinary floor of 1-inch boards, together with the usual 2 × 12 inch joists, 12 inches apart from centre to centre, is from 9 to 12 lbs. per square foot. White pine timber, if dry, may be considered to weigh about 25 lbs., northern yellow pine 35 lbs., and southern yellow pine 45 lbs. per cubic foot; if wet, add from 20 to 50 per cent. Oak may be reckoned at from 40 to 50 lbs. per cubic foot; cast iron at 450 lbs. per cubic foot; wrought iron at 480 lbs. per cubic foot.

The allowance to be made for the weight of snow will depend upon the latitude; from 12 to 15 lbs. per square foot of roof will suffice for most places. In some situations snow may accumulate in considerable quantities, becoming saturated with water and turning to ice; but snow saturated with water will generally slide off from roofs of ordinary pitch. The weight of a cubic foot varies much; freshly fallen snow may weigh from 5 to 12 lbs.; snow and hail, sleet or ice may weigh from 30 to 50 lbs. per cubic foot, but the quantity on a roof will usually be small.

✓ 95. **Action of Materials under Stress.**—After the stresses in the frame are determined, the several parts must be designed to withstand them. While it is not the purpose of this work to explain minutely the method of proportioning the members of a truss and of working out the details, a few suggestions as to safe stresses and some points which should be borne in mind in designing may not be amiss.

As materials, if repeatedly strained to an amount at all approaching the breaking strain, will fail sooner or later, the

severe action weakening them, and as we must provide for unforeseen and unknown defects of material and workmanship, as well as for more or less of shock and vibration, it is customary to so proportion the several parts of a structure that they will be able to resist without failure much larger forces than those obtained from the stress diagrams. The smaller the load or stress on a piece the greater number of applications and removals before the piece is injured or broken. If the stress is reduced so much by increase of cross-section of the member that the piece will safely sustain an indefinitely great number of repetitions of it, such cross-section will be the proper one for a piece in a bridge or machine.

The stress arising from a stationary load, such as the weight of the structure, which is constant, is not so trying as repeated application and release of the same stress. The heavy wind-stresses determined in the previous chapters are not likely to occur more than once or twice, if at all, in the life of the structure. Hence good practice will authorize the employment of stresses some fifty per cent. in excess of those considered allowable in first-class bridge structures and those subjected to frequent change of load, to shock and vibration.

96. Allowable Stresses.—In accordance with this view, the following values may be used, where the wind-pressure of Chapter IV. has been allowed for.

Material.	Bending Stress.	Tension.	Compression with grain.	Compression across grain.	Shear with grain.
White Oak.....	1,600	1,500	1,400	400	180
Long-leaf Southern Pine....	1,600	1,400	1,400	300	150
Oregon Pine or Fir.....	1,600	1,800	1,300	250	200
White Pine (Eastern).....	1,400	800	1,200	200	100
Spruce	1,200	1,200	1,200	200	100
			Compression		Shear
Wrought Iron.	10,000	12,000	10,000		8,000
“ “ best quality..	12,000	15,000	12,000		10,000

The quality of the iron employed materially affects the force which it may safely resist

The above values must not be applied to parts subjected to moving loads, such as floor-beams and suspending rods for same, unless the load is moderate in total amount and very gradually applied and removed. For bridge work they must be reduced from 20 to 33 per cent.

97. Tension Members.—Pieces in tension will be liable to break at the smallest cross-section. It is therefore economical to enlarge the screw-ends of long iron rods and bolts so that the cross-section at the bottom of the threads shall be at least as large as at any other point. It is desirable that the centre of resistance of the cross-section of struts and ties shall coincide with the centre of figure, as a deviation from that position greatly weakens the piece. To calculate the net or smallest cross-section of a tension member where the pull is axial or central it is sufficient to divide the force by the safe working tensile stress. Allowance must be made for diminution of cross-section by any cutting away, bolt or rivet holes.

98. Compression Members.—For very short pieces or blocks in compression, whose lengths do not exceed six times the least dimension, the same process may be followed. But as the length increases the strut has a tendency to yield sideways when compressed, and the cross-section must be increased. The most comprehensive formula for such pieces is that known as Gordon's Formula. Letting l denote the length of the piece in inches, h its least external diameter in inches, S the cross-section in square inches, P the given force to be resisted in pounds, f the safe working compressive stress, and a a certain constant, this formula, for pieces with flat, securely bedded ends, or ends fixed in direction by bolting or riveting, may be written

$$P = \frac{fS}{1 + a \frac{l^2}{h^2}},$$

where $a = \frac{1}{4800}$ for rectangular timber struts, and $\frac{1}{30000}$ for rectangular wrought-iron struts.

If the struts are jointed at their ends by pin connections, or are so narrow as to readily yield sideways at these points, use $2a$ in place of a ; if one end is firmly fixed in direction while the other end is jointed use $\frac{3}{4}a$ in place of a .

It is convenient to assume h and compute S . If the other dimension then comes smaller than h , a less value must be taken for h than before, and the calculation made anew. For cross-sections in wrought iron not rectangles, such as L, T, and H sections, $1\frac{1}{2}l$ may be used in place of l .

Pieces subjected alternately to tension and compression should have a materially larger section than would be required for either stress alone.

Cast iron is not in favor with the best designers for any but short compression pieces, packing blocks and pedestals, although it is still employed for columns. The values in Gordon's Formula for cast iron may be $f = 15,000$; $a = \frac{1}{867}$.

99. Beams.—The values of f to be used in the moment of resistance, for pieces subjected to bending, are marked bending stress in the preceding table. In determining the moment of resistance of a piece exposed to bending, or in calculating the cross-section required at the point of maximum bending moment, allowance must be made for portions cut away on the tension side in attaching fastenings, bolting or riveting together parts, and also on the compression side unless the holes, etc., are so tightly filled that the compression can be fairly considered as resisted by those portions also.

Those pieces which resist both a bending moment and a direct stress may first be designed to safely carry the bending moment, and then the dimension transverse to that in which the piece will bend may be so much increased that the added slice will resist the direct pull or thrust. If that force is thrust, it will be well to test the size of the piece by Gordon's Formula also.

100. Pins and Eyes.—A reasonable rule for proportioning pins and eyes of tension bars is as follows:—Make the diameter of the pin from three-fourths to four-fifths of the width

of the bar in flats, and one and one-fourth times the diameter of the bar in rounds, giving the eye a sectional area of fifty per cent. in excess of that of the bar. The thickness of flat bars should be at least one-fourth of the width in order to secure a good bearing surface on the pin, and the metal at the eyes should be as thick as the bars. As the bending moment on a pin generally determines its diameter, pieces assembled on a pin should be packed closely, and those having opposing stresses should be brought into juxtaposition if possible.

101. Details.—Very close attention must be given to all minor details; to so proportion all the parts of a joint that it will be no more likely to yield in one way than another; to weaken as little as possible the pieces connected at a splice; to give sufficient bearing surface so as to bring the intensity of the compression on the surface within proper limits; to distribute rivets and bolts so as to give the greatest resistance with the least cutting away of other parts; to keep the action line of every piece as near its axis as possible; and to examine all sections and parts for tension, compression, and shear. The failure of a joint or connection is as fatal to a frame as to have a member too small for the stress upon it.

The pocket and hand-books issued by the different iron companies, for the use of their patrons, give the sections and weights of the various shapes of rolled iron, the safe loads for beams of different spans, data for columns, details of construction, and much miscellaneous useful information.



INDEX.

	PAGE		PAGE
ACTION of wind	23	Horizontal thrust indeterminate..	48
Allowable stresses.....	74	" " trusses with....	44
Analysis, order of.....	5	Howe truss.....	20
BEAMS, designing.....	76	IMAGINARY forces, use of.....	50, 58
Bending moment.....	61	Inclined forces, trusses under..	22, 33
" " formula.....	63	" reactions.....	7, 8
" " from equilibri-		Irregular section, moment of re-	
um polygon.....	66	sistance of.....	69
Bending moment on rafter.....	59	I section, moment of resistance,	70
Bracket truss.....	12	JOINTS, loads between.....	50, 59
CAMBERING lower tie, effect of...	14	" loads on all.....	14, 19
Change of diagonal.....	18, 42	KING-POST truss.....	9
Compression and tension, to dis-		LATERAL bracing.....	72
tinguish between.....	6, 32	Load and details.....	72
Compression members, designing	75	" between joints.....	50, 59
Curb roof, truss for.....	33	" on all joints.....	14, 19
" " without roller.....	34	Lower tie, effect of cambering....	14
" " with roller.....	36	MATERIALS under stress, action of,	73
Curved members.....	46	" weight of.....	72
Curved roof-truss.....	37	Method of trial and error.....	55
DETAILS.....	77	Moment, bending.....	61
Diagonal, change of.....	18, 42	Moment, formula for bending....	63
" reversal of.....	53	" from equilibrium poly-	
Diagonals in same quadrilateral,		gon.....	66
two.....	18	Moment of resistance.....	62
Distribution of load.....	15	" " " irregular sec-	
EQUILIBRIUM polygon.....	64 to 67	tion.....	69
Eyebars.....	76	Moment of resistance, I section..	70
Example, general.....	56	" " " rectangular	
FLAT roofs, trusses for.....	16	section.....	67
Fink truss.....	13, 54	Moment of resistance, T section..	68
Forces not applied to joints....	50, 59	Moments, reactions found by ..	17, 21
HAMMER-BEAM truss.....	46, 55	Moving load.....	21
" " " amount of		NOTATION.....	2
horizontal thrust.....	47	ORDER of analysis.....	5

	PAGE		PAGE
PINS.....	77	Triangular truss.....	7
Polonceau truss.....	18, 54	Truss conforming to shape of equilibrium polygon.....	16, 39
Polygon, equilibrium.....	64, 65	Truss for curb or mansard roof...	33
Pratt truss.....	20	Truss, Fink.....	13, 54
Pressure, wind.....	23	“ Hammer-beam.....	46, 55
Principle of reciprocity.....	6	“ Howe.....	20
QUEEN-POST truss.....	16	“ King-post.....	9
Rafter, bending moment on.....	59	“ Pratt.....	20
Railroad-station roof.....	53, 56	“ Polonceau.....	18, 54
Reactions found by moments, 17, 21, 24		“ Queen-post.....	16
Reactions from wind.....	24, 28, 35	“ Scissor.....	44
Reciprocity, principle of.....	6	“ Trapezoidal.....	16
Rectangle, moment of resistance,	67	“ Warren.....	19
Resistance, moment of.....	62	Truss to conform to roof.....	19
“ “ “ for various sections.....	67 to 70	“ with roller bearing.....	27, 36
Reversal of diagonal.....	53	Trusses for flat roofs.....	16
Roller bearing, effect of.....	27, 36	“ “ halls.....	18
Roof, truss to conform to.....	19	“ under vertical forces.....	7, 16
Roof-truss, wooden.....	10	“ “ inclined “.....	22, 33
SCISSOR truss.....	44	“ with horizontal thrust....	44
Snow diagram.....	33	T section, moment of resistance,	68
Snow, weight of.....	73	USE of two diagonals in quadri- lateral.....	18, 40
Special solutions.....	53	VERTICAL forces, trusses under..	7, 16
Stress, action of materials under..	73	WARREN girder.....	19
“ determining kind of.....	6, 32	Weight of materials.....	72
Stresses, allowable.....	74	Wind, action of.....	22
“ in triangular frame.....	3, 4	“ diagram, reactions, 24, 28, 29, 41	
Superfluous pieces.....	11	“ “ stresses, 25, 30, 34, 43	
TENSION and compression, to dis- tinguish between.....	6, 32	“ on alternate sides, change of stress.....	26, 31, 43
Tension members, designing.....	75	Wind-pressure.....	23
Three forces unknown.....	13, 53	“ “ on curb or man- sard roofs.....	33
Trapezoidal truss, equal loads....	16	Wind-pressure on curved roofs..	37
“ “ unequal loads, 16		“ “ “ pitched or ga- ble roofs.....	23
Trial and error, method of.....	55	Wooden roof-truss.....	10
Triangle of forces.....	1		
“ “ external forces.....	2, 4		

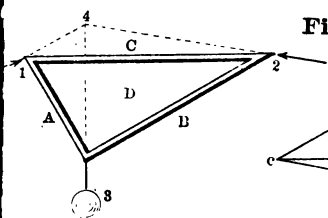
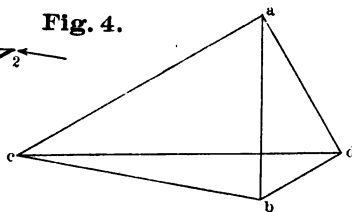
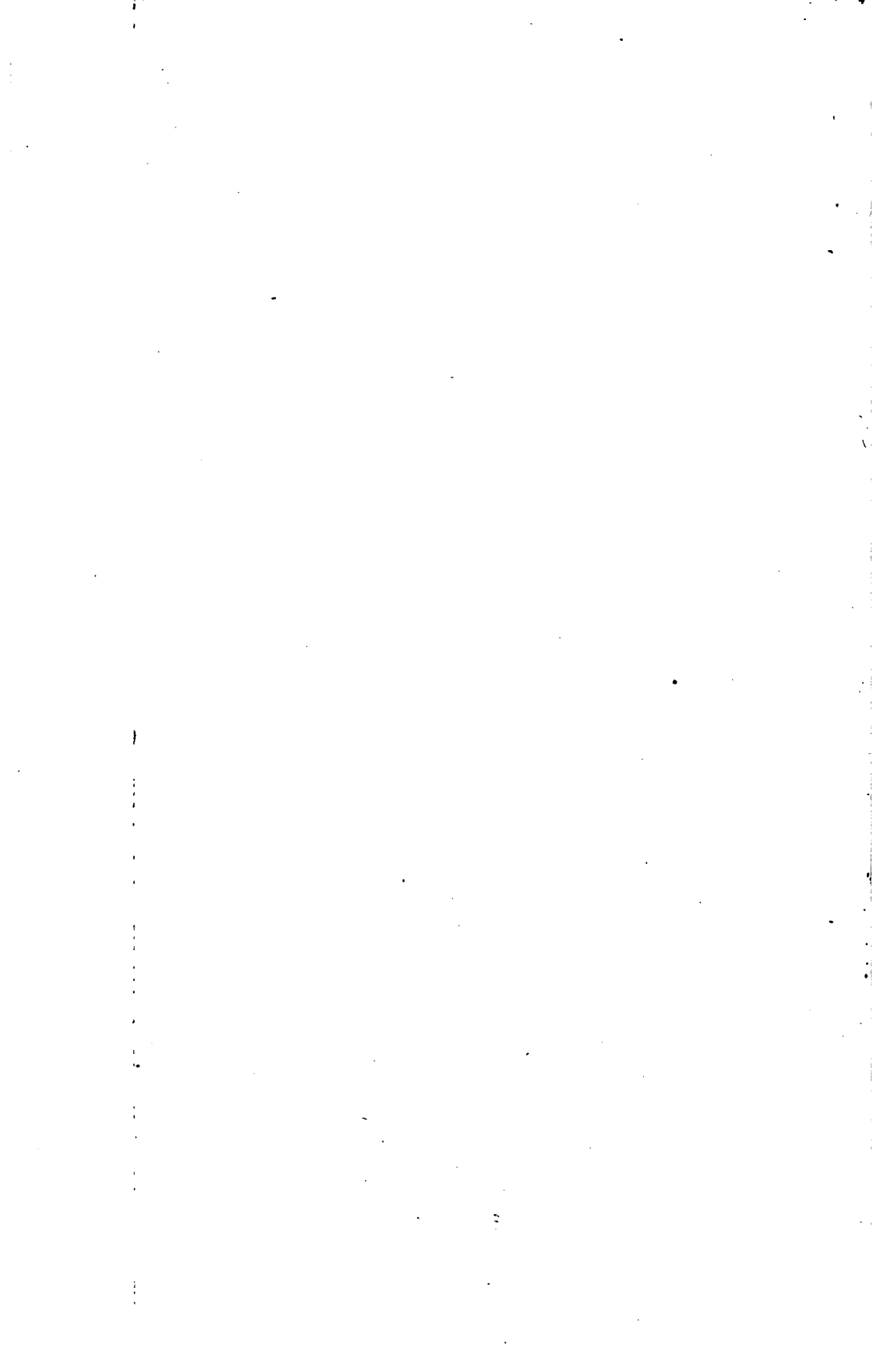


Fig. 4.





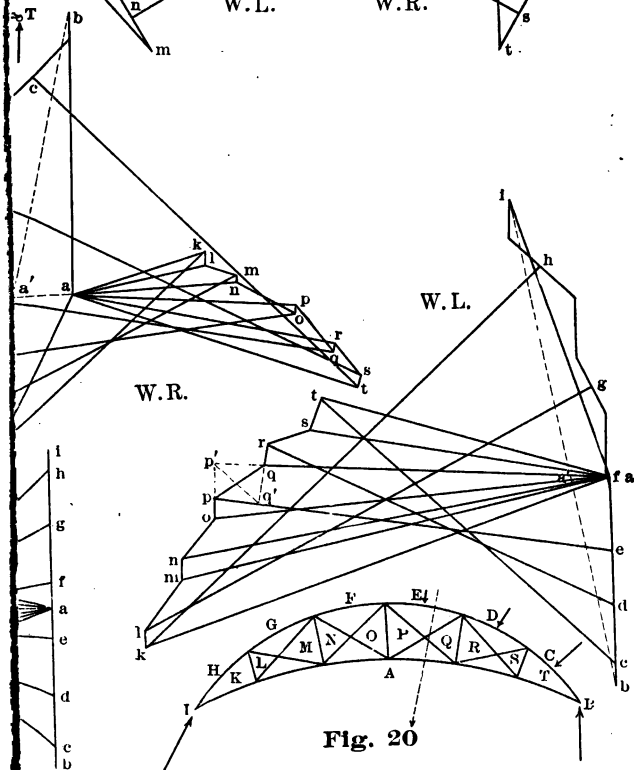


Fig. 20



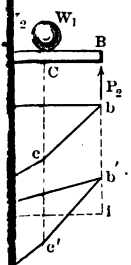
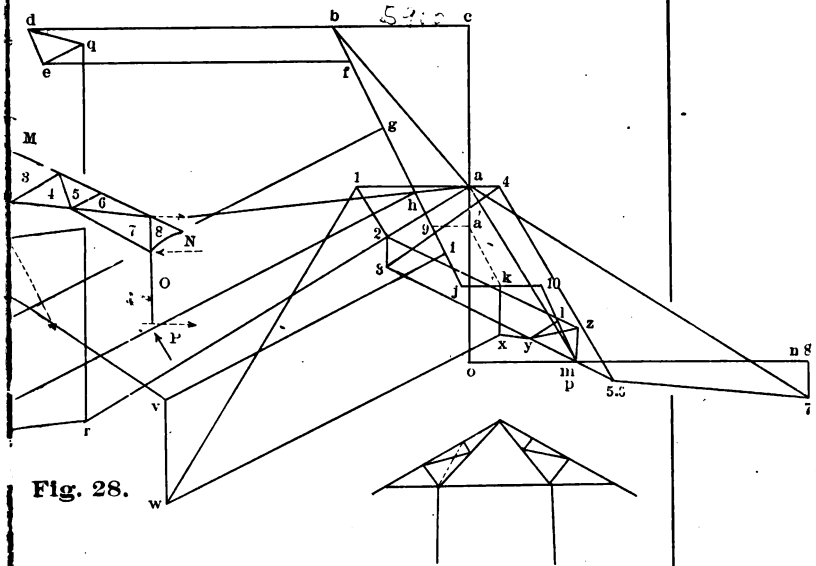
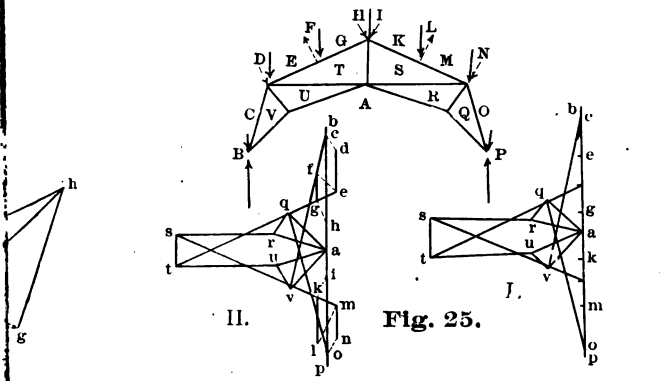
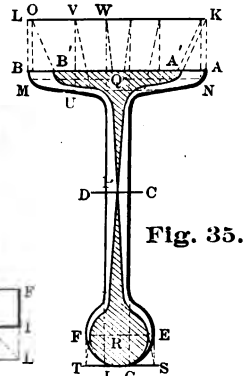
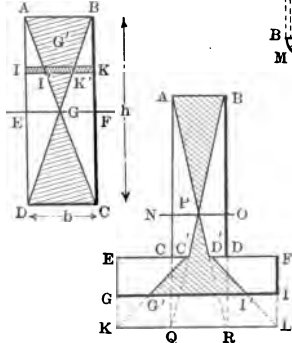
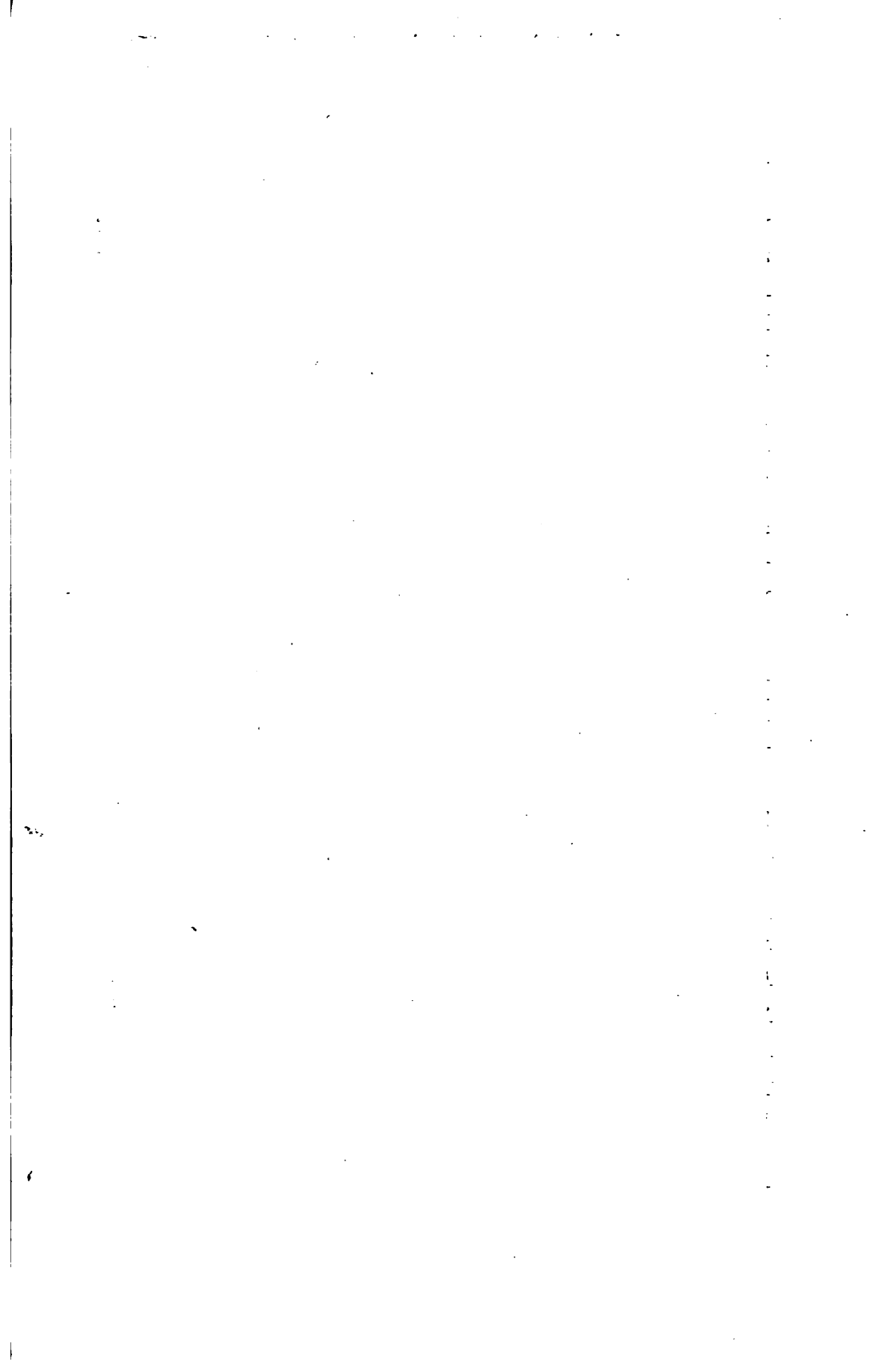
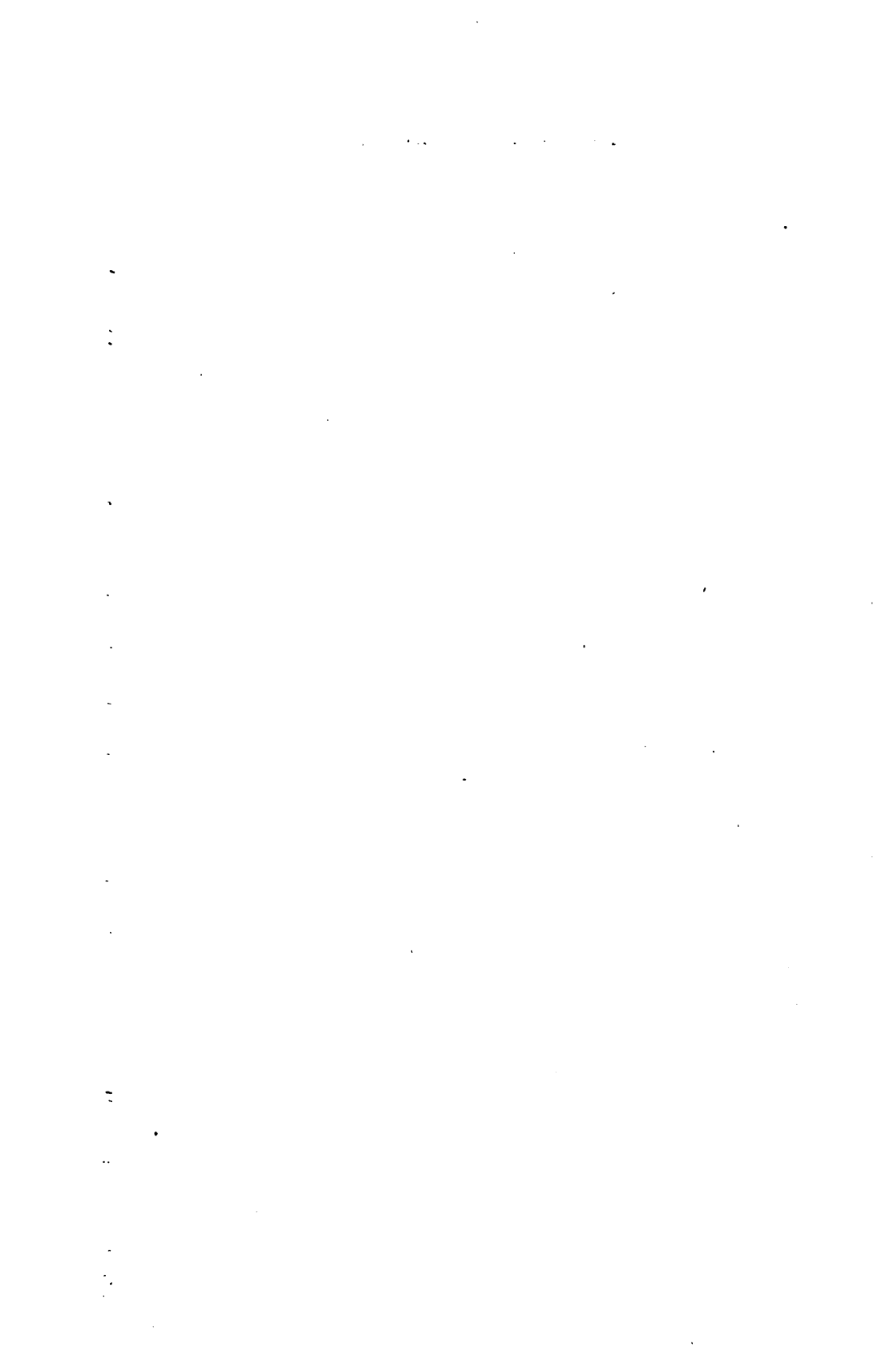
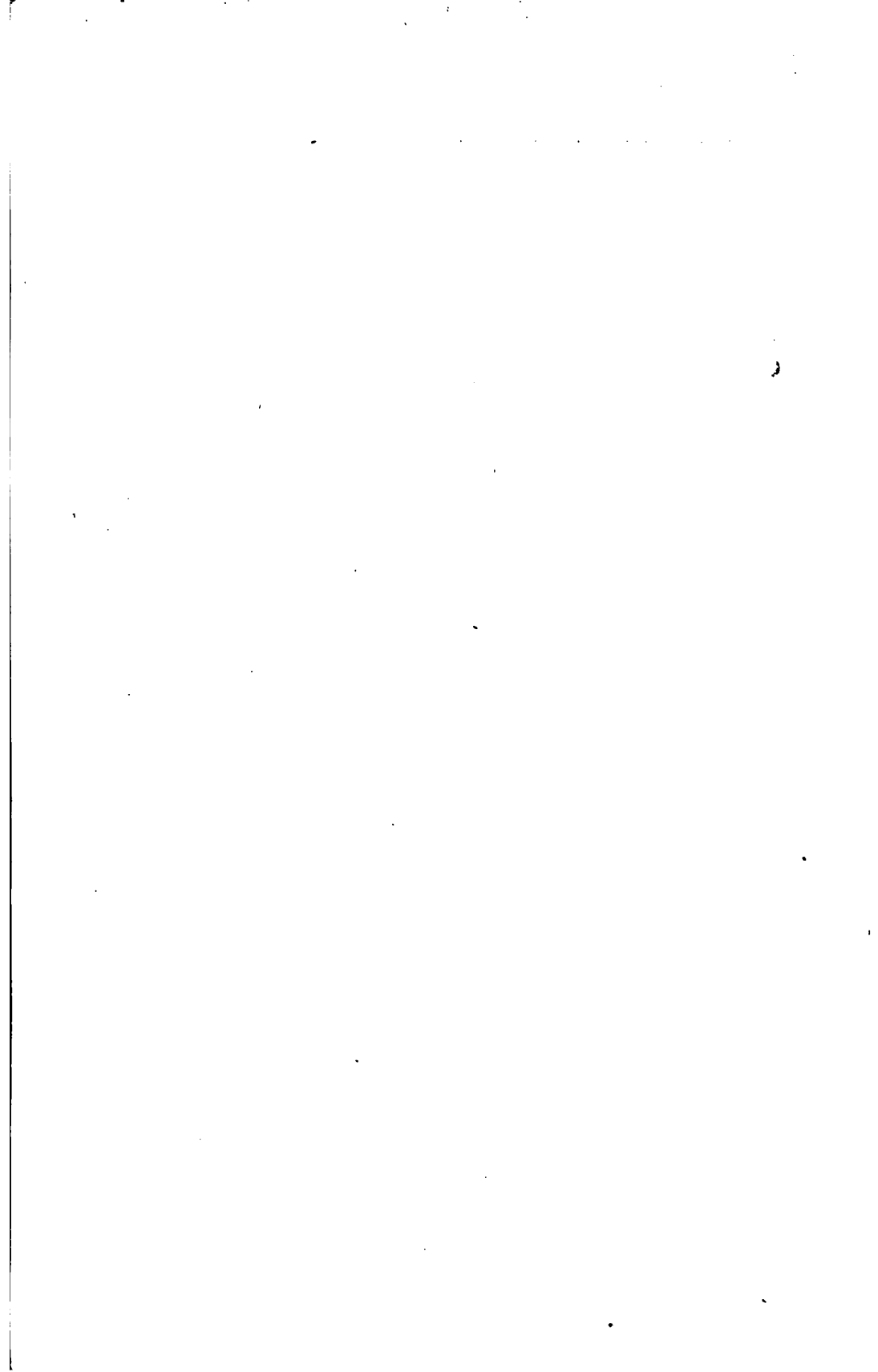


Fig. 33.











SHORT-TITLE CATALOGUE

OF THE
PUBLICATIONS
OF
JOHN WILEY & SONS,
NEW YORK.

LONDON: CHAPMAN & HALL, LIMITED.

ARRANGED UNDER SUBJECTS.

Descriptive circulars sent on application.

Books marked with an asterisk are sold at *net* prices only.

All books are bound in cloth unless otherwise stated.

AGRICULTURE.

CATTLE FEEDING—DAIRY PRACTICE—DISEASES OF ANIMALS— GARDENING, ETC.

Armsby's Manual of Cattle Feeding.....	12mo,	\$1 75
Downing's Fruit and Fruit Trees.....	8vo,	5 00
Grotenfelt's The Principles of Modern Dairy Practice. (Woll.)		
	12mo,	2 00
Kemp's Landscape Gardening.....	12mo,	2 50
Maynard's Landscape Gardening.....	12mo,	1 50
Steel's Treatise on the Diseases of the Dog.....	8vo,	3 50
“ Treatise on the Diseases of the Ox.....	8vo,	6 00
Stockbridge's Rocks and Soils.....	8vo,	2 50
Woll's Handbook for Farmers and Dairyemen.....	12mo,	1 50

ARCHITECTURE.

BUILDING—CARPENTRY—STAIRS—VENTILATION—LAW, ETC.

Berg's Buildings and Structures of American Railroads.....	4to,	7 50
Birkmire's American Theatres—Planning and Construction.....	8vo,	3 00
“ Architectural Iron and Steel.....	8vo,	3 50
“ Compound Riveted Girders.....	8vo,	2 00
“ Skeleton Construction in Buildings.....	8vo,	3 00

Birkmire's Planning and Construction of High Office Buildings.

	8vo,	\$3 50
Briggs' Modern Am. School Building	8vo,	4 00
Carpenter's Heating and Ventilating of Buildings.....	8vo,	3 00
Freitag's Architectural Engineering.....	8vo,	2 50
" The Fireproofing of Steel Buildings	8vo,	2 50
Gerhard's Sanitary House Inspection.....	16mo,	1 00
" Theatre Fires and Panics.....	12mo,	1 50
Hatfield's American House Carpenter.....	8vo,	5 00
Holly's Carpenter and Joiner..	18mo,	75
Kidder's Architect and Builder's Pocket-book...16mo, morocco,		4 00
Merrill's Stones for Building and Decoration.....	8vo,	5 00
Monckton's Stair Building—Wood, Iron, and Stone.....	4to,	4 00
Wait's Engineering and Architectural Jurisprudence.....	8vo,	6 00
	Sheep,	6 50
Worcester's Small Hospitals—Establishment and Maintenance, including Atkinson's Suggestions for Hospital Archi- tecture... ..	12mo,	1 25
World's Columbian Exposition of 1893.....	Large 4to,	2 50

ARMY, NAVY, Etc.

MILITARY ENGINEERING—ORDNANCE—LAW, ETC.

* Bruff's Ordnance and Gunnery.....	8vo,	6 00
Chase's Screw Propellers.....	8vo,	3 00
Cronkhite's Gunnery for Non-com. Officers....	32mo, morocco,	2 00
* Davis's Treatise on Military Law.....	8vo,	7 00
	Sheep,	7 50
* " Elements of Law.....	8vo,	2 50
De Brack's Cavalry Outpost Duties. (Carr.)...	32mo, morocco,	2 00
Dietz's Soldier's First Aid.....	16mo, morocco,	1 25
* Dredge's Modern French Artillery....	Large 4to, half morocco,	15 00
" Record of the Transportation Exhibits Building, World's Columbian Exposition of 1893..	4to, half morocco,	10 00
Durand's Resistance and Propulsion of Ships.....	8vo,	5 00
Dyer's Light Artillery.....	12mo,	3 00
Hoff's Naval Tactics.....	8vo,	1 50
* Ingalls's Ballistic Tables.....	8vo,	1 50

Ingalls's Handbook of Problems in Direct Fire.....	8vo,	\$4 00
Mahan's Permanent Fortifications. (Mercur.).....	8vo, half morocco,	7 50
* Mercur's Attack of Fortified Places.....	12mo,	2 00
* " Elements of the Art of War	8vo,	4 00
Metcalf's Ordnance and Gunnery.....	12mo, with Atlas,	5 00
Murray's A Manual for Courts-Martial.....	16mo, morocco,	1 50
" Infantry Drill Regulations adapted to the Springfield Rifle, Caliber .45.....	32mo, paper,	10
* Phelps's Practical Marine Surveying.....	8vo,	2 50
Powell's Army Officer's Examiner.....	12mo,	4 00
Sharpe's Subsisting Armies.....	32mo, morocco,	1 50
Wheeler's Siege Operations.....	8vo,	2 00
Winthrop's Abridgment of Military Law.....	12mo,	2 50
Woodhull's Notes on Military Hygiene.....	16mo,	1 50
Young's Simple Elements of Navigation.....	16mo, morocco,	2 00
" " " " " first edition.....		1 00

ASSAYING.

SMELTING—ORE DRESSING—ALLOYS, ETC.

Fletcher's Quant. Assaying with the Blowpipe.....	16mo, morocco,	1 50
Furman's Practical Assaying.....	8vo,	3 00
Kunhardt's Ore Dressing.....	8vo,	1 50
O'Driscoll's Treatment of Gold Ores.....	8vo,	2 00
Ricketts and Miller's Notes on Assaying.....	8vo,	3 00
Thurston's Alloys, Brasses, and Bronzes.....	8vo,	2 50
Wilson's Cyanide Processes.....	12mo,	1 50
" The Chlorination Process.....	12mo,	1 50

ASTRONOMY.

PRACTICAL, THEORETICAL, AND DESCRIPTIVE.

* Craig's Azimuth.....	4to,	3 50
Doolittle's Practical Astronomy.....	8vo,	4 00
Gore's Elements of Geodesy.....	8vo,	2 50
Hayford's Text-book of Geodetic Astronomy.....	8vo.	3 00
* Michie and Harlow's Practical Astronomy.....	8vo,	3 00
* White's Theoretical and Descriptive Astronomy.....	12mo,	2 00

BOTANY.

GARDENING FOR LADIES, ETC.

Baldwin's Orchids of New England.....	Small 8vo,	\$1 50
Thomé's Structural Botany.....	16mo,	2 25
Westermaier's General Botany. (Schneider.).....	8vo,	2 00

BRIDGES, ROOFS, Etc.

CANTILEVER—DRAW—HIGHWAY—SUSPENSION.

(See also ENGINEERING, p. 8.)

Boller's Highway Bridges.....	8vo,	2 00
* " The Thames River Bridge.....	4to, paper,	5 00
Burr's Stresses in Bridges.....	8vo,	3 50
Crehore's Mechanics of the Girder.....	8vo,	5 00
Dredge's Thames Bridges.....	7 parts, per part,	1 25
Du Bois's Stresses in Framed Structures.....	Small 4to,	10 00
Foster's Wooden Trestle Bridges.....	4to,	5 00
Greene's Arches in Wood, etc.....	8vo,	2 50
" Bridge Trusses.....	8vo,	2 50
" Roof Trusses.....	8vo,	1 25
Howe's Treatise on Arches	8vo,	4 00
Johnson's Modern Framed Structures.....	Small 4to,	10 00.
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part I., Stresses.....	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part II., Graphic Statics.....	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part III., Bridge Design.....	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part IV., Continuous, Draw, Cantilever, Suspension, and		
Arched Bridges.....	8vo,	2 50
* Morison's The Memphis Bridge.....	Oblong 4to,	10 00
Waddell's Iron Highway Bridges.....	8vo,	4 00
" De Pontibus (a Pocket-book for Bridge Engineers).		
	16mo, morocco,	3 00
Wood's Construction of Bridges and Roofs.....	8vo,	2 00
Wright's Designing of Draw Spans. Parts I. and II..	8vo, each	2 50
" " " " Complete.....	8vo,	3 50

CHEMISTRY—BIOLOGY—PHARMACY.

QUALITATIVE—QUANTITATIVE—ORGANIC—INORGANIC, ETC.

Adriance's Laboratory Calculations.....	12mo,	\$1 25
Allen's Tables for Iron Analysis.....	8vo,	3 00
Austen's Notes for Chemical Students.....	12mo,	1 50
Bolton's Student's Guide in Quantitative Analysis.....	8vo,	1 50
Boltwood's Elementary Electro Chemistry.....	(In the press.)	
Classen's Analysis by Electrolysis. (Herrick and Boltwood.)	8vo,	3 00
Cohn's Indicators and Test-papers.....	12mo	2 00
Crafts's Qualitative Analysis. (Schaeffer.).....	12mo,	1 50
Davenport's Statistical Methods with Special Reference to Bio- logical Variations.....	12mo, morocco,	1 25
Drechsel's Chemical Reactions. (Merrill.).....	12mo,	1 25
Fresenius's Quantitative Chemical Analysis. (Allen.).....	8vo,	6 00
“ Qualitative “ “ (Johnson.).....	8vo,	3 00
“ “ “ “ (Wells.) Trans.		
16th German Edition.....	8vo,	5 00
Fuertes's Water and Public Health.....	12mo,	1 50
Gill's Gas and Fuel Analysis.....	12mo,	1 25
Hammarsten's Physiological Chemistry. (Mandel.).....	8vo,	4 00
Helm's Principles of Mathematical Chemistry. (Morgan.)	12mo,	1 50
Ladd's Quantitative Chemical Analysis.....	12mo,	1 00
Landauer's Spectrum Analysis. (Tingle.).....	8vo,	3 00
Löb's Electrolysis and Electrosynthesis of Organic Compounds. (Lorenz.).....	12mo,	1 00
Mendel's Bio-chemical Laboratory.....	12mo,	1 50
Mason's Water-supply.....	8vo,	5 00
“ Examination of Water.....	12mo,	1 25
Meyer's Radicles in Carbon Compounds. (Tingle.).....	12mo,	1 00
Miller's Chemical Physics.....	8vo,	2 00
Mixter's Elementary Text-book of Chemistry.....	12mo,	1 50
Morgan's The Theory of Solutions and its Results.....	12mo,	1 00
“ Elements of Physical Chemistry.....	12mo,	2 00
Nichols's Water-supply (Chemical and Sanitary).....	8vo,	2 50
O'Brine's Laboratory Guide to Chemical Analysis.....	8vo,	2 00
Perkins's Qualitative Analysis.....	12mo,	1 00
Pinner's Organic Chemistry. (Austen.).....	12mo,	1 50

Poole's Calorific Power of Fuels	8vo,	\$3 00
Ricketts and Russell's Notes on Inorganic Chemistry (Non-metallic).....	Oblong 8vo, morocco,	75
Ruddiman's Incompatibilities in Prescriptions.....	8vo,	2 00
Schimpf's Volumetric Analysis.....	12mo,	2 50
Spencer's Sugar Manufacturer's Handbook.....	16mo, morocco,	2 00
Handbook for Chemists of Beet Sugar Houses.	16mo, morocco,	3 00
Stockbridge's Rocks and Soils.....	8vo,	2 50
* Tillman's Descriptive General Chemistry.....	8vo,	3 00
Van Deventer's Physical Chemistry for Beginners. (Boltwood.)	12mo,	1 50
Wells's Inorganic Qualitative Analysis.....	12mo,	1 50
" Laboratory Guide in Qualitative Chemical Analysis.	8vo,	1 50
Whipple's Microscopy of Drinking-water.....	8vo,	3 50
Wiechmann's Chemical Lecture Notes.....	12mo,	3 00
" Sugar Analysis.....	Small 8vo,	2 50
Wulling's Inorganic Phar. and Med. Chemistry.....	12mo,	2 00

DRAWING.

ELEMENTARY—GEOMETRICAL—MECHANICAL—TOPOGRAPHICAL.

Hill's Shades and Shadows and Perspective.....	8vo,	2 00
MacCord's Descriptive Geometry.....	8vo,	3 00
" Kinematics.....	8vo,	5 00
" Mechanical Drawing	8vo,	4 00
Mahan's Industrial Drawing. (Thompson.).....	2 vols., 8vo,	3 50
Reed's Topographical Drawing. (H. A.).....	4to,	5 00
Reid's A Course in Mechanical Drawing.....	8vo.	2 00
" Mechanical Drawing and Elementary Machine Design.	8vo. (<i>In the press.</i>)	
Smith's Topographical Drawing. (Macmillan.).....	8vo,	2 50
Warren's Descriptive Geometry.....	2 vols., 8vo,	3 50
" Drafting Instruments.....	12mo,	1 25
" Free-hand Drawing	12mo,	1 00
" Linear Perspective.....	12mo,	1 00
" Machine Construction	2 vols., 8vo,	7 50

Warren's Plane Problems.....	12mo,	\$1 25
“ Primary Geometry.....	12mo,	75
“ Problems and Theorems.....	8vo,	2 50
“ Projection Drawing.....	12mo,	1 50
Warren's Shades and Shadows.....	8vo,	3 00
“ Stereotomy—Stone-cutting.....	8vo,	2 50
Whelpley's Letter Engraving.....	12mo,	2 00

ELECTRICITY AND MAGNETISM.

ILLUMINATION—BATTERIES—PHYSICS—RAILWAYS.

Anthony and Brackett's Text-book of Physics. (Magie.) Small		
	8vo,	3 00
Anthony's Theory of Electrical Measurements.....	12mo,	1 00
Barker's Deep-sea Soundings.....	8vo,	2 00
Benjamin's Voltaic Cell.....	8vo,	3 00
" History of Electricity.....	8vo,	3 00
Classen's Analysis by Electrolysis. (Herrick and Boltwood.)	8vo,	3 00
Crehore and Squier's Experiments with a New Polarizing Photo- Chronograph.....	8vo,	3 00
Dawson's Electric Railways and Tramways. Small, 4to, half		
	morocco,	12 50
* Dredge's Electric Illuminations... 2 vols., 4to, half morocco,		25 00
" " " Vol. II.....	4to,	7 50
Gilbert's De magnete. (Mottelay.).....	8vo,	2 50
Holman's Precision of Measurements.....	8vo,	2 00
" Telescope-mirror-scale Method.....	Large 8vo,	75
Löb's Electrolysis and Electrosynthesis of Organic Compounds. (Lorenz.).....	12mo,	1 00
* Michie's Wave Motion Relating to Sound and Light.....	8vo,	4 00
Morgan's The Theory of Solutions and its Results.....	12mo,	1 00
Niaudet's Electric Batteries. (Fishback.).....	12mo,	2 50
Pratt and Alden's Street-railway Road-beds.....	8vo,	2 00
Reagan's Steam and Electric Locomotives.....	12mo,	2 00
Thurston's Stationary Steam Engines for Electric Lighting Pur- poses.....	8vo,	2 50
* Tillman's Heat.....	8vo,	1 50

ENGINEERING.

CIVIL—MECHANICAL—SANITARY, ETC.

(See also BRIDGES, p. 4; HYDRAULICS, p. 9; MATERIALS OF ENGINEERING, p. 10; MECHANICS AND MACHINERY, p. 12; STEAM ENGINES AND BOILERS, p. 14.)

Baker's Masonry Construction.....	8vo,	\$5 00
“ Surveying Instruments.....	12mo,	3 00
Black's U. S. Public Works.....	Oblong 4to,	5 00
Brooks's Street-railway Location.....	16mo, morocco,	1 50
Butts's Civil Engineers' Field Book.....	16mo, morocco,	2 50
Byrne's Highway Construction.....	8vo,	5 00
“ Inspection of Materials and Workmanship.....	16mo,	3 00
Carpenter's Experimental Engineering	8vo,	6 00
Church's Mechanics of Engineering—Solids and Fluids....	8vo,	6 00
“ Notes and Examples in Mechanics.....	8vo,	2 00
Crandall's Earthwork Tables.....	8vo,	1 50
“ The Transition Curve.....	16mo, morocco,	1 50
*Dredge's Penn. Railroad Construction, etc. Large 4to,		
	half morocco,	20 00
* Drinker's Tunnelling.....	4to, half morocco,	25 00
Eissler's Explosives—Nitroglycerine and Dynamite.....	8vo,	4 00
Folwell's Sewerage.....	8vo,	3 00
Fowler's Cofferdam Process for Piers.....	8vo,	2 50
Gerhard's Sanitary House Inspection.....	12mo,	1 00
Godwin's Railroad Engineer's Field-book.....	16mo, morocco,	2 50
Gore's Elements of Geodesy.....	8vo,	2 50
Howard's Transition Curve Field-book.....	16mo, morocco,	1 50
Howe's Retaining Walls (New Edition.).....	12mo,	1 25
Hudson's Excavation Tables. Vol. II.....	8vo,	1 00
Hutton's Mechanical Engineering of Power Plants.....	8vo,	5 00
“ Heat and Heat Engines.....	8vo,	5 00
Johnson's Materials of Construction.....	Large 8vo,	6 00
“ Theory and Practice*of Surveying.....	Small 8vo,	4 00
Kent's Mechanical Engineer's Pocket-book.....	16mo, morocco,	5 00
Kiersted's Sewage Disposal.....	12mo,	1 25
Mahan's Civil Engineering. (Wood.).....	8vo,	5 00
Merriman and Brook's Handbook for Surveyors....	16mo, mor.,	2 00
Merriman's Precise Surveying and Geodesy.....	8vo,	2 50
“ Retaining Walls and Masonry Dams.....	8vo,	2 00
“ Sanitary Engineering.....	8vo,	2 00
Nagle's Manual for Railroad Engineers.....	16mo, morocco,	3 00
Ogden's Sewer Design.....	12mo,	2 00
Patton's Civil Engineering.....	8vo, half morocco,	7 50

Patton's Foundations.....	8vo,	\$5 00
Pratt and Alden's Street-railway Road-beds.....	8vo,	2 00
Rockwell's Roads and Pavements in France.....	12mo,	1 25
Searles's Field Engineering	16mo, morocco,	3 00
" Railroad Spiral.....	16mo, morocco,	1 50
Siebert and Biggin's Modern Stone Cutting and Masonry...8vo,		1 50
Smart's Engineering Laboratory Practice.....	12mo,	2 50
Smith's Wire Manufacture and Uses.....	Small 4to,	3 00
Spalding's Roads and Pavements.....	12mo,	2 00
" Hydraulic Cement.....	12mo,	2 00
Taylor's Prismoidal Formulas and Earthwork.....	8vo,	1 50
Thurston's Materials of Construction.....	8vo,	5 00
* Trautwine's Civil Engineer's Pocket-book....	16mo, morocco,	5 00
* " Cross-section.....	Sheet,	25
* " Excavations and Embankments.....	8vo,	2 00
* " Laying Out Curves.....	12mo, morocco,	2 50
Waddell's De Pontibus (A Pocket-book for Bridge Engineers).		
	16mo, morocco,	3 00
Wait's Engineering and Architectural Jurisprudence.....	8vo,	6 00
	Sheep,	6 50
" Law of Field Operation in Engineering, etc.....	8vo.	
Warren's Stereotomy—Stone-cutting.....	8vo,	2 50
Webb's Engineering Instruments. New Edition. 16mo, morocco,		1 25
Wegmann's Construction of Masonry Dams.....	4to,	5 00
Wellington's Location of Railways.....	Small 8vo,	5 00
Wheeler's Civil Engineering.....	8vo,	4 00
Wolf's Windmill as a Prime Mover.....	8vo,	3 00

HYDRAULICS.

WATER-WHEELS—WINDMILLS—SERVICE PIPE—DRAINAGE, ETC.

(See also ENGINEERING, p. 8.)

Bazin's Experiments upon the Contraction of the Liquid Vein. (Trautwine.).....	8vo,	2 00
Bovey's Treatise on Hydraulics.....	8vo,	4 00
Coffin's Graphical Solution of Hydraulic Problems.....	12mo,	2 50
Ferrel's Treatise on the Winds, Cyclones, and Tornadoes...8vo,		4 00
Folwell's Water Supply Engineering.....	8vo,	4 00
Fuertes's Water and Public Health.....	12mo,	1 50
Ganguillet & Kutter's Flow of Water. (Hering & Trautwine.)		
	8vo,	4 00
Hazen's Filtration of Public Water Supply.....	8vo,	3 00
Herschel's 115 Experiments	8vo,	2 00

Kiersted's Sewage Disposal.....	12mo,	\$1 25
Mason's Water Supply.....	.8vo,	5 00
" Examination of Water.....	12mo,	1 25
Merriman's Treatise on Hydraulics.....	.8vo,	4 00
Nichols's Water Supply (Chemical and Sanitary).....	.8vo,	2 50
Wegmann's Water Supply of the City of New York.....	4to,	10 00
Weisbach's Hydraulics. (Du Bois.).....	.8vo,	5 00
Whipple's Microscopy of Drinking Water.....	.8vo,	3 50
Wilson's Irrigation Engineering.....	.8vo,	4 00
" Hydraulic and Placer Mining.....	12mo,	2 00
Wolff's Windmill as a Prime Mover.....	.8vo,	3 00
Wood's Theory of Turbines.....	.8vo,	2 50

MANUFACTURES.

BOILERS—EXPLOSIVES—IRON—STEEL—SUGAR—WOOLLENS, ETC.

Allen's Tables for Iron Analysis.....	.8vo,	3 00
Beaumont's Woollen and Worsted Manufacture.....	12mo,	1 50
Bolland's Encyclopædia of Founding Terms.....	12mo,	3 00
" The Iron Founder.....	12mo,	2 50
" " " " Supplement.....	12mo,	2 50
Bouvier's Handbook on Oil Painting.....	12mo,	2 00
Eissler's Explosives, Nitroglycerine and Dynamite.....	.8vo,	4 00
Ford's Boiler Making for Boiler Makers.....	18mo,	1 00
Metcalf's Cost of Manufactures.....	.8vo,	5 00
Metcalf's Steel—A Manual for Steel Users.....	12mo,	2 00
* Reisig's Guide to Piece Dyeing.....	.8vo,	25 00
Spencer's Sugar Manufacturer's Handbook.....	16mo, morocco,	2 00
" Handbook for Chemists of Beet Sugar Houses.....	16mo, morocco,	3 00
Thurston's Manual of Steam Boilers.....	.8vo,	5 00
Walke's Lectures on Explosives.....	.8vo,	4 00
West's American Foundry Practice.....	12mo,	2 50
" Moulder's Text-book.....	12mo,	2 50
Wiechmann's Sugar Analysis.....	Small 8vo,	2 50
Woodbury's Fire Protection of Mills.....	.8vo,	2 50

MATERIALS OF ENGINEERING.

STRENGTH—ELASTICITY—RESISTANCE, ETC.

(See also ENGINEERING, p. 8.)

Baker's Masonry Construction.....	.8vo,	5 00
Beardslee and Kent's Strength of Wrought Iron.....	.8vo,	1 50
Bovey's Strength of Materials.....	.8vo,	7 50
Burr's Elasticity and Resistance of Materials.....	.8vo,	5 00

Byrne's Highway Construction.....	8vo,	\$5 00
Church's Mechanics of Engineering—Solids and Fluids.....	8vo,	6 00
Du Bois's Stresses in Framed Structures.....	Small 4to,	10 00
Johnson's Materials of Construction.....	8vo,	6 00
Lanza's Applied Mechanics.....	8vo,	7 50
Martens's Testing Materials. (Henning.).....	2 vols., 8vo,	7 50
Merrill's Stones for Building and Decoration.....	8vo,	5 00
Merriman's Mechanics of Materials.....	8vo,	4 00
“ Strength of Materials.....	12mo,	1 00
Patton's Treatise on Foundations.....	8vo,	5 00
Rockwell's Roads and Pavements, in France.....	12mo,	1 25
Spalding's Roads and Pavements.....	12mo,	2 00
Thurston's Materials of Construction.....	8vo,	5 00
“ Materials of Engineering.....	3 vols., 8vo,	8 00
Vol. I., Non-metallic.....	8vo,	2 00
Vol. II., Iron and Steel.....	8vo,	3 50
Vol. III., Alloys, Brasses, and Bronzes.....	8vo,	2 50
Wood's Resistance of Materials.....	8vo,	2 00

MATHEMATICS.

CALCULUS—GEOMETRY—TRIGONOMETRY, ETC.

Baker's Elliptic Functions.....	8vo,	1 50
Barnard's Pyramid Problem.....	8vo,	1 50
*Bass's Differential Calculus.....	12mo,	4 00
Briggs's Plane Analytical Geometry.....	12mo,	1 00
Chapman's Theory of Equations.....	12mo,	1 50
Compton's Logarithmic Computations.....	12mo,	1 50
Davis's Introduction to the Logic of Algebra.....	8vo,	1 50
Halsted's Elements of Geometry.....	8vo,	1 75
“ Synthetic Geometry.....	8vo,	1 50
Johnson's Curve Tracing.....	12mo,	1 00
“ Differential Equations—Ordinary and Partial.		
	Small 8vo,	3 50
“ Integral Calculus.....	12mo,	1 50
“ “ “ Unabridged. Small 8vo.		
	(In the press.)	
“ Least Squares.....	12mo,	1 50
*Ludlow's Logarithmic and Other Tables. (Bass.).....	8vo,	2 00
* “ Trigonometry with Tables. (Bass.).....	8vo,	3 00
*Mahan's Descriptive Geometry (Stone Cutting).....	8vo,	1 50
Merriman and Woodward's Higher Mathematics.....	8vo,	5 00
Merriman's Method of Least Squares.....	8vo,	2 00
Rice and Johnson's Differential and Integral Calculus,		
	2 vols. in 1, small 8vo,	2 50

Rice and Johnson's Differential Calculus.....	Small 8vo,	\$3 00
“ Abridgment of Differential Calculus.		
	Small 8vo,	1 50
Totten's Metrology.....	.8vo,	2 50
Warren's Descriptive Geometry.....	2 vols., 8vo,	3 50
“ Drafting Instruments.....	12mo,	1 25
“ Free-hand Drawing.....	12mo,	1 00
“ Linear Perspective.....	12mo,	1 00
“ Primary Geometry.....	12mo,	75
“ Plane Problems.....	12mo,	1 25
“ Problems and Theorems.....	.8vo,	2 50
“ Projection Drawing.....	12mo,	1 50
Wood's Co-ordinate Geometry.....	.8vo,	2 00
“ Trigonometry.....	12mo,	1 00
Woolf's Descriptive Geometry.....	Large 8vo,	3 00

MECHANICS—MACHINERY.

TEXT-BOOKS AND PRACTICAL WORKS.

(See also ENGINEERING, p. 8.)

Baldwin's Steam Heating for Buildings.....	12mo,	2 50
Barr's Kinematics of Machinery.....	.8vo,	2 50
Benjamin's Wrinkles and Recipes	12mo,	2 00
Chordal's Letters to Mechanics.....	12mo,	2 00
Church's Mechanics of Engineering.....	.8vo,	6 00
“ Notes and Examples in Mechanics.....	.8vo,	2 00
Crehore's Mechanics of the Girder.....	.8vo,	5 00
Cromwell's Belts and Pulleys.....	12mo,	1 50
“ Toothed Gearing.....	12mo,	1 50
Compton's First Lessons in Metal Working.....	12mo,	1 50
Compton and De Groodt's Speed Lathe.....	12mo,	1 50
Dana's Elementary Mechanics	12mo,	1 50
Dingey's Machinery Pattern Making.....	12mo,	2 00
Dredge's Trans. Exhibits Building, World Exposition.		
	Large 4to, half morocco,	10 00
Du Bois's Mechanics. Vol. I., Kinematics8vo,	3 50
“ “ Vol. II., Statics.....	.8vo,	4 00
“ “ Vol. III., Kinetics.....	.8vo,	3 50
Fitzgerald's Boston Machinist.....	18mo,	1 00
Flather's Dynamometers.....	12mo,	2 00
“ Rope Driving.....	12mo,	2 00
Hall's Car Lubrication.....	12mo,	1 00
Holly's Saw Filing	18mo,	75
Johnson's Theoretical Mechanics. An Elementary Treatise.		
(In the press.)		
Jones's Machine Design. Part I., Kinematics.....	.8vo,	1 50

Jones's Machine Design. Part II., Strength and Proportion of Machine Parts.....	8vo,	\$3 00
Lanza's Applied Mechanics	8vo,	7 50
MacCord's Kinematics.....	8vo,	5 00
Merriman's Mechanics of Materials.....	8vo,	4 00
Metcalf's Cost of Manufactures.....	8vo,	5 00
*Michie's Analytical Mechanics.....	8vo,	4 00
Richards's Compressed Air.....	12mo,	1 50
Robinson's Principles of Mechanism.....	8vo,	3 00
Smith's Press-working of Metals.....	8vo,	3 00
Thurston's Friction and Lost Work.....	8vo,	3 00
" The Animal as a Machine	12mo,	1 00
Warren's Machine Construction.....	2 vols., 8vo,	7 50
Weisbach's Hydraulics and Hydraulic Motors. (Du Bois.)..	8vo,	5 00
" Mechanics of Engineering. Vol. III., Part I., Sec. I. (Klein.).....	8vo,	5 00
Weisbach's Mechanics of Engineering. Vol. III., Part I., Sec. II. (Klein.).....	8vo,	5 00
Weisbach's Steam Engines. (Du Bois.).....	8vo,	5 00
Wood's Analytical Mechanics.....	8vo,	3 00
" Elementary Mechanics.....	12mo,	1 25
" " " Supplement and Key.....	12mo,	1 25

METALLURGY.

IRON—GOLD—SILVER—ALLOYS, ETC.

Allen's Tables for Iron Analysis.....	8vo,	3 00
Egleston's Gold and Mercury.....	Large 8vo,	7 50
" Metallurgy of Silver.....	Large 8vo,	7 50
* Kerl's Metallurgy—Copper and Iron.....	8vo,	15 00
* " " Steel, Fuel, etc.....	8vo,	15 00
Kunhardt's Ore Dressing in Europe.....	8vo,	1 50
Metcalf's Steel—A Manual for Steel Users.....	12mo,	2 00
O'Driscoll's Treatment of Gold Ores.....	8vo,	2 00
Thurston's Iron and Steel.....	8vo,	3 50
" Alloys.....	8vo,	2 50
Wilson's Cyanide Processes.....	12mo,	1 50

MINERALOGY AND MINING.

MINE ACCIDENTS—VENTILATION—ORE DRESSING, ETC.

Barringer's Minerals of Commercial Value....	Oblong morocco,	2 50
Beard's Ventilation of Mines.....	12mo,	2 50
Boyd's Resources of South Western Virginia.....	8vo,	3 00
" Map of South Western Virginia.....	Pocket-book form,	2 00
Brush and Penfield's Determinative Mineralogy. New Ed.	8vo,	4 00

Chester's Catalogue of Minerals.....	8vo,	\$1 25
“ “ “ “	Paper,	50
“ Dictionary of the Names of Minerals.....	8vo,	3 00
Dana's American Localities of Minerals.....	Large 8vo,	1 00
“ Descriptive Mineralogy. (E. S.) Large 8vo. half morocco,		12 50
“ First Appendix to System of Mineralogy. ...	Large 8vo,	1 00
“ Mineralogy and Petrography. (J. D.).....	12mo,	2 00
“ Minerals and How to Study Them. (E. S.).....	12mo,	1 50
“ Text-book of Mineralogy. (E. S.)...New Edition. 8vo,		4 00
* Drinker's Tunnelling, Explosives, Compounds, and Rock Drills.		
	4to, half morocco,	25 00
Egleston's Catalogue of Minerals and Synonyms.....	8vo,	2 50
Eissler's Explosives—Nitroglycerine and Dynamite.....	8vo,	4 00
Hussak's Rock-forming Minerals. (Smith.).....	Small 8vo,	2 00
Ihlseng's Manual of Mining.. ..	8vo,	4 00
Kunhardt's Ore Dressing in Europe.....	8vo,	1 50
O'Driscoll's Treatment of Gold Ores.....	8vo,	2 00
* Penfield's Record of Mineral Tests.....	Paper, 8vo,	50
Rosenbusch's Microscopical Physiography of Minerals and		
Rocks. (Iddings.).....	8vo,	5 00
Sawyer's Accidents in Mines.....	Large 8vo,	7 00
Stockbridge's Rocks and Soils.....	8vo,	2 50
Walke's Lectures on Explosives.....	8vo,	4 00
Williams's Lithology.....	8vo,	3 00
Wilson's Mine Ventilation.....	12mo,	1 25
“ Hydraulic and Placer Mining.....	12mo,	2 50

STEAM AND ELECTRICAL ENGINES, BOILERS, Etc.

STATIONARY—MARINE—LOCOMOTIVE—GAS ENGINES, ETC.

(See also ENGINEERING, p. 8.)

Baldwin's Steam Heating for Buildings.....	12mo,	2 50
◀Clerk's Gas Engine.....	Small 8vo,	4 00
Ford's Boiler Making for Boiler Makers.....	18mo,	1 00
Hemenway's Indicator Practice.....	12mo,	2 00
Hoadley's Warm-blast Furnace.....	8vo,	1 50
Kneass's Practice and Theory of the Injector	8vo,	1 50
MacCord's Slide Valve.....	8vo,	2 00
Meyer's Modern Locomotive Construction.....	4to,	10 00
Peabody and Miller's Steam-boilers.....	8vo,	4 00
Peabody's Tables of Saturated Steam.....	8vo,	1 00
“ Thermodynamics of the Steam Engine.....	8vo,	5 00
“ Valve Gears for the Steam Engine.....	8vo,	2 50
Pray's Twenty Years with the Indicator.....	Large 8vo,	2 50
Pupin and Osterberg's Thermodynamics:.....	12mo,	1 25

Reagan's Steam and Electric Locomotives.....	12mo,	\$2 00
Röntgen's Thermodynamics. (Du Bois.).....	8vo,	5 00
Sinclair's Locomotive Running.....	12mo,	2 00
Snow's Steam-boiler Practice.....	8vo,	3 00
Thurston's Boiler Explosions.....	12mo,	1 50
" Engine and Boiler Trials.....	8vo,	5 00
" Manual of the Steam Engine. Part I., Structure and Theory.....	8vo,	6 00
" Manual of the Steam Engine. Part II., Design, Construction, and Operation.....	8vo,	6 00
	2 parts,	10 00
Thurston's Philosophy of the Steam Engine.....	12mo,	75
" Reflection on the Motive Power of Heat. (Carnot.)	12mo,	1 50
" Stationary Steam Engines.....	8vo,	2 50
" Steam-boiler Construction and Operation.....	8vo,	5 00
Spangler's Valve Gears.....	8vo,	2 50
Weisbach's Steam Engine. (Du Bois.).....	8vo,	5 00
Whitham's Constructive Steam Engineering.....	8vo,	6 00
" Steam-engine Design.....	8vo,	5 00
Wilson's Steam Boilers. (Flather.).....	12mo,	2 50
Wood's Thermodynamics, Heat Motors, etc.....	8vo,	4 00

TABLES, WEIGHTS, AND MEASURES.

FOR ACTUARIES, CHEMISTS, ENGINEERS, MECHANICS—METRIC TABLES, ETC.

Adriance's Laboratory Calculations.....	12mo,	1 25
Allen's Tables for Iron Analysis.....	8vo,	3 00
Bixby's Graphical Computing Tables.....	Sheet,	25
Compton's Logarithms.....	12mo,	1 50
Crandall's Railway and Earthwork Tables.....	8vo,	1 50
Egleston's Weights and Measures.....	18mo,	75
Fisher's Table of Cubic Yards.....	Cardboard,	25
Hudson's Excavation Tables. Vol. II.....	8vo,	1 00
Johnson's Stadia and Earthwork Tables.....	8vo,	1 25
Ludlow's Logarithmic and Other Tables. (Bass.).....	12mo,	2 00
Totten's Metrology.....	8vo,	2 50

VENTILATION.

STEAM HEATING—HOUSE INSPECTION—MINE VENTILATION.

Baldwin's Steam Heating.....	12mo,	2 50
Beard's Ventilation of Mines.....	12mo,	2 50
Carpenter's Heating and Ventilating of Buildings.....	8vo,	3 00
Gerhard's Sanitary House Inspection.....	12mo,	1 00
Wilson's Mine Ventilation.....	12mo,	1 25

MISCELLANEOUS PUBLICATIONS.

Alcott's Gems, Sentiment, Language.....	Gilt edges,	\$5 00
Davis's Elements of Law.....	.8vo,	2 00
Emmon's Geological Guide-book of the Rocky Mountains..	.8vo,	1 50
Ferrel's Treatise on the Winds.....	.8vo,	4 00
Haines's Addresses Delivered before the Am. Ry. Assn....	12mo,	2 50
Mott's The Fallacy of the Present Theory of Sound..	Sq. 16mo,	1 00
Richards's Cost of Living.....	12mo,	1 00
Ricketts's History of Rensselaer Polytechnic Institute....	.8vo,	3 00
Rotherham's The New Testament Critically Emphasized.	12mo,	1 50
“ The Emphasized New Test. A new translation.	Large 8vo,	2 00
Totten's An Important Question in Metrology.....	.8vo,	2 50
* Wiley's Yosemite, Alaska, and Yellowstone	4to,	3 00

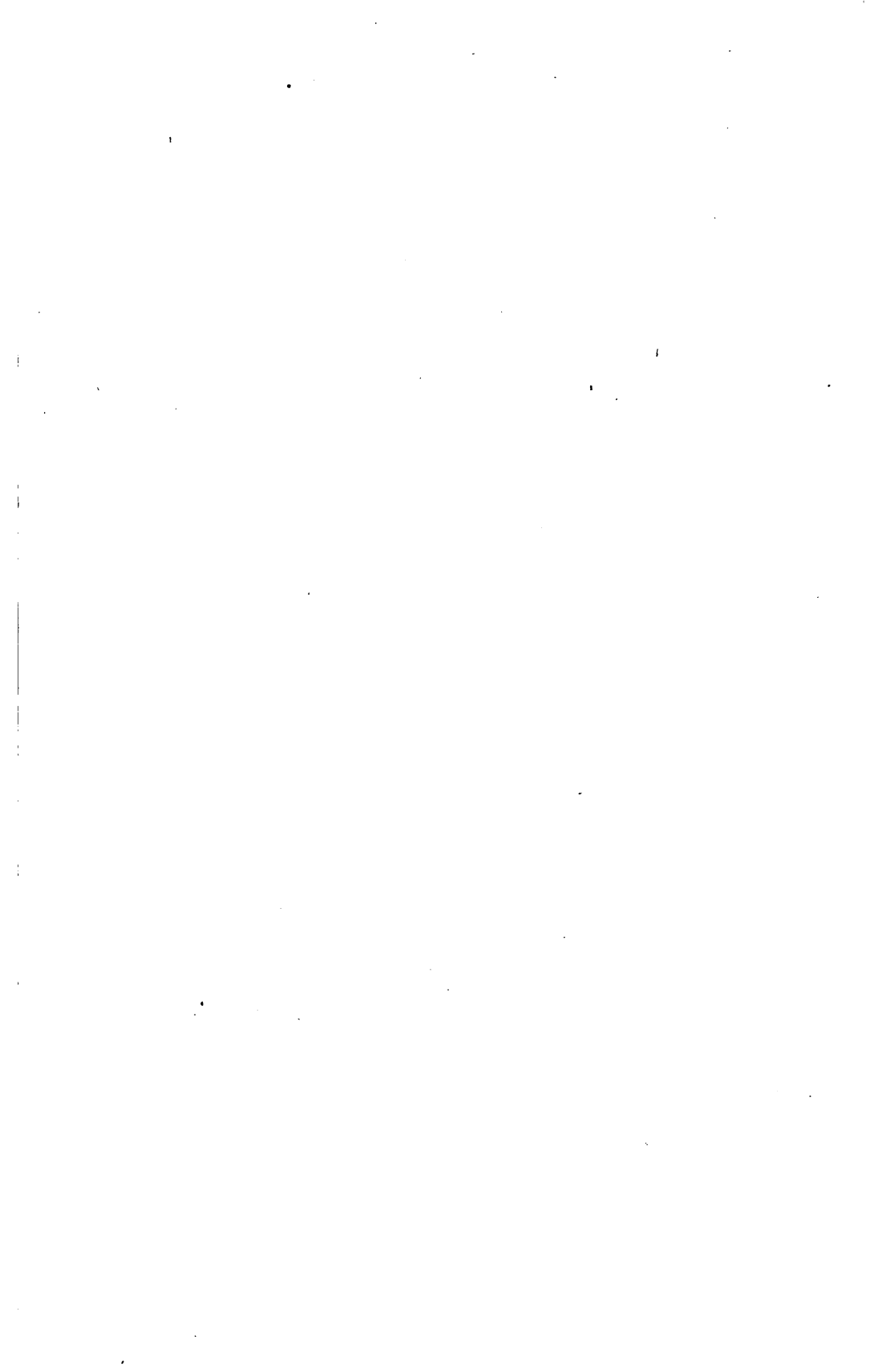
HEBREW AND CHALDEE TEXT-BOOKS.

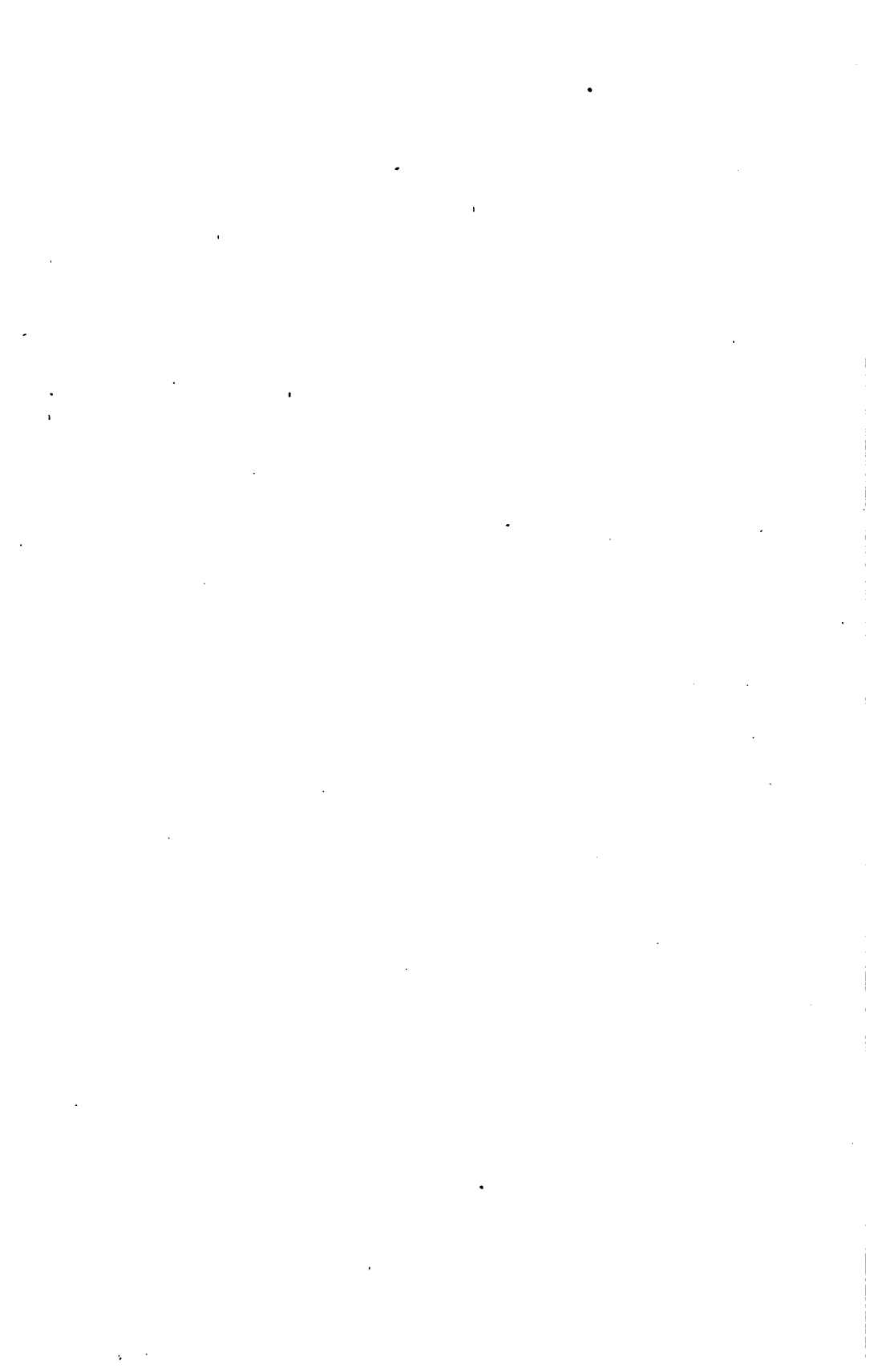
FOR SCHOOLS AND THEOLOGICAL SEMINARIES.

Gesenius's Hebrew and Chaldee Lexicon to Old Testament. (Tregelles.).....	Small 4to, half morocco,	5 00
Green's Elementary Hebrew Grammar.....	12mo,	1 25
“ Grammar of the Hebrew Language (New Edition).	.8vo,	3 00
“ Hebrew Chrestomathy.....	.8vo,	2 00
Letteris's Hebrew Bible (Massoretic Notes in English).	8vo, arabesque,	2 25

MEDICAL.

Hammarsten's Physiological Chemistry. (Mandel.).....	.8vo,	4 00
Mott's Composition, Digestibility, and Nutritive Value of Food.	Large mounted chart,	1 25
Ruddiman's Incompatibilities in Prescriptions.....	.8vo,	2 00
Steel's Treatise on the Diseases of the Ox....	.8vo,	6 00
“ Treatise on the Diseases of the Dog.....	.8vo,	3 50
Woodhull's Military Hygiene.....	16mo,	1 50
Worcester's Small Hospitals—Establishment and Maintenance, including Atkinson's Suggestions for Hospital Archi- tecture.....	12mo,	1 25





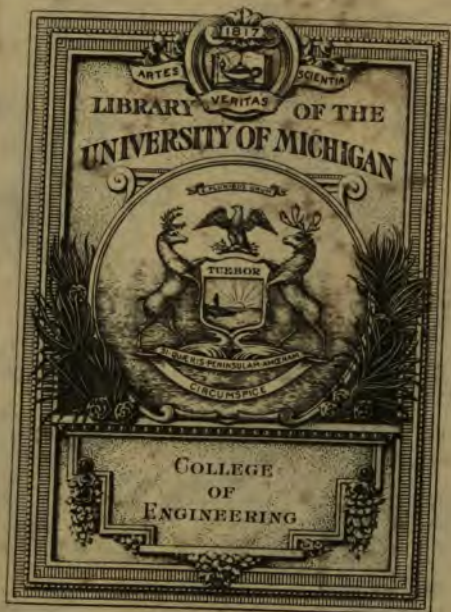
125





Gift

B 465293





When you have passed the first
point which has positive area,
the paper, since you have passed
the first point which has a zero

Engin. Library

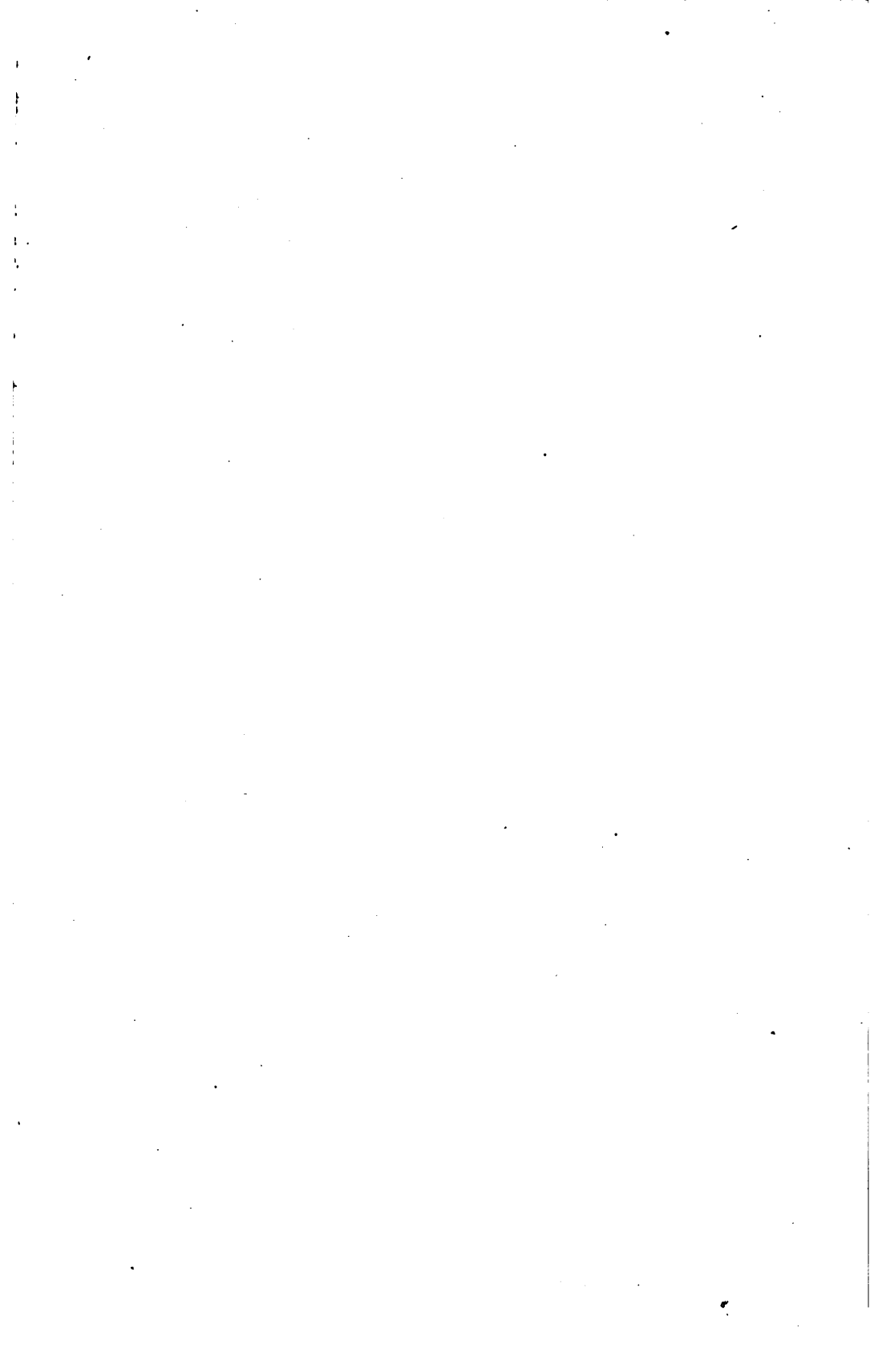
TG

270

.08

1900





WORKS OF PROF. C. E. GREENE

PUBLISHED BY

JOHN WILEY & SONS.

Graphics for Engineers, Architects, and Builders.

A Manual for Designers, and a Text-book for Scientific Schools.

Trusses and Arches. Analyzed and Discussed by Graphical Methods. In THREE PARTS.

PART I. Roof Trusses. Diagrams for Steady Load, Snow, and Wind. 8vo, cloth, \$1.25.

PART II. Bridge Trusses. Single, Continuous, and Draw Spans; Single and Multiple Systems; Straight and Inclined Chords. 8vo, cloth, \$2.50.

PART III. Arches in Wood, Iron, and Stone. For Roofs, Bridges, and Wall Openings; Arched Ribs and Braced Arches; Stresses from Wind and Change of Temperature. 8vo, cloth, \$2.50.

Structural Mechanics: The Action of Materials Under Stress. A work on the Strength and Resistance of Materials and the Elements of Structural Design. Ann Arbor, Mich., 1897. Printed for the author. 8vo, 300 pp., 100 illustrations. Price \$3.00.

Graphics for Engineers, Architects, and Builders :
A MANUAL FOR DESIGNERS, AND A TEXT-BOOK FOR SCIENTIFIC SCHOOLS.

TRUSSES AND ARCHES

ANALYZED AND DISCUSSED BY GRAPHICAL METHODS

BY
CHARLES E. GREENE, A.M.,
PROFESSOR OF CIVIL ENGINEERING, UNIVERSITY OF MICHIGAN.

IN THREE PARTS.

I.

ROOF-TRUSSES: DIAGRAMS FOR STEADY LOAD, SNOW, AND WIND.

II.

BRIDGE-TRUSSES: SINGLE, CONTINUOUS, AND DRAW SPANS; SINGLE AND MULTIPLE SYSTEMS; STRAIGHT AND INCLINED CHORDS.

III.

ARCHES, IN WOOD, IRON, AND STONE, FOR ROOFS, BRIDGES, AND WALL-OPENINGS; ARCHED RIBS AND BRACED ARCHES; STRESSES FROM WIND, AND CHANGE OF TEMPERATURE.

PART II.

BRIDGE-TRUSSES.

TEN FOLDING PLATES.

FIFTH EDITION REVISED.

FIRST THOUSAND.

NEW YORK
JOHN WILEY & SONS
LONDON
CHAPMAN & HALL, LTD.

1899

COPYRIGHT, 1879,
BY CHARLES E. GREENE.

Braunworth, Munn & Barber
Printers and Binders.
Brooklyn, N. Y.

eng. in. lib.
helps
9-25-1935
add. copy.

GENERAL PREFACE.

THERE is not the necessity which would have existed a few years ago to write a few words explanatory of graphical methods of analysis, and to urge their accuracy, convenience, and adaptability to all types of structure. If, however, some readers now glance at such subjects for the first time, it will be sufficient to point out, that, as forces may be represented by straight lines of definite magnitude and direction, the same operations which are performed by mathematical analysis may be more easily carried out by geometrical construction upon the drawing-board; and, as the whole system is founded upon the parallelogram of forces, the results deduced by a brief chain of reasoning are theoretically accurate, and only depend for their numerical exactness upon the scale of the diagrams, and the care with which they are drawn. Any draughtsman who gives the method a fair trial, and then takes the trouble to compare the results with those obtained by mathematical formulæ, will be surprised to see how closely the two agree. The difference is much less than can be taken account of in designing the proportions of the several parts of the structure. The simplest tools alone are needed, — drawing-board, T-square, triangles, and scale.

This work is intended for office use by any designer who may have occasion to prepare plans for a structure intended to support a load above any opening,—a roof, or bridge-truss, or an arch,—of any type, span, and distribution of load. It is also hoped that these three parts may be found serviceable as a text-book for engineering and architectural students, and may aid in clearing away some of the obscurity which is apt to surround certain parts of the subjects treated in the following pages, especially those of continuous bridges and arches. From his own experience as an engineer and as an instructor, the author has been led to go more into detail than an ordinary expansion of the methods would require, and in some cases to repeat what has been explained earlier; for he has found that both the practical man, taking up the subjects at intervals as required for use, and the student, finding so much that is new, need a certain amount of repetition to fix the principles clearly in mind. It will be found that students generally grasp readily, and evince a strong liking for, graphical methods. Mathematical and graphical analysis go well hand in hand, the latter aiding the student to more clearly apprehend the meaning of the former. Indeed, the mathematical formulæ may be, if desired, deduced directly from the diagrams. If sufficient time is allowed between the class-room exercises for the construction in the drawing-room of problems suggested by the text, the results accruing from this study will be most satisfactory.

Some special remarks in regard to subjects treated will be found prefacing the several parts.

C. E. G.

PREFACE TO PART II.

THE general method of analysis called "The Method by Area Moments," which is the foundation of the following pages, was first printed in 1874 in "Graphical Method for the Analysis of Bridge-Trusses," &c., having been discovered and taught by the author the year preceding. Part I., discussing roof-trusses, was founded upon what is known as Professor Clerk-Maxwell's method of drawing diagrams, and has little which is original, except the carrying-out of details. It is thought that Part II. will be found to contain much which is new, and, it is hoped, valuable. That this part is not a second edition, and simply a reprint, of the book above referred to, will be seen, when it is noticed that there is more than twice the amount of text, and sixty-three in place of fifteen illustrative figures. Chapter III. is devoted to an analysis of various types of trusses with horizontal chords having single systems of bracing, and is almost all new. The treatment of multiple systems of bracing, comprising Chapter IV., is entirely new. The construction for the bowstring girder, and the chapter on deflection of beams, are here introduced for the first time: partially continuous trusses and pivot-draws, with turn-table tipping or stable, have also been added.

As it is now four years since this method of area moments for the analysis of continuous girders was first given to the public by the author, and as no statement that such a method can be found in any other place has appeared, the author feels warranted in putting forth a claim for priority of discovery and originality. The attention of the reader is asked to the extreme simplicity and the generality of the formulæ here deduced for pier moments in continuous girders and draw-spans, and to the facility and brevity with which all the usual formulæ of the text-books, simple or intricate, for the slope and deflection of beams of one or more spans, are derived without the use of the integral calculus, as usually understood. These results have never been obtained, so far as known, in this way before. If the reader knows the position of the centre of gravity of a triangle, he need here accept nothing on faith. Following this investigation, it will be seen that the *Three Moment Theorem* in its special and general forms is deduced directly from the equation of *Area Moments* by simple substitution. The truth of all these formulæ is now for the first time made clear to the reader who has no knowledge of the higher mathematics. Indeed, it may not be amiss to state that all the propositions here advanced may be understood by one who possesses a very moderate knowledge of mathematics and mechanics.

The endeavor has been made to take nothing for granted: hence graphical proof has been offered in regard to the extent of load which produces maximum bending moment and maximum shear, in regard to the effect of inclination of chords on web-stresses, in regard to absence of stress in the braces of the parabolic girder, &c. Some minor points will be found to possess novelty: prominent among these is the construction for

web-stresses in the bowstring, parabolic truss, which will be found to be extremely simple in application. Whether the special constructions for trusses with multiple systems of bracing will be useful or not, depends upon the frequency with which they are needed: some of them are thought to be elegant; they are all short in construction. The method here set forth of drawing the line which limits the maximum shear ordinates in a single span truss, and which was given in the former edition without proof, is here shown by a mathematical demonstration to be theoretically exact.¹ A simple and convenient diagram for the effect of a locomotive, or a load of greater than average intensity, is inserted. Very many of the details are thought to be new, and a comparison is invited for accuracy and brevity between these constructions and those found in other books.

The author would also ask a candid comparison of his method of Area Moments with the German method, as a means of solving problems in continuous girders, believing that, for the small number of operations, and hence simplicity and accuracy, as well as comprehensiveness, it will be found at least equally convenient.

The attempt has been made, by the use of capitals for the lettering of the trusses, accented capitals for the moment diagrams, small letters for the shear diagrams, and numerals for the load line, &c., to render a reference to the several diagrams of each figure easy.

Part III., on arches of all types, is in an advanced stage of preparation, about one-half of it having been once already in

¹ This demonstration in its essential features, as here given, was the work of Mr. Charles A. Marshall, at the time a student in the University of Michigan.

type, and will follow this part during the coming season. It is hoped that it will be found equally simple and intelligible with the previous parts.

C. E. G

ANN ARBOR, MICH., July 5, 1878.

NOTE TO FIFTH EDITION.

THE Post and triple quadrangular trusses have been omitted from this edition, and the analysis of the double quadrangular and Warren trusses has been simplified. Room has thus been made for the treatment of skew bridges and inclined trusses, and for what the writer hopes will be found very satisfactory and workable diagrams for trusses subjected to prescribed wheel-weights. The chord-stress diagram was in the appendix of the previous edition; the shear diagram now added is not original, but one or two modifications have been made, to render calculations unnecessary. An exceedingly simple way of calculating area moments, when weights are at equidistant panel joints, is here printed for the first time.

The principal changes are on pages 51-52, 58-78, and 97-99. Plate III. has been redrawn, and some cantilever bridges added to Plate X.

ANN ARBOR, February 9, 1895.

TABLE OF CONTENTS.

CHAPTER I.

INTRODUCTION.

SECTION.	PAGE.
1. Triangle of Forces	15
2. Equilibrium Polygon; Stress Diagram	16
3. Equilibrium Polygon for Bridge-Trusses	17
4. Shearing Force	17
5. Bending Moment	18
6. Example of Equilibrium Polygon for a Beam; Supporting Forces .	20
7. Shear Diagram	21
8. Moment Diagram	22
9. Relation between Shear and Moment Diagrams	23
10. Moment due to any Force; Centre of Gravity of Weights . .	23
11. Maximum Bending Moment at a Section	24
12. To bring the Closing Line Horizontal	25
13. Maximum Shearing Force at a Section	25
14. Diagram of Maximum Shear	26

CHAPTER II.

SINGLE-SPAN TRUSSES WITH HORIZONTAL CHORDS. — GENERAL TREATMENT.

15. Equilibrium Polygon applied to a Bridge; Data	27
16. Polygon for a Partial Load	27
17. Shear Diagram for the Same Load	28
18. Effect on Diagrams of Movement of Load	29
19. Maximum Moments and Shears	29
20. Curve for Maximum Shears	30

SECTION.	PAGE
21. Proof; General Formula for Shear	31
22. Equation of the Shear Line	32
23. Diagonals and their Stresses	33
24. Stresses in the Verticals	35
25. The Middle Vertical	35
26. Stresses in the Chords	36
27. Chord-Stresses (<i>continued</i>)	37
28. Parabola for Chord-Stresses	38
29. Sections of Pieces	39
30. Moment Diagram for Partial Moving Load	40
31. Shear Diagram for Partial Moving Load	41
32. Stresses due to Locomotive	42

CHAPTER III.

TRUSSES WITH HORIZONTAL CHORDS. — SINGLE SYSTEMS. — VARIOUS TYPES.

33. General Example	44
34. Howe Truss	45
35. Pratt or Single Quadrangular Truss	46
36. Comparison of Trusses	47
37. Warren or Triangular Truss; every Joint loaded	48
38. The Same; alternate Joints loaded	49
39. Comparison of Trusses	50
40. Baltimore Bridge Company's Patent Truss	51
41. Kellogg's Patent Truss	53
42. Plate-Girders	53
43. Truss of Odd Number of Panels	55
44. Trapezoidal Truss	56
45. Fink Truss	56
46. Bollman Truss	57
47. Wind Bracing	57
48. Shear shown by Moment Diagram	58
49. Skew Bridges	59
50. Trusses on a Grade	59
51. Inclined Trusses, supported on Loaded Chord	60
52. Inclined Trusses, supported on Unloaded Chord	61
53. Shear Diagram for Concentrated Loads	61
54. Maximum Bending Moments from Concentrated Loads	63

CHAPTER IV.

TRUSSES WITH HORIZONTAL CHORDS.—MULTIPLE SYSTEMS.

SECTION.	PAGE.
55. Double Quadrangular Truss	66
56. Stresses in Diagonals and Verticals	68
57. Inclined End-Posts	69
58. Odd Number of Panels	69
59. Double Triangular or Warren Truss	71
60. Lattice Girders	73
61. Effect of Locomotive on Double System	75

CHAPTER V.

TRUSSES WITH INCLINED CHORDS.

62. General Remarks; Effect on Chord-Stresses	78
63. Effect on Stresses in Diagonals	79
64. Stresses in the Verticals	80
65. Remarks	81
66. Construction for Horizontal Components	81
67. Stresses when Shorter Segment is loaded	82
68. Maximum Equilibrium Polygon Sufficient	83
69. Rolling Load to extend to the Panel for Maximum Stress in the Brace	85
70. Strut Diagonals; Load on Top Chord	85
71. Bowstring Girder	86
72. Chord-Stresses	87
73. No Stress in Diagonals for a Complete Load; Tension in Ver- ticals	88
74. Chord-Stresses when Verticals will not transmit Tension	89
75. Maximum Stresses in Braces	90
76. Stresses in the Verticals	92
77. Recapitulation	93
78. Triangular Bracing	94
79. Bowstring Girder with Circular Bow	95
80. General Remarks	95
81. Extent of Continuous Load to produce Maximum Stress in a Diago- nal of any Truss	96
82. Construction for Web-Stresses	97

CHAPTER VI.

FLEXURE AND DEFLECTION OF BEAMS.

SECTION.	PAGE.
83. Flexure of Beams, &c.	100
84. Change of Inclination	100
85. Modulus of Elasticity	101
86. Moment of Inertia	101
87. Formula for Change of Inclination	102
88. Deflection; Area Moments	103
89. Mathematical Solution	104
90. Applications	105
91. Beam fixed at One End, loaded at Other	105
92. Beam fixed at One End, and uniformly loaded	106
93. Both Cases combined	107
94. Beam supported at Both Ends, Single Weight in Middle	107
95. Beam supported at Both Ends, and uniformly loaded	107
96. Beam supported at Both Ends, Single Eccentric Weight	108
97. Beam on Two Supports, but overhanging	110
98. Beam supported and fixed at Both Ends, Weight in Middle	111
99. Beam fixed at Both Ends, and uniformly loaded	113
100. Beam fixed at One End, supported at Other; Weight Eccentric	113
101. Beam fixed at One End, supported at Other; uniformly loaded	115
102. Beam of Two Spans; Special Device	115

CHAPTER VII.

CONTINUOUS TRUSS OF TWO SPANS.

103. General Principle of Continuity	117
104. Abutment Deflections	118
105. Areas and Centres of Gravity	119
106. Value of the Pier Ordinate	120
107. Remarks	121
108. Shear Diagram	122
109. Discussion	122
110. Formula for Pier Moment for a Continuous Load	123
111. Extent of Load to produce Maximum Moments	123
112. Example	124
113. Load on One Span only	126
114. Load again shifted	126

TABLE OF CONTENTS.

13

SECTION.	PAGE.
115. Discussion of Results	127
116. Length of Chord under each Kind of Stress	128
117. Partial Load on One Span	129
118. Completion of Shear Diagram; Analysis	130
119. Remarks	132
120. Checks on the Accuracy of Diagrams	133
121. Maximum Negative Moments	133
122. Closing Remarks	134

CHAPTER VIII.

CONTINUOUS TRUSS OF MANY SPANS

123. Truss of Four Spans; General Treatment	136
124. Pier Ordinates	137
125. Solution of Equations	136
126. Positive Pier Moment	136
127. Shear Diagram	136
128. Clapeyron's Formula for Uniform Loads	139
129. Three-Moment Theorem for Single Weight	140
130. Piers not on the Same Horizontal Line	141
131. Example: Three Spans	142
132. Points of Contra-flexure; Chord-Stresses	144
133. Joints requiring Special Treatment	145
134. Shear Diagram	148
135. Extent of Chord subject to One Stress	148
136. Deflection of a Continuous Girder	149

CHAPTER IX.

PARTIALLY-CONTINUOUS TRUSS.

137. Settlement of Point of Support	150
138. Partially-Continuous Girders	152
139. Fixing Points of Contra-flexure by Hinges	153
140. Element of Indetermination in Multiple Systems of Bracing	155
141. Weighing the Reactions	156
142. Conclusion	150

CHAPTER X.

PIVOT OR DRAW SPANS.

SECTION	PAGE
143. Draw-Spans	158
144. Draw as Two Single Spans	158
145. Draw as a Two-Span Continuous Girder	159
146. Draw balanced on Pivot when closed	161
147. Action of a Rolling Load	161
148. Condition that Draw shall rest on Three Points	162
149. Shear Diagram, and Points of Contra-flexure	163
150. Draw with Locked Ends	164
151. Draw with Ends partially lifted	165
152. Remarks on the Preceding Cases	165
153. Example	166
154. Discussion	168
155. Web-Stresses	169
156. Remarks	170
157. Changes by Omission of Bolt	170
158. Draw of Three Spans	171
159. Values of Pier Ordinates	172
160. Special Treatment	172
161. Circular Girder carried by Pivot; Draw of Two Spans with Tipper	174
162. First Condition for Pier Ordinates	174
163. Second Condition for Pier Ordinates	175
164. Special Treatment.	176
165. Remarks	177

APPENDIX.

Bending Moments on Pins	178
Retaining Wall or Abutment	180
Maximum Stresses from Uniform Loads	186

BRIDGE-TRUSSES.

CHAPTER I.

INTRODUCTION.

1. **Triangle of Forces.**— We know, from one of the fundamental theorems of mechanics, that, if three forces not parallel act upon a body and are in equilibrium, their directions must intersect at a common point, and that these forces must be proportional to the sides of a triangle drawn parallel to their directions; and that, conversely, if three forces are parallel to the three sides of any triangle, and proportional in magnitude to the same sides, with the direction of the several forces taken in the order obtained by passing over the sides of the triangle in succession, these three forces applied at one point must be in equilibrium.

If the weight W , Fig. 1, be hung from two points, A and B , by the cords AC and CB , we may find the pull or tension on each of the cords by drawing a vertical line ab , equal, by any convenient scale, to the given weight W , and then drawing the lines ac and bc , from the extremities of ab , parallel to AC and BC , intersecting at c . Then will ac and bc represent, by the same scale by which W was laid off, the pull on the cords AC and BC . The arrows on the cords represent the directions of the stresses relatively to the point C , and the arrows on the triangle abc must follow one another in order round the triangle.

If W and the two forces bc and ca had been given, we might have reversed the problem, and have found the direction and length of two cords, which, while supporting W , would have exerted the given tensions on the points A and B . As it is possible to draw any number of triangles on the line ab , we may have a like number of arrangements of cords, from A and B , to carry W .

2. Equilibrium Polygon; Stress Diagram.— If we have several weights suspended from the points A and B , Fig. 2, by means of a cord whose weight is either neglected, or considered as included in the given weights, we may find the tensions in the several portions of the cord by successive applications of the above process. The weight at C is balanced by the tensions of AC and CD , and a triangle of forces may be drawn for the point C . Next, taking the weight W_2 , and the tension, just found, in CD , we may draw a triangle for the point D .

But since, when we draw a triangle for each loaded point, each portion of the cord will be represented by a side in two of the triangles, and since all of the triangles will have one vertical side, they may be brought together into one figure by the following construction: Draw a vertical line 1-2, and lay off on the same, $W_1 = 1-3$, $W_2 = 3-4$, $W_3 = 4-5$, and $W_4 = 5-2$. Draw 1-0 parallel to AC , and 2-0 parallel to BF ; connect the point of intersection 0 with the other points of division on the vertical line: then will 1-0, 3-0, 4-0, 5-0, and 2-0 be equal to the stresses in AC , CD , DE , EF , and FB ; and the portions of the cord must be parallel to the lines radiating from 0, or otherwise the cord will not be in equilibrium.

The figure 012 is styled by many authors the *force polygon*; but we prefer to call it the *stress diagram*, from its analogy to the stress diagram of a frame. The vertical line 1-2 will be called the *load line*; and the cord from A to B , which hangs in what is known as a *funicular polygon*, we shall have occasion hereafter to designate as the *equilibrium* or *moment polygon*. The point 0 is called the *pole* of the stress diagram.

3. Equilibrium Polygon for Bridge-Trusses. — If the weights, their horizontal distances from the point A, and the horizontal distance of B from A, were alone given, we might draw an *equilibrium polygon* to satisfy these conditions by assuming the pole 0 in any convenient position, drawing the radiating lines to the several points on 1-2, then drawing a line parallel to 1-0, from A, to meet the vertical through W_1 , — from the point C, so found, drawing a line parallel to 3-0 to meet a vertical through W_2 , —and so on; the last line parallel to 2-0 determining the position of B, since its horizontal distance from A was previously given. This latter simple application of the method for finding an equilibrium polygon will suffice for the investigation of bridge-trusses which is to follow.

4. Shearing Force. — If all of the forces which act on a structure lie in one vertical plane, they may be resolved into horizontal and vertical components in that plane. All the external forces considered in this part are vertical; viz., the weight of the structure, the imposed load, and the reactions of the points of support. If a beam or truss supported in any manner, as in Fig. 4 or 6, is cut by an imaginary vertical plane, we shall have on one side of the plane of section the resultant of all the external vertical forces which act upon the portion of the beams or truss upon that side; and, on the other side of the section, another resultant of the external forces upon the second side: since there is perfect equilibrium, these two resultants must be equal and opposite to one another. As it is proved in mechanics that a force at any distance from a given point is equivalent, so far as its action in regard to that point is concerned, to the same force at the point and a moment equal to the product of the force by its perpendicular distance from the point, the two resultants above will, in the first place, give the two equal and opposite forces at the section, represented by the arrows in the panel A C D B of Fig. 3. These forces tend to move the two portions of the beam or truss in opposite directions; and there must be a resisting stress in the fibres of the beam at the section, or a brace in the panel of the truss, to

prevent the movement. From the nature of the movement, this stress is called a *shearing stress*; and, as it must be equal to the resultant of vertical forces on *one side* of the plane of section, this resultant is usually styled the *shearing force* or shear. If it acts upward on the left of a section, as here shown, we call it *positive*; if downward, *negative* shear.

A reference to the panel A C D B of Fig. 3 will, we think, enable the reader to see how the direction of the shear determines the kind of stress in the bracing or web members. At the section in this panel the shear is supposed to be upward on the left side of the section denoted by the dotted line, and downward on the right side. If we imagine the horizontal members or *chords* to be severed, it is evident that a strut in the diagonal C B, drawn in the sketch, or a tie in the diagonal A D of the rectangle, will be needed to prevent one part of the truss from falling. Or if we suppose the points of intersection of the chords with the web members to be joints free to turn, as is the case with pin-connected parts, it is clear, that, if no diagonal member exists, the rectangle A' C' D' B' will be distorted into a parallelogram, one diagonal shortening, and the other lengthening. This movement also shows that the chords alone, if jointed, have no power to resist shear, or convey a vertical force, but that a strut must be introduced in the diagonal which tends to shorten, or a tie in that one which tends to lengthen, if the distortion and falling of the truss are to be prevented. *The web members, therefore, resist the shearing forces.*

If, in the same or any other panel, the shear is in the opposite direction, a strut in the diagonal A D, or a tie in the diagonal C B, will be necessary. The arrows in the panels of the truss E F are supposed to represent the direction of the shear on the *left side* of a section in the successive panels. The diagonals for the given shears are struts, and change in direction as soon as the shear is reversed.

5. **Bending Moment.** — Further, as indicated in the last section, the same external vertical forces on one side of the plane

of section have *moments* about any point of the section ; and the resultant moment will be obtained by multiplying the resultant previously referred to by the horizontal distance of its point of application from the section, or by multiplying each external force by its horizontal distance, or lever arm, from the plane of section, adding those products which have a tendency to produce rotation in one direction, and subtracting the sum of those products which tend to produce rotation in a contrary direction. The remainder is called the *bending moment* at the section, and would cause rotation of the structure about the section in the direction shown by the larger sum, if it were not exactly balanced, as required by the condition that equilibrium exists, by an equal and opposite moment on the other side of the section. These two equal moments neutralize one another by means of the *moment of resistance* in the given vertical section of the beam or truss, arising from the resistances to extension and compression of the fibres or pieces in the two portions, upper and lower, of the beam or truss ; each stress being multiplied by its lever arm from any convenient point in the section. These horizontal resisting stresses are due to the attempt of the external forces to bend the beam.

Since the external forces are vertical, their only effects at the plane of section must be those due to a vertical force and a moment, called, as stated, the *shear* and the *bending moment* : and the opposing resistances in the material at the section will be, first, a vertical shearing stress ; and, second, horizontal tensions and compressions. Thus we have at any section the horizontal and vertical components of the stresses in the parts cut by the section, and are able to derive from them the direct stresses in the pieces. It will then be convenient to determine the shearing forces and bending moments at all points of the structure as a preliminary step. If, from any consideration, we know the stress in one or more pieces at a section, the remainder of the horizontal and vertical components must exist in the remaining pieces. As the external forces which are here treated are vertical, the horizontal components of the stresses at any section must balance among themselves.

6. Example of Equilibrium Polygon for a Beam; Supporting Forces.—Suppose that a beam, such as is represented in Fig. 4, is supported at the two ends, and has four unequal weights situated, as in the sketch, at unequal intervals upon it. Let the weight of the beam be included in the imposed weights. It is required to find the *supporting forces*, or reactions, at the abutments A and B, and the shearing force and bending moment at all points. Draw a stress diagram, as before, by laying off on a vertical line 1-2, Fig. 4, the weights W_1 , W_2 , &c. Assume the pole 0 at a convenient distance from 1-2, and draw radiating lines from 0 to all points of division on 1-2. Draw vertical lines through the points of support and the loaded points A, C, D, E, F, and B. Commence at a convenient point, A', in the vertical through A, and draw A' C' parallel to 0-1. From the point C', where this line cuts the vertical from the first weight, draw C' D' parallel to 0-3. Continue the same process until the last line F' B', drawn parallel to 0-2, meets the vertical line let fall from B, the second point of support.

If A' C' D' E' F' B' were a cord, fastened at A' and B', and under the action of the given weights in their given positions, it would be in equilibrium, as already shown. The pull on A' would be in the direction A' C' and of the amount 0-1. The pull on B' would be in the direction B' F' and of the amount 2-0. If the two ends of the cord, in place of being fastened at A' and B', were attached to the two ends of a rigid bar, A' B', the whole system might be suspended by two cords from the points A and B without disturbing the equilibrium of the polygon; for, if we draw 0-5 in the stress diagram parallel to B' A', we can see that the inclined pulls at A' and B' are, by the introduction of the bar A' B', each decomposed into a thrust along the bar and a force acting vertically downwards, as shown by the arrows. At the point A' we have three forces, which, in the stress diagram, must make the triangle whose sides are 0-1, 1-5, and 5-0. In the same way, for the point B', we have the sides of the triangle 0-5, 5-2, and 2-0. The thrust 5-0 at A' balances the thrust 0-5 at B'; and there remain 1-5 and 5-2, or

P_1 and P_2 , the forces exerted by this system on the points of support A and B. Any other system of framing, loaded and supported in the same way, must give the same pressure on the points of support; and thus the upward reactions of the abutments for the given loaded beam will be determined.

Therefore, to find the supporting forces: Having drawn an equilibrium polygon, connect A' with B', draw 0-5 through 0, parallel to this closing line; and the two portions into which the load line is thus divided will be the forces required.

7. Shear Diagram.—As the shearing force at any section is the resultant of the vertical forces on one side of the plane of section, the shearing force at any point between A and C will be P_1 , or 1-5, the only vertical force on the left of C, and acting upwards on the left side of the section. Between C and D the shearing force will be $P_1 - W_1$, or 3-5; between D and E it will be $P_1 - W_1 - W_2$, or 4-5; and so on, being that part of load included by the lines of the stress diagram parallel to the two lines of the moment polygon cut by a vertical section.

But if we draw a horizontal line ab equal to AB; then at a lay off ag equal to P_1 , and upwards, as denoting the direction of P_1 ; next draw from g a line gi parallel to ab ; then at the point i , vertically over c and under C, measure off ik equal to W_1 , downwards in the direction of action of the weight; draw kl horizontally, then lm downwards equal to W_2 , and finally $mnpqrs$,—we shall have a broken line, the ordinate to which, from any point of ab , will be the shear on the *left* of the section at the corresponding point of AB. When the W 's which have been subtracted exceed P_1 , the broken line passes below ab ; and finally, on arriving at f , having subtracted all the W 's, we have a shearing force equal but opposite to P_2 . So, at any section, the shear at one side of the plane of section, obtained by the subtraction just described, will always be equal and opposite to the shear on the other side of the plane of section, obtained by working from B, with the supporting force P_2 as the minuend. Such a result is required to fulfil the condition of equilibrium.

If the load is continuous, in place of being concentrated at a certain number of points, the successive ordinates will vary in length, diminishing with the amount of load passed; so that the bounding line will not be horizontal in the loaded portions. If the load is of uniform intensity, the upper extremities of the ordinates will be bounded by a straight inclined line.

8. **Moment Diagram.**—Lastly, to find graphically the bending moment at any point. Take the point S. The bending moment on the beam at S is, when we take the moments of external forces on the left of S,

$$P_1 \cdot AS - W_1 \cdot CS - W_2 \cdot DS.$$

Drop a perpendicular from S, cutting the polygon A' E' B' at I and K. Produce C' D' and A' C' to meet this perpendicular at L and M. Also draw C' N horizontally. In the stress diagram draw a horizontal line, 0-6, through 0, to meet the load line. Call this line H. It is the horizontal projection of the stress in each side of the equilibrium polygon, which projection is well known to be *constant* for a system of vertical loads. The triangles C' N M and 0 6 1, having their sides parallel, are similar, and we have the proportion,

$$MN : C' N = 6-1 : 0-6.$$

From the similar triangles C' N L and 0 6 3 we have

$$LN : C' N = 6-3 : 0-6:$$

hence

$$\frac{MN - LN}{C' N} = \frac{(6-1) - (6-3)}{0-6},$$

or

$$ML \cdot (0-6) = (3-1) \cdot C' N.$$

But

$$0-6 = H; 3-1 = W_1;$$

therefore

$$H \cdot ML = W_1 \cdot C' N = W_1 \cdot CS.$$

In the same way

$$H \cdot LK = W_2 \cdot DS, \text{ and } H \cdot IM = P_1 \cdot AS;$$

therefore the bending moment

$$M = H (IM - ML - LK) = H \cdot IK.$$

Hence the bending moment at any point of the beam is *proportional* to the ordinate, from $A'B'$ to the equilibrium polygon, vertically below that point, and is equal to the product of that ordinate by H , the constant horizontal component of the tensions in the polygon.

The ordinate must be measured by the scale to which the beam is drawn; and the line which represents H , by the scale to which the load line is measured off: but the two figures may be of the same scale, that is, number of tons and feet to the inch, or of different scales, whichever is more convenient. If the pole O be taken at such a distance from the load line that $O-6$ shall measure ten or one hundred units of weight in length, the bending moment will be readily obtained by scaling the ordinate, and moving the decimal point one or two places to the right, as the case may be.

9. Relation between Shear and Moment Diagrams.—It will be noticed that the line which limits the ordinates for shear crosses the horizontal line, or, in other words, the shear changes sign, at that point of the beam where the ordinate to the equilibrium polygon, and hence the bending moment, is a maximum. This relation between the shear and moment diagrams always exists; for, since $0-5$ was drawn parallel to the closing line $A'B'$ of the equilibrium polygon, the maximum ordinate will be found at E' , where the two sides $D'E'$ and $E'F'$, which make the angle E' , are parallel to lines from O on opposite sides of $0-5$; and at that place sufficient weights will have been subtracted from the reaction $1-5$ to reduce it to zero, and cause the shear line to pass the horizontal line at e .

10. Moment due to any Force; Centre of Gravity of Weights.—It has been proved in § 8, that the portion LM of the ordinate at S , intercepted between the lines $C'M$ and $C'L$, is proportional to the moment of W_1 about S ; that LK , cut off by $D'L$ and $D'K$, is proportional to the moment of the weight which is over D' ; and that IM , included between the lines from A' , is proportional to the moment of P_1 about S : hence it follows that the particular portion of the bending moment

at any point of a beam which is due to one or more of the external forces is readily ascertained by dropping a vertical from the point in question, and prolonging those two sides of the equilibrium polygon which include between them the given forces. The portion of the vertical line thus cut off, when multiplied by H , will be the desired quantity. This property of the equilibrium polygon will be of service later.

If $A' C'$ and $B' F'$ are prolonged to an intersection at R , and the rest of the polygon is removed, the two sides $A' R$ and $B' R$ will be in equilibrium, and will undergo the tensions 0-1 and 0-2, if the entire load 1-2 is concentrated at R . Since the two reactions will be unchanged, a vertical line drawn through R gives the position of the resultant of the applied weights of Fig. 4. Similarly, if any two sides of the equilibrium polygon are prolonged until they meet, the resultant of the included weights will lie in the vertical through that intersection. Thus the resultant of W_1 , W_2 , and W_3 , must pass through T . This operation may also be termed finding the position horizontally of the centre of gravity of the weights.

11. Maximum Bending Moment at a Section. — If the beam AB , of Fig. 5, carries at first only the weight W at C , the bending moments at all points of the beam will be proportional to the ordinates from AB to $A' C' B$, and all of the moments will be of the same kind, tending to make the beam concave on the upper side: such moments we prefer to call positive, since they are the ones with which we are most familiar. In the stress diagram, 3-0 will be the accompanying value of H , and 2-3 the supporting force at A . In the same manner, an equal W at D gives the triangle $A' D' B$, with 0' 2 1 in the stress diagram. The other two equal W 's give the remaining parts of the diagram. If all the values of H are equal, as they have been made here, the several ordinates below each point of the beam may be added arithmetically, since the bending moments are alike in kind, producing the figure $A'' C'' E'' B''$, whose ordinates will be multiplied by 0-3, or any other equal line representing H , to produce the bending moments. Exactly

the same figure would be obtained by laying off the whole load on a vertical line, taking the pole at the above distance from the vertical, and proceeding as usual.

It will be evident, from the preceding steps, that, as every additional weight increases the bending moments at all points of a beam or truss of one span supported at both ends, *the greatest possible bending moments at all points*, in case such a structure is subject to a moving load, *will be found when the span is entirely covered with the rolling load.*

12. To bring the Closing Line Horizontal. — Since the closing line $A''B''$ is parallel to the line from the pole to that point on the load line which divides it into the two supporting forces or reactions, if it is desired that A'' and B'' , the extremities of the funicular polygon, shall fall on the same horizontal line as in this figure, it is necessary to divide the load line into the two reactions, and then to assume the pole O in the same horizontal line with the point of division. It is generally of no consequence that $A''B''$ should be horizontal: the equilibrium polygon has the same properties in any position.

13. Maximum Shearing Force at a Section. — Returning again to the case where the weight W alone acted upon the beam of Fig. 5 at C , we know that the shear diagram will be represented by $acghib$, where $ac = P_1 = 2-3$, and $gh = W$, which is subtracted at the point vertically below C . On the left of any section between A and C the shear is positive or upward, and equal to P_1 ; while at any point between C and B the shear is negative on the left of a section, and equal to $-P_2$. In the same way, when W alone is placed upon the beam at D , we obtain the shear diagram $adklmb$, in which the shear again changes in sign at the weight. The other two shear diagrams belong to the remaining weights. If, then, it is possible to have some or all of these weights on the beam at one time, it is evident that the maximum positive shear on the left of a section, at all points between A and C , will be found when all of the weights are placed upon the beam; for we shall then have a shear whose value is the sum of the separate positive ordi-

notes of the four shear diagrams just drawn. For any section between C and D it is manifest that the positive shear will be greatest when W is removed from C. For a section between D and E the two weights on the right will give positive shears, and anywhere between E and F a positive shear on the left of a section can only be obtained from the weight at F. It is also plain that the four weights together will make the maximum negative shear on the left of a section between B and F, and that the weights must be removed in succession as we pass them to find the greatest negative shears at different points.

Hence follows the rule, that *the greatest positive or upward shear on one side of any section will exist, for a beam or truss of one span supported at both ends, when all possible moving load is placed upon the other segment only of the span; and the greatest negative shear, when the moving load covers the segment on the same side.*

14. Diagram of Maximum Shear.—These maximum positive values of shear may be grouped in one figure, when they will produce the diagram $a' c' d' e' f' b'$; but it must be remembered that *the shears represented by the ordinates of this diagram are not co-existent*, but occur in succession as the loads are added from one end. They are, however, useful, since they give the greatest shears, which must be guarded against in the structure. If the reader will draw a shear diagram for a complete load, and then for loads over a less extent of span, he can readily compare the diagrams with the last one of Fig. 5, and see the difference, and at the same time the agreement of the *maximum values* at successive sections.

If the load is uniformly distributed, the broken line $c' d' e' f'$ will become a continuous curve, which can be proved to be parabolic; and, if a simple method of constructing the diagram is given, the maximum shear at each point for such a load can be readily found.

CHAPTER 11.

SINGLE-SPAN TRUSSES WITH HORIZONTAL CHORDS. — GENERAL TREATMENT.

15. Equilibrium Polygon applied to a Bridge; Data. — Let the method, as thus far developed, be applied to a bridge-truss with parallel chords, under a moving load of given intensity. The truss is represented by A B C D, Fig. 6, supported at A and D. In order to deal with moderate dimensions, we will suppose that the span is 80 feet, height of truss 10 feet, fixed load $\frac{1}{2}$ ton per running foot, moving load $\frac{1}{2}$ ton per running foot, each for one truss. The scales of the figure and diagrams are, as shown below them, 30 feet = 1 inch, and 30 tons = 1 inch. The small weights represent the fixed load arising from the truss and platform, as if concentrated on the joints of the lower chord; the larger weights represent the moving load. As each panel is 10 feet long, the load from the bridge will be $2\frac{1}{2}$ tons at each joint, and from the rolling load 5 tons. The points A and D will each carry one-half of a panel weight: these weights will cause no stress in the truss, and might be neglected altogether; but it will be found convenient to plot them on the load line, as thus the total weight of the truss will be accounted for, and the shearing force will be more readily obtained.

16. Polygon for a Partial Load. — Suppose that the rolling load extends from the abutment D to the joint G inclusive. Draw a vertical line, and lay off 1-2 equal to $\frac{1}{2}$ ($2\frac{1}{2} + 5$) tons, the weight at D; next 2-3 = $7\frac{1}{2}$ tons at L, 3-4 = $7\frac{1}{2}$ tons at K, and so on to 6-7 at G; then 7-11 = $2\frac{1}{2}$ tons at F, 11-12 = $2\frac{1}{2}$

tons at E, and finally $1\frac{1}{2}$ tons at A, reaching a point 13, midway between 12 and 14. Next assume the pole O, which is so taken here that H measures 20 tons. Leave out of consideration 1-2, the weight at D; and starting at D', a convenient distance vertically below D, draw D'L', L'K', . . . E''A'' parallel respectively to 2-0, 3-0, 4-0, &c.; the last line, E''A'', being drawn parallel to 12-0. The equilibrium polygon will be completed by drawing the closing line from A'' to D'.

17. Shear Diagram for the Same Load. — Draw in the stress diagram 0-9 parallel to A''D': 13-9 and 9-1 will be the supporting forces at A and D. The shearing force at any section will be represented by the ordinates to a broken line, constructed similarly to *giklm*, &c., of Fig. 4; the ordinate at *a* being equal to 12-9. If, however, the concentrated loads of this truss are supposed to be distributed over the horizontal line AD with an intensity of one-fourth ton per foot from A to a point half way between F and G, and of three-fourths ton per foot for the remaining distance, the total load will be unchanged, the reactions will be the same; but the ordinate *ag* will equal 13-9, *dn* will equal —(9-1), and the ordinates at all points will be limited by *gpn*. The line *gp* must have a declivity or slope corresponding to the intensity of $\frac{1}{4}$ ton per foot, or $2\frac{1}{2}$ tons per panel; while *pn* inclines at three times that rate. The angle *p* occurs in the middle of a panel, where the intensity of the load changes. The difference between the ordinate at *a* to the inclined line *gp* and the one which would have been plotted for the broken line, or between 13-9 and 12-9, is the half-panel weight directly supported at A. Then as, in reaching the middle of successive panels, we shall have passed beyond just as much distributed load as the amount of concentrated load which is here carried at the joints, the broken line which falls by steps, and the inclined lines *gpn* of the figure, will intersect one another in the middle of each panel. A small portion of the broken line is sketched near *g*, and, for another case, near *i*: hence, as it is easier to draw *gpn* than the broken line, we may find the shear in each panel by measuring the ordinate to *gpn* in the middle of each panel.

18. Effect on Diagrams of Movement of Load.— If the rolling load retires, so that J is the last fully loaded point, we shall have three loads of $7\frac{1}{2}$ tons each, and the points from E to I will carry only $2\frac{1}{2}$ tons each. We may make available a considerable portion of the polygon already drawn. Since 4-5 represents the load on J, we can lay off the smaller loads below; the one on E falling at 6-16, and the half load at A reaching a little beyond 16. Constructing a second equilibrium polygon as we did the first, we shall get $D'K'I'G''F''A'''$; and, drawing 0-8 parallel to $A'''D'$, we get the new supporting forces for this position of the rolling load: hence we find jr and ri as we did gp and pn . Do not forget that $di = 1-8$, and not 2-8, and similarly for aj ; for the inclined lines always start from the end of an ordinate which represents the entire abutment reaction, including the weight directly supported there.

If the load extends over the whole truss, our load line will be 1-10, and the equilibrium polygon will be $A'E'F'G'I' \dots D'$. As the pole 0 was taken opposite the middle of 1-10, and as, for an entire uniform load, the two supporting forces are equal, the line $A'D'$ is horizontal. The shear diagram for this case is $abvcd$.

19. Maximum Moments and Shears.— Without carrying out in detail the construction for every possible position of the moving load, we see indications which will render needless the drawing of so many diagrams. First, as pointed out in § 11, the bending moment at any point of the truss is greatest when the whole bridge is loaded. The polygon $A'G'J'D'$ will give ordinates of the greatest length, and these ordinates multiplied by H give the bending moments. Second, as in § 13, the positive shearing force at any point is greatest, when, of the two portions into which the point divides the span, that segment is fully loaded which lies on the opposite side of the plane of section to the shear considered. For example, the shear in the panel FG, when the rolling load extends from D to G, is hp . If the rolling load is less, or covers a less extent, the supporting force at A will be less than ag , and the line parallel to

gp will therefore pass below p . On the other hand, if the moving load advances farther, the supporting force at D will be greater than dn , and the line parallel to np will again pass below p . The ordinate hp is, therefore, equal to the greatest possible positive shear in the panel FG .

20. Curve for Maximum Shears.—It is evident that the greatest value of the reaction at either abutment will be one-half of the weight of truss and full load, and that the least value will be one-half of the weight of the unloaded truss. In the first case the shear diagram will be $abvcd$; and, in the second case, $alvkd$; al and ab being respectively the half weight of truss and the half weight of truss and load, and the two inclined lines cutting ad at the middle of the span. From an inspection of the two diagrams already drawn, whose angles fall at p and r , it will be seen, that for loads coming on at D , and extending gradually across the bridge, the vertex r of the angle of the shear diagram will, for successive advances of load, take a series of positions from m in the last panel, on the line lk , through r and p , to t in the first panel, on the line bc , when the rolling load has finally reached E . If, then, we know the path described from t to m , and can construct it, all the maximum ordinates can be readily obtained. If the rolling load were applied uniformly foot by foot to a beam, the desired locus would be a *parabola*, extending from b to k , described on bv and vk as tangents; but as our locus is to extend from t to m , for loads concentrated at intervals, the lines bv and vk will not be tangents. The construction will now be shown: the proof of its truth will follow.

Draw a horizontal line AB , Fig. 7, equal to the span of the truss. At one end of it, as A , erect a perpendicular AC , equal to one-half weight of truss fully loaded, to the scale of load line used; and at B draw on the opposite or negative side of AB a perpendicular BD , equal to one-half weight of truss alone. From the extremities of these lines draw CO and DO to the middle point O of AB . Divide CO and DO each into the same number of equal parts as there are panels in the truss.

Number the points of division in the same direction on each, beginning at C and O with 0. Draw straight lines 1-1, 2-2, &c., between the points having the same numbers. Then the vertical ordinates from A B to the intersections of consecutive lines in the series (which ordinates will come at the middle of the panels of A B, if that line be divided into panels to match the truss) will be the *exact shears* required in the panels in which they occur for loads concentrated at joints as here taken, and already described in Fig. 6.

As verticals dropped from the panel points of the truss will divide the lines C O and D O, or $b v$ and $v k$ of Fig. 6, into half the desired number of parts, it is only necessary to bisect these parts to obtain the desired points. The reader can complete this figure on a large scale, and notice the intersections of the several lines between C O and D O. Any consecutive lines, such as 3 and 4, will intersect vertically above the number found by adding their numbers, in this case 7, always an odd number; and the intersection thus occurs exactly in the middle of a panel.

21. Proof; General Formula for Shear.—First, to deduce a formula for the shear in any panel. Let

N = number of panels in the truss; l = length of truss in feet;

$\frac{l}{N}$ = length of one panel; W = fixed load at a joint;

W' = moving load at a joint; n = number of any joint in the bottom chord from the left abutment, the abutment being numbered 0;

F_n = shear in the n th panel, between the n th and $(n+1)$ st joints.

If the joints from $n+1$ to $N-1$ inclusive are each loaded with $W + W'$, and the others with W only, the shear from steady load in the n th panel will be the supporting force $\frac{1}{2}(N-1)W$ less the loads, nW , between the left abutment and the section, or $[\frac{1}{2}(N-1) - n]W$. The amount of rolling load on the truss is $(N-1-n)W'$; and the shear from this load at the section in the n th panel will be the same as the supporting force at A, which is equal to the above rolling load multiplied by $\frac{N-n}{2} \cdot \frac{l}{N}$, the distance of its centre of gravity from the opposite abutment, and divided by the span l , or $(N-1-n) \frac{N-n}{2N} W'$. Therefore

$$F_n = \frac{1}{2}(N-2n-1)W + (N-n-1) \frac{N-n}{2N} W'.$$

22. Equation of the Shear Line.—Second, let us take the origin of co-ordinates in Fig. 7 at O, A B being the axis of abscissas, and the ordinates being measured vertically. Let v and m be the co-ordinates of any point of division P, v being vertical, and measured to the scale by which A C and B D were laid off, and m being horizontal, expressed in units of a half-panel length. The equation of the line O C is

$$v = -\frac{1}{2} m (W + W') :$$

the equation of the line O D is

$$v = -\frac{1}{2} m W.$$

Let P' P'' be any line of the series previously described. The co-ordinates of the point P'' are

$$m'', \text{ and } v'' = -\frac{1}{2} m'' W.$$

The co-ordinates of the point P' are

$$m' = m'' - N, \text{ and } v' = -\frac{1}{2} m' (W + W') = -\frac{1}{2} (m'' - N) (W + W').$$

From analytic geometry we have the general equation of a line in terms of co-ordinates of two points in it,

$$v - v' = \frac{v' - v''}{m' - m''} (m - m').$$

Substituting the value of the other co-ordinates in terms of m'' , as given above, we obtain

$$v + \frac{1}{2} (m'' - N) (W + W') = \frac{-\frac{1}{2} (m'' - N) (W + W') + \frac{1}{2} m'' W}{m'' - N - m''} (m - m'' + N) \\ = -\frac{1}{2} (m - m'' + N) (W + W') + m'' (m - m'' + N) \frac{W'}{2N},$$

$$\text{or} \quad v = -\frac{1}{2} m (W + W') + m'' (m - m'' + N) \frac{W'}{2N}. \quad (a)$$

which is the equation of the line P' P'', in terms of m'' .

Writing in this equation $m'' + 1$ for m'' , we shall obtain the equation of the consecutive line in the series, which is

$$v = -\frac{1}{2} m (W + W') + (m'' + 1) (m - m'' - 1 + N) \frac{W'}{2N} \\ = -\frac{1}{2} m (W + W') + (m'' + 1) (m - m'' + N) \frac{W'}{2N} - (m'' + 1) \frac{W'}{2N}. \quad (b)$$

If we eliminate m'' from equations (a) and (b), we shall have the equation of the locus of the intersection of the consecutive lines. To do so, subtract (a) from (b), obtaining

$$0 = (m - m'' + N) \frac{W'}{2N} - (m'' + 1) \frac{W'}{2N}; \text{ or}$$

$$0 = m - 2m'' + N - 1:$$

whence

$$m'' = \frac{1}{2} (N + m - 1).$$

Substitute this value in (a), and

$$\begin{aligned} v &= -\frac{1}{2} m (W + W') + \frac{1}{2} (N + m - 1) \left[m - \frac{1}{2} (N + m - 1) + N \right] \frac{W'}{2N} \\ &= -\frac{1}{2} m (W + W') + \frac{1}{2} (N + m - 1) (N + m + 1) \frac{W'}{2N}, \quad (c) \end{aligned}$$

which is the equation of the locus required.

The points of division at the ends of $P'P''$ are both numbered m'' , or the horizontal distance of one end from O to the right is the same as the horizontal distance of the other end from C to the right; that is, the projection of the moving line is a constant quantity.

The value of m is shown above to be

$$m = -N + 2m'' + 1.$$

It will be remembered that m is the abscissa, measured in half-panel lengths, of the point of intersection of the two lines whose extremities are at m'' and $m'' + 1$: hence, to find where the ordinate to this point comes on the axis of abscissas, measure from A , which is $-N$ half-panel lengths from O , $2m'' + 1$ half-panel lengths. Putting this fact in general terms, we may say that the ordinate to the intersection of the n th line with the $(n+1)$ st line of the series comes at a distance of $2n + 1$ half-panel lengths to the right from A , or in the middle of the n th panel.

To find the value of this ordinate in terms of n , we write in (c) $m'' = n$, and $m = -N + 2n + 1$ thus producing

$$\begin{aligned} v &= \frac{1}{2} (N - 2n - 1) (W + W') + \frac{1}{2} n (-N + 2n + 1 + 1 + N) \frac{W'}{2N} \\ &= \frac{1}{2} (N - 2n - 1) W + (N - n - 1) \frac{N - n}{2N} W', \end{aligned}$$

which was the value of the shear F deduced in § 21.

23. Diagonals and their Stresses.—The shearing force in any panel is then obtained, for example, in the panel EF , Fig. 6, by erecting the ordinate es to the *shear curve* (if we may so style it) at the middle e of the panel, the curve having been constructed by the method of Fig. 7. As the panel would change from its rectangular form in case the diagonal were removed, as already explained in § 4, the diagonal SF alone must resist the shear, and the diagonal will be a tie. The ordinate zs being the vertical component of the tension in this tie, we need only draw sf parallel to SF , from s , until it meets the horizontal line, when sf will be the tension in SF , according to the scale of loads. It is well to bear in mind that a horizon-

tal member can transmit no vertical force, unless it acts as a beam, which it can only do when it has a force or load imposed directly upon it; but a direct force of tension or compression applied at one end of a piece passes through without change to the next joint.

A similar line from p will give the tension in $R G$, and so on to r . The point r , under the middle of the panel $I J$, will be the last one where we shall find any positive or upward shear on the *left* of the plane of section, which occurs when the load extends from D to J . As shear of an opposite kind calls for a diagonal, if a tie, sloping in the other direction, in the panel $I J$ will be found the last necessary tie parallel to $B E$.

The remaining panels from J to D would have, therefore, the diagonals represented in the sketch; but a rolling load coming on at A , and extending towards D , would cause shears similar to those given by ordinates between r and m , but greater in amount than those stresses. The diagram would be completed by drawing on lv and vc another shear curve similar to the one previously constructed; but, as the figure would be exactly the reverse of $btsprmk$, it is sufficient to find the stresses from A to J , and then make the truss from D to G symmetrical with $A J$. The heavier the rolling load compared with the fixed load, the farther will the diagonals sloping one way pass beyond the middle of the span. Those diagonals which are between the centre and the abutment to which they convey their load are usually called *main* braces: those beyond the centre are termed *counterbraces*, or simply *counters*. Fig. 6 has but two counters. Some designers, for constructive reasons, place counters in many or all of the remaining panels.

The occurrence of two diagonals in one panel should lead to no ambiguity as to which one is under strain. They must both be tension, or both compression members; and the points developed in § 4 will enable one to see which diagonal will act for a given shear. As one diagonal tends to shorten, the brace occupying that place, if a compression member, will resist the tendency; or, if a tie, will spring or buckle under a slight force,

leaving the distortion to be resisted by the other brace. As the other diagonal tends to lengthen, the brace in that line, if a tie, will come into action; but, if it is a strut with abutting ends, it cannot exert tension, and the first diagonal must supply the force. If, however, one diagonal is a tie and the other a strut in the same panel, or if both diagonals are competent to carry either kind of stress, they will both act together; and the distribution of the given shear between them is indeterminate, except on some assumption or condition. No good design will be so constructed.

24. Stresses in the Verticals.—Since es is the vertical component of the stress in SF , it must be the compression in SE ; for the tension in SF , upon reaching S , is decomposed, the horizontal component compressing SR , and the vertical component compressing SE . The vertical force of SE will next pass over EB , with the addition of that part of the weight at E which may properly pass that way. As these things are true of any set of diagonals and verticals, it follows that *the ordinate to the shear curve at the middle of a panel determines the stress in that diagonal and vertical, which, taken together, may be considered to connect two loaded joints*. If, therefore, we imagine the load to be upon the top chord in place of the bottom chord, se will give the compression on RF instead of SE ; for, considering the vertical force passing from R to S over the pieces RF and FS , there is then no applied load between R and S to alter its amount. In Fig. 6 the stress in BA equals the ordinate at t : were the load on the top chord, the stress would be increased by the half load at B , so that it would become ba . A change of load from one chord to the other affects the stresses in no pieces except the verticals, and only changes the *amount* of those stresses, but not the *kind*.

25. The Middle Vertical.—The middle vertical alone may sometimes offer an exception to the preceding way of determining the maximum stress in it. It will be seen, that, when the load is upon the upper chord of Fig. 6, and the entire span is covered with rolling load, the ties which meet at I in the lower

chord will be in action, since the bridge is symmetrically loaded; and that the vertical at I must then transmit from the top to the bottom joint the entire weight which rests at its top, or one panel weight of steady and rolling load. If this amount is more than the ordinate which the shear diagram gives as belonging to this vertical, the former must be taken in place of the latter. The same remark holds if the load is on the bottom chord, and the main braces lie in the other diagonals; in which case they will be struts. The verticals will then be tension members. If the truss is as here represented, or if, when strut diagonals are used, the load is on the top chord, no special attention need be paid to the middle vertical, as it then follows the general rule.

26. Stresses in the Chords.—As the equilibrium polygon $A' F' I' K' D'$ alone is needed for determining the maximum bending moments at all points, we have simply to multiply each ordinate under a joint by the value of H from the stress diagram, and divide by the height of the truss, to obtain the chord-stresses. For suppose that we pass a vertical plane of section through the joint F . It follows from § 5 that the moments of the stresses in all of the pieces cut by that section, when taken about any point in the section, must balance, or, in other words, equal, the bending moment at that section. If the origin of moments is at F in the lower chord, and the moment of resistance of all the stresses on the left of the section is desired, as balancing the bending moment of the external forces on the right, we see that the vertical $R F$, the diagonal $S F$, and the chord piece $E F$, terminate at the proposed moment axis or point F ; and hence, having no arms, their stresses have no moments. There only remains the compression in the piece $S R$ of the top chord, multiplied by $R F$, the perpendicular to it from F . As this moment must resist the bending moment, we divide the latter by $R F$ to find the stress in $S R$. Hence follows the opening statement of this section. Similarly, taking moments round R , and conceiving them, as taken on the right of the section, to balance the equal bending moment on the left, all the pieces which meet at R have no moments; and there

remains the tension in FG multiplied by RF , its lever arm. The same steps will give the chord-stresses in the other panels. As, with a uniform load from A to D , no counters are required, the shear diagram being simply $abvcd$, or the tendency of the panels to change their shape being such that the ties which slope from G and J toward the centre will slacken, the counters may be disregarded in determining chord-stresses.

27. Chord-Stresses (*continued*). — It follows that the compression in SR equals the tension in FG ; and the same equality is true of BS and EF and of the other portions of the chords. This fact may also be readily seen if we pass an oblique plane of section through SR and FG : the only other piece cut by the section will be RF ; and as this vertical cannot carry a horizontal stress, and the external forces are all vertical, the stresses in SR and FG must be equal and of opposite kinds, so that those pieces of top and bottom chord which lie between two diagonals in action will have the same amount of stress. That the upper chord is in compression, and the lower chord in tension, follows from the tendency of the load to make the truss concave on the upper side, or to shorten the top chord and lengthen the bottom one. Of course the compression or tension is constant for a panel-length, and the stresses are symmetrically distributed with regard to the middle of the span. As there is no bending moment at the abutments, there is no stress in AE or LD ; and these pieces are unnecessary for the equilibrium of the truss. If the bridge-seats came directly under the top chord, AE and LD might be left out; but, as it is, they lend aid to the bottom lateral bracing in stiffening the bridge against wind and vibration. The counters being neglected, it is evident that two panels of the top chord at the middle have the maximum stress, and that no equal tension occurs in the bottom chord. The reverse is true if the diagonals are struts.

To find, therefore, the tension and compression in the bottom and top chords between any two adjacent diagonals which incline the same way, it is only necessary to select the ordinate to

the maximum equilibrium polygon under the common panel joint of the two chord pieces, multiply by H , and divide by the height of truss. Since H is constant, and the height of the truss, in trusses with parallel chords, is also constant, if H is numerically some simple factor or multiple of the height of the truss, the whole operation can be performed by changing the scale by which the ordinate is measured.

28. Parabola for Chord-Stresses.—Availing ourselves of the last suggestion, we are able to draw a figure for chord-stresses at once. That the vertices of the equilibrium polygon for an equal load on each joint of a bridge-truss lie on the parabola which is the limit of the polygon when its sides become infinite in number, or, in other words, when the load is uniformly distributed over the truss, may be proved by taking moments at any joint. If, then, we draw a parabola below the truss whose middle ordinate is equal to the maximum stress in the chord at the centre of the span when the load is uniformly distributed over the span, the ordinates at the several joints will give all the chord-stresses directly.

As the bending moment at the middle of a beam which is uniformly loaded with a total load W'' is equal to the supporting force at one abutment, $\frac{1}{2} W''$, multiplied by its arm, one-half the span $= \frac{1}{2} l$, minus the weight on the half span, $\frac{1}{2} W''$, multiplied by its arm reckoned from its centre of gravity, $\frac{1}{4} l$, or

$$M = \frac{1}{2} W'' \cdot \frac{1}{2} l - \frac{1}{2} W'' \cdot \frac{1}{4} l = \frac{1}{8} W'' l,$$

or is equal to one-eighth of the total load multiplied by the span, the maximum stress in one chord of a truss of the height k will be, at the mid-span,

$$\frac{W'' l}{8k},$$

and this will be the value of the middle ordinate.

Therefore proceed as follows: On a horizontal line $A B$, Fig. 8, equal to the length of the span, lay off the panel joints D, E, F, G , &c. Draw $C I$ and $A P$ vertically, each equal to $\frac{W'' l}{8k}$, $C I$ being at the middle of the span. Divide $A P$ into

the same number of equal parts with AC ; thus finding the points K, L, N , and O . Draw IO, IN, IL, IK . The points R, S, T , and U , where these lines cut the verticals dropped from the panel joints, will determine the desired ordinates CI, GR, FS , &c., which are the stresses in the chords from the middle to the abutment. Since the lines which radiate from I cut from the vertical PA the distances $PO, 2PO, 3PO$, &c., and the points R, S, T , &c., occur at horizontal distances of $CG, 2CG, 3CG$, &c., from I , it is easy to see that the vertical distances of R, S, T , &c., from I , vary as the square of their horizontal distances, and that these points will, therefore, lie in a parabola. The other half of the figure may be completed if desired.

Figs. 7 and 8 will thus give at once all of the desired vertical and horizontal forces in parallel-chord trusses with one system of bracing. The diagonal stresses will be obtained by drawing lines as described in Fig. 7, parallel to the braces. For the small number of necessary lines, and the exactness of intersections and measurements, it is believed that these two diagrams will compare very favorably with any others. In Fig. 11 they are given with no repetitions or useless lines, to show how small is the required work.

29. Sections of Pieces.—Each part of the truss which undergoes tensile stress should have its effective or smallest section equal to the quotient of the maximum stress it must exert divided by the safe working stress on the square inch. The cross-section of pieces in compression should be determined by Gordon's Formula, or some similar method. The working out of details does not come within the scope of these pages, and we will refer the reader to books which treat of the strength and resistance of materials. Some additional matter which appropriately comes in this connection may be found in Part I., "Roofs," Chap. X.¹ Pieces liable to alternate compression and extension should be made of larger cross-section than for

¹ Any reader who is taking up this subject for the first time, and unassisted, will find much valuable information, presented in a simple form, in Boller's *Iron Highway Bridges*, published by Wiley and Sons, New York.

one stress only, as the safe working stress on the square inch is less. This safe stress is also taken as a smaller quantity for web members near the middle of the span (or for any others which experience the maximum stress, especially if accompanied by shock, every time a load passes across the bridge) than for those members which only have the greatest stress when most or all of the bridge is loaded with the heaviest possible load; a contingency which occurs less often in large spans than in those of moderate length.

30. Moment Diagram for Partial Moving Load.—If a load W is placed at any point C , Fig. 9, distant x from one support of a beam AB whose span is l , the equilibrium polygon will be $A'C'B'$, and the stress diagram will be drawn on the line 1-2. The supporting force at A , obtained from similarity of triangles in the stress diagram and the equilibrium polygon, is $W \frac{l-x}{l}$, and the bending moment at C will be $\frac{W}{l} (l-x) x$.

As this moment varies as the product of x and $l-x$, the two segments into which C divides the span, and as the maximum ordinate occurs at the weight, the maximum moment at each point of the beam, as the weight W rolls across it, will be represented by the ordinate to a parabola, drawn through A' and B' ,

and having its vertex at a distance $\frac{W l}{4 H}$ or $\frac{1}{4} W l$ vertically below the middle of the beam, according as the ordinate is or is not to be multiplied by H to give the bending moment. This parabola is shown by the dotted curve $A'D'B'$.

If the load is distributed over a certain space CF , in place of being concentrated at a single point, we can place the load first on the middle of the span, and, by a stress diagram or otherwise, draw $A'D'B'$. The diagram is $2'1'0'$. The lines $2'-0'$ and $1'-0'$ will give $B'D'$ and $A'D'$, which, as explained in § 10, will meet on the vertical dropped from the centre of gravity of the imposed weights. If the centre of gravity of these weights is placed at the middle of the span, and they then occupy the positions C , E , D , and F , the true equilibrium poly-

gon will be $A'E'G'F'B'$, of which the part $E'F'$ is readily constructed from the points of division on $2'-1'$. The maximum ordinate is at G' , and is less than that for the concentrated weight by the quantity $G'D'$. When the loads are moved until their centre of gravity is over I , the polygon passes through K , a constant vertical distance within the parabola $A'D'$, by § 10, depending upon the amount of load on one side of D' , giving the curve shown within on the right. If the maximum ordinate, at G , lies to the left (or right) of D' , draw lines such as $A'C'$, $A'I$ (or $B'I$, $B'C'$), etc., and set off vertically above these lines, at the horizontal distance from C' , I , etc., that the maximum ordinate lies to the left (or right) of D' , the distance by which the maximum ordinate falls short of the line $A'D'$ (or $B'D'$). The absolute maximum ordinate occurs when the middle of the beam bisects the horizontal distance between G and D' . For another solution see § 54, p. 63. When the load begins to pass off the span at either end, the two curves approach one another.

Such a moment diagram as Fig. 9 may be useful when the requirements for a highway bridge are, that it shall support a certain distributed load, or a certain other load concentrated on the wheels of a wagon. The diagram also applies to a locomotive in connection with a lighter train: it will be referred to again.

31. Shear Diagram for Partial Moving Load.—The diagram for shear for a single load at the middle of the span will be the two equal rectangles $aighd$ and $dghkb$ of Fig. 10, the height ai and kb of each being one-half the load. If the load is moved to c , we have $tsuv$. If it is put at one-quarter of the span from one end, one rectangle will be one-quarter of the load in height, and the other three-quarters of the load: hence we see, that, as the load passes across the span, the maximum shear at successive points will be found by drawing ordinates to an inclined line which reaches from one extremity b of the span to a point l , at a distance al equal to the load, above the other extremity a . If the load is distributed according to any law over a definite area, the shear line will fall, according to the

intensity of the load, over that portion of the span which is loaded. Thus, for the four concentrated loads at *c*, *e*, *d*, and *f*, corresponding to those of Fig. 9, the shear diagram changes to *irqp onmk*. If this load had been spread over the portions from *c* to *f*, the dotted line would have taken the place of the broken line; and, as before, it can be used for the broken line, if the ordinates are measured at points midway between the weights. The maximum ordinate is now just before *c*, but still equals *ai* or *dh*, the ordinate at the centre of gravity. Hence, when the system of loads is moved, as there is always a definite amount of load in advance of the centre of gravity, the maximum ordinate will be found by drawing a line parallel to *bl*, and at a distance below it determined by the position of the end of the load, as in this figure at *rs*.

32. Stresses due to Locomotive.—While it has thus far been considered that the load on a truss, or the weight of a train, is uniform per foot, it is manifest that the locomotives at the head of a train are heavier per foot than the cars, and that such additional weight should be allowed for, more especially in designing the bracing. If we find the *excess* of such load over the previously-considered uniformly-distributed load, and then draw moment and shear diagrams for this excess of load, distributed over its proper space as it moves across the span, these diagrams can be added to the ones previously obtained, and the solution will thus be completed. As the maximum moment for a concentrated load occurs when the weight is placed at the middle of the span, there is not so much need of adding its diagram to that for uniform load. At the time the load in question is in the middle of the span, the train can cover but half of the bridge; and, by the time the train covers the greater part or the whole of the span, the locomotive is just leaving the bridge, and the ordinate for the weight in that position will be very small. If, however, some car of the train is liable to carry a heavy load, — such as a locomotive *in transitu*, heavy castings, or ordnance, — this diagram is very properly added. The shear diagram for concentrated weight is particularly applicable to railroad bridges.

These two diagrams are added to the usual ones in Fig. 11. In adding them to figures already drawn, we must see that the moment diagram has the same value of H as the large one to which it is joined; or, if the ordinates themselves represent chord-stresses, that the scales shall be the same, and that the two shear diagrams shall have the same vertical scale.

Modern methods of designing call for a consideration of the maximum and minimum stresses in the several pieces. The equilibrium polygon for steady load will give the minimum chord stresses, or they may be found by proportion from the stresses for full load. A shear curve like $t r m$, Fig. 6, may be constructed on $l v$ and $v c$, which will determine the minimum shears in the successive panels from the left, or the ordinates to $r m$ in the right-hand panels may be used on the left with their sign reversed.

CHAPTER III.

TRUSSES WITH HORIZONTAL CHORDS. — SINGLE SYSTEMS. — VARIOUS TYPES.

38. General Example. — In the illustrations annexed to Fig. 11, the span is taken as 120 feet, height of truss 15 feet, rolling load for one truss 1,000 lbs. per foot, steady load for same 500 lbs. per foot. The maximum ordinate for chord-stresses will be, by § 28,

$$\frac{(60,000 + 120,000) 120}{8 \times 15} = 180,000 \text{ lbs.}$$

N O is made equal to this amount at the middle of the span, and P Q equal to the same at the abutment. P Q is divided into the same number of equal parts (here four) as there are panels in the half-span; and lines from O to these points of division cut off the chord-stresses at N O and the points S, U, and W.

The abutment reaction of truss and complete load will be half $(60,000 + 120,000) = 90,000$ lbs.; and of truss alone, half $60,000 = 30,000$ lbs. Lay off these quantities at st and pq respectively, connect the points t and q with o , the middle of sp , drop verticals from panel joints to these inclined lines, bisect the parts so obtained, connect the points of division on to and oq as directed for Fig. 7, and the shear curve is completed for the above load.

If the train may contain or be drawn by an engine having 56,000 lbs. on three pairs of drivers, 14 feet wheel base, the load for one truss will be 28,000 lbs. on 14 feet; from which we must deduct 14,000 lbs. mean rolling load already consid-

ered, and we have 14,000 lbs. extra weight to be allowed for. We may consider this load as concentrated on one panel joint, or as distributed to the adjacent joints. The first supposition will give the most stress. The chord-stress at the middle will be, by § 30,

$$\frac{14,000 \times 120}{4 \times 15} = 28,000 \text{ lbs.},$$

which is plotted above the horizontal line, at N Y, and the parabola completed. The combined chord-stresses are thus obtained by scaling from the vertices of one polygon to the corresponding vertices of the other, Y O, R S, T U, and V W.

For shear, lay off the excess, 14,000 lbs., at $s r$, below the line $s p$, and draw a straight line from r to p . As the ordinates to the shear curve give stresses in the web when the load extends from the right abutment to the panel in question, it is apparent that this construction will give at the panel joint the additional amount of shear due to the engine at the head of the load. As the shear is measured in the middle of each panel, move $r p$ parallel to itself until the point p on $r p$ falls in the middle of the first panel on the right: the alteration is too small to be seen in this figure. By laying the diagram off below the horizontal or base line, while it is still considered positive, the two ordinates at any panel are at once combined. Lines drawn parallel to the braces, from the upper ends of these ordinates, and limited by horizontal lines from the lower ends of the ordinates, as shown in the figure, will give the stresses in the braces. It will be seen that the distance over which braces that incline one way extend is prolonged farther beyond the middle; the limit now being fixed by the point u , where the shear curve crosses the line $r p$.

Remember, that, if the *additional* load is not concentrated at one joint, the new lines for chord-stresses and shear will be a certain constant, vertical distance within the ones just constructed, as explained in §§ 30 and 31.

✓ 34. **Howe Truss.**—Truss I of Fig. 11 is generally called,

from the inventor who built it in wood, the Howe truss. The diagonals are struts, and the verticals ties (see § 4). The load is upon the bottom chord. If the verticals are iron rods, and the remainder of the bridge is built of wood, this type of truss is well adapted for such materials. Iron bars used in the bottom chord bring the truss into the class usually known as combination bridges. The end posts and end pieces of the top chord have no truss-stresses, but are useful in connecting the top lateral bracing by a stiff frame with the abutment. As generally constructed, the joint at B has not sufficient rigidity for this purpose; so that a pier or abutment panel is introduced. Some designers, however, make a special connection at B, and carry the top chord no farther, the truss then ending as shown at the right.

The stresses in the bottom chord, in the successive panels from the abutment to the middle, will be equal to VW , TU , RS , and YO . In the top chord the compressions will be, from B to the middle, VW , TU , and RS . The compressions in the diagonals which have corresponding letters are ab , cd , ef , gi , and kl . For convenience in screwing up the bridge, and to stiffen the main braces by bolting to the counters where they intersect, most builders carry the counters through all of the panels: they may be of small cross-section. As these bridges are generally covered from the weather, and have, in some rare instances, been blown from the abutments by violent gales, they should be bolted to the abutments, when the counters will resist any sudden gust of wind from beneath. The tensions in the verticals BC , DE , FG , and IK , will be the stresses represented by the vertical lines at b , d , f , and i . The middle vertical, when the truss is completely loaded, and GI and IM are under stress, must carry one panel weight $= 15 \times 1,500 = 22,500$ lbs. This amount is less than the ordinate at i , and therefore of no consequence. By symmetry we get the stresses on the other half of the truss.

35. Pratt or Quadrangular Truss.—Truss II differs from the truss of Fig. 6 in being loaded on the top chord. The

modification at the right end of this truss may be introduced with advantage, in case it is not too expensive to carry the masonry for the bridge seats to such a height. If the bridge seat is at the level of the lower chord, the end post will carry one-half the *total* weight of truss and rolling load, or rt ; for the half-weight at A will come upon this post in addition to the shear in the first panel.

Truss III differs from the truss of Fig. 6 at the ends only. The end diagonal is a strut, which transmits the same amount of force as do the diagonals in the first panels of the preceding trusses. This introduction of *inclined end posts* is very common in iron bridges, and is economical. The first vertical now becomes a tie, which manifestly can carry only a panel weight of steady and rolling load. The stress in the bottom chord will be uniform from A to D, and equal to VW , the stress which would have been found in BD of the truss above.

Authors and engineers are not agreed upon the name by which the type of truss illustrated by Fig. 6 and Trusses II and III shall be designated: it is often named after some designer who invented a modification of one or another detail of construction, with which details a skeleton diagram has nothing to do. Some of the names which have been given are Pratt, Whipple (see § 55), Murphy-Whipple, Linville, &c. We prefer the first title, if it is to be named after any engineer, but would rather style it a *single quadrangular truss*, by which shall be understood a truss with horizontal chords, inclined ties which extend but one panel each, and vertical struts. The modification of Truss III is understood when the qualifying term *inclined end posts* is added. If the rolling load comes upon the lower chord joints, the bridge is called an *over-grade* or *through* bridge; if upon the upper chord, an *under-grade* or *deck* bridge.

36. Comparison of Trusses.—Truss IV, a Howe truss, is seen to differ from Truss I in having its load upon the upper chord; otherwise no separate remarks are necessary. The Howe truss is evidently the reverse of the Pratt or quadrangular truss.

As stated before, it will be seen from these examples that no change in the magnitude of the stresses in *chords* or *diagonals* occurs when the load is shifted from the bottom to the top chord; and hence, so far as they are concerned, it makes no difference whether the steady or rolling load is considered as applied to either or both at once. This truth may also be recognized from the fact that neither the bending moment nor the shear can be changed by moving a load vertically. The stresses on the *verticals*, however, will be altered by such a change, as may be seen by remembering that the vertical and diagonal, which together connect two adjacent weights, transmit the same amount of vertical force.

37. Warren or Triangular Truss; every Joint Loaded. —

The truss marked V is usually known as the Warren Girder; although the name Triangular Truss is often used, and is appropriate. Every joint of the bracing is loaded; the loads on the bottom chord at B, E, &c., being transferred to C, F, &c., by the vertical suspending rods B C, E F, &c. These rods carry whatever load can be placed at their lower ends. No part of one chord has the same stress as any part of the other chord; but a section at successive joints, by the usual analysis, necessarily determines the stress in the opposite pieces of the chord: thus the ordinate of the diagram for chord-stresses, which comes above C, will be the tension in A D; the ordinate above D will be the compression in C F, &c. The stresses in the braces will be found as usual; but, by reason of the alternating inclinations of the pieces, they will be successively compression and tension from the abutment to the middle. The same rule will hold beyond the middle as far as counters may be necessary; so that a certain number of the inclined members must be struts and ties alternately, as they change from main to counter-braces under a passing load. In this particular example one piece each side of the middle must be designed to resist tension as a main brace, and a small amount of compression as a counter-brace. As it is of doubtful economy to have one piece repeatedly undergo a reversal of stress, some builders use a hollow

strut with light tension rods within it when a main-compression member has to act as a counter-tension member. Tension bars are stiffened against a moderate compression by a light lattice bracing of flat bars riveted to the edges.

Sometimes, though rarely, this truss is loaded upon the top chord, when this figure inverted will represent the case: B C, E F, &c., must then be replaced by vertical struts, and the series of inclined members will begin with a tie.

38. The Same ; alternate Joints Loaded. — When the verticals are omitted, the load will come upon alternate joints, as seen in Truss VI. The shear ordinate will be measured midway between the loaded points, as usual, and belongs to the diagonals on each side of it; so that the diagonals in pairs will have the same amount of stress: that is, the compression in A B equals the tension in B C, the compression in C D equals the tension in D E, &c. For this truss also, K L will be a strut, and L M a tie, when the rolling load extends from the right abutment to M: but, when the load covers A K only, these pieces undergo stresses equal to those in G I and I K for a load on the right segment up to K; K L will then be a tie, and L M a strut. Web members near the middle of the truss must therefore be adapted to both compression and tension. Lines from *b*, *d*, *f*, *i*, and *l*, in the shear diagram, parallel to these braces, will give the amount of stress in each, as usual.

The stresses in the pieces of the top chord will be given by the ordinates over the corresponding joints of the bottom chord, as was done for both chords of Truss V. But, to find the tension in A C of the bottom chord, the plane of section must pass through B, and the ordinate will therefore be the one at B' in the diagram below the truss. The tension in C E will be given by the ordinate at D': hence the ordinates at B', C', D', E', &c., will be the stresses in those pieces of the chords which lie above them. The funicular polygon is drawn as usual, with angles under the loaded joints; but additional ordinates are drawn to the middle points of the sides.

When the top chord is to be loaded, the truss may stand as

here sketched, or be inverted, and the web system will then begin with a tie. Shifting the loads from the bottom to the top chord moves them laterally half a panel, if the truss is not inverted; and hence the chord-stresses are changed: the method of analysis is not affected, however.

89. Comparison of Trusses.—We show in the annexed table, in one view, those parts of the six trusses which have the same stress. The first portion of the table is devoted to the chord pieces which correspond to the ordinates V W, T U, R S, and Y O; and below them will be found those web members of each truss whose stresses have *vertical components* equal to $b v$, $d w$, $f x$, $i y$, and $l z$. The span and the load are in each case the same.

Truss	I.	II.	III.	IV.	V.	VI.
Ordinate.	{ A C	{ A C	A D	{ A C	A D	B D
V W =	{ B D	{ B D		{ B D		
T U =	{ C E	{ C E	{ C E	{ C E	C F	D F
	{ D F	{ D F	{ D F	{ D F		
R S =	{ E G	{ E G	{ E G	{ E G	D G	F I
	{ F I	{ F I	{ F I	{ F I		
Y O =	G K	G K	G K	G K	F K	I L
$b v =$	{ A B	{ A B	A C	A B	A C	{ A B
	{ B C	{ B C				{ B C
$d w =$	{ C D	{ C D	C D	{ B C	C D	{ C D
	{ D E	{ D E		{ C D		{ D E
$f x =$	{ E F	{ E F	{ D E	{ D E	D F	{ E F
	{ F G	{ F G	{ E F	{ E F		{ F G
$i y =$	{ G I	{ G I	{ F G	{ F G	F G	{ G I
	{ I K	{ I K	{ G I	{ G I		{ I K
$l z =$	K L	K L	{ I K	{ I K	{ F G	{ K L
			{ K L	{ K L	{ G K	{ L M

We have added to the trusses just discussed two other types, resulting from efforts of the designers to invent trusses which shall carry additional loads without an increase in the number of panels; that is, when the panel joints become so far removed from one another, by reason of the height of the truss combined with the desired angle of inclination of the diagonals, that it is thought best to concentrate loads at intermediate points, they

have introduced short auxiliary pieces to support such loads. Modifications of Truss VII. are becoming very common for bridges of long span. (The following section may be omitted until part of Chapter V. has been read.)

omit 40. **Baltimore Bridge Co.'s Truss.**—Pratt or Single Quadrangular Truss with Intermediate Braces.—Each short vertical of Truss VII. transfers the load at its foot to its upper end. The amount of force in any short brace, its horizontal component, and its vertical component or portion of the load on the short vertical carried by the short brace, may be found thus: At the point M meet four pieces, M A, M B, M N, and L M. The weight at N is the tension in M N; lay it off downwards in as , Fig. 13. Proceeding in left-hand rotation about M, draw sq indefinitely, parallel to M B; return on the same line, parallel to M A, to a point t , so that ta , parallel to L M, will close on a . The compression in L M is ta . The short brace in each panel will therefore have a vertical component of one-half of whatever load is at N, P, R, &c. The horizontal projection of ta will be the horizontal component of the force in the short brace, and will be the difference between the compression in A C and the tension in B D, or in C E and D F, &c.

At any vertical or oblique section of a truss under vertical forces, the sum of the horizontal components or forces in the pieces cut by the section must be zero, and the sum of the vertical components on one side of any vertical section must equal the shear. Also, a member which really assists in carrying positive shear must, if a strut, slant *down*, and, if a tie, slant *up* toward the left abutment, and *vice versa* for negative shear. If a member inclines in the other direction, the remaining inclined members cut by the vertical section must resist the shear and the component in the first member besides. Figs. 30 and 31 will illustrate this matter later.

Drawing the diagrams marked Fig. 12, we see that A C, C E, E G, and G K of the top chord of Truss VII. will have compressions equal to the ordinates B' F', C' G', D' I', and E' K', here drawn on the right to save space. The tension in L B will be

the horizontal component of the thrust in LM for a full load at N , or the horizontal distance from t to a , Fig. 13, if $a's$ is a panel weight of truss and rolling load. The tensions in BD , DF , and FI will exceed the stresses in AC , CE , and EG by this same amount. Therefore set off this distance vertically above $A'E'$, draw a horizontal line, and measure the bottom chord ordinates to it.

From the shear diagram of Fig. 12 it is evident that bn , dp , fr , it , and lv will be the vertical components of the tensions in MB , OD , QF , SI , and UW . From am , eo , eq , and gs should be subtracted half a full panel weight to give the vertical components of the maximum tensions in AM , CO , EQ , and GS . A horizontal line at the proper distance above wu will cut off the four ordinates at once. The compressions in the verticals AL , CB , ED , and GF will also be given by this step. As IU is part of the main diagonal tie, it will not carry positive shear, and KU will have ku for its vertical component, which will also be the compression in KI . UW and FS will sometimes be in tension, as indicated above. When UW is carrying lv , the diagonal vertically above it is slack.

Truss VIII. is Truss VII. inverted. The short verticals become struts and carry a panel weight; the short diagonals become ties and sustain half a panel weight. Fig. 13 is equally applicable. Beginning at the left abutment, we see that the first half of the main diagonal tie has am for its vertical component. The lower half will carry, in addition to bn , a half panel weight arising from the action of the short tension diagonal; for the *algebraic sum* of the vertical components in these two inclined pieces must equal the shear bn , as explained in the second paragraph of this section. The horizontal line must therefore be drawn below wu in this case. The stresses in the top chord will be greater than in panels of the bottom chord lying between the same main diagonals by the amount of the horizontal component of the stress in the short tension diagonal, and the top chord ordinates will now be measured to the horizontal line above $A'E'$.

The inclined portal at the right end of Truss VII. is a modifi-

cation which affects only the end panel. The chord-stress is constant for two long panels, and equal to that in B D.

omit 41. **Kellogg's Patent Truss.** — In this truss, marked IX, the tension in M N will be given by the ordinate for that joint, or B' F'; the compression in C E and the tension in D O will equal C' G'; and in the same way D' I' applies to E G and F P. The middle ordinate belongs to the middle panels of the top chord. The ordinate am will be the shear in A C; dp , the shear in C D; fr , the shear in E F; it , that in G I; but ku gives the shear in the counter K L. The vertical component of the stress in M C, C N, E O, and G P, is the amount of weight at M, N, &c. The horizontal component will be resisted by the lower member. The tension in A M will be the horizontal component of the compression in A C; the stress in M N has already been indicated; that in N D will be greater than the tension in M N by the horizontal component of the stress in N C. The stress in O F is increased by the same amount over that in D O. The force in B C equals the weight at its foot. The compressions in D E, F G, and I K, are given by eq , gs , and ku . A right-angled triangle whose hypotenuse is parallel to C N, and whose altitude equals a panel weight at N, may be drawn to give the stress in C N and the horizontal component to be added for the stress in N D, or the three forces may be compared by the similar triangle B C N.

Any other parallel-chord trusses with single systems of bracing can be analyzed in a similar way.

essential 42. **Plate-Girders.** — Where, from lack of room, we are obliged to make the ratio of the height of a truss to the span small, and where the load per foot to be carried is large, as is the case with short span railway bridges required to carry a locomotive whose drivers will cover a considerable portion of the span, a plate girder may be the most desirable form of bridge. In this case it may be conceived that the web members have been made so thin in the dimension transverse to the span, that they occupy with their other dimension the whole of the web. It is still supposed that the plate of which the web is

now formed withstands all of the shear, and that the moment of the stresses in the two flanges resists the bending moment. As the web is quite thin, this assumption involves no material error. The web is usually of one thickness throughout the span, and hence should be thick enough to withstand the shear at the abutments: it should also be thick enough to resist compression in a direction of 45° with the horizon, of an amount at each section equal to the shear at the same section. If the web, computed by Gordon's Formula, as a column of 1.4 times the height of the girder, and of a thickness required to resist the shear, is not thick enough to carry this thrust, vertical stiffening ribs of angle or T iron are usually riveted to the web at such a distance apart as shall reduce the length of the compressed piece, measured at an angle of 45° between two verticals, to the proper amount for the given thickness and compression. It results that usually no verticals will be required near the middle of the span, but that there will be a less and less interval between these stiffening ribs as we approach the abutments. From practical considerations, the web is generally not less than $\frac{1}{4}$ inch to $\frac{3}{8}$ inch thick, — an amount usually enough to resist the shear.

In determining the cross-section of the flanges, as the girder is frequently of a uniform height from end to end, and the flanges of a constant width, we see that the thickness of the plates of the flange at each section will be directly proportioned to the bending moment or chord-stress. They may, therefore, be laid off on the parabola for bending moment, and their lengths determined. Thus, suppose that the tension or compression in one flange at mid-span, from a uniform load, is 100,000 lbs., that the flange is 8 inches wide, and that the safe stress on the square inch is 8,000 lbs. Every 16,000 lbs. height of ordinate will require $\frac{1}{4}$ inch thickness of flange. If, therefore, we draw the parabola of Fig. 14 to represent the chord-stresses, and draw horizontal lines at intervals, in height, of 16,000 lbs., the length of the successive plates of the flanges may be easily measured off on the scale of the base-line which represents the

span. The thickness of the plates may be made $\frac{1}{4}$ inch, $\frac{3}{8}$ inch, and $\frac{1}{2}$ inch, or as judgment dictates. The lengths of the pieces can now be easily arranged to break joint, and the required cover plates added above to supply the deficiency at joints. When cover plates come near together, they may be combined in one supplementary plate.

Fig. 15 is intended to show how the several pieces of the flanges and web should break joint. The rectangle at A represents the web 3' 6" deep, and $\frac{3}{8}$ " thick, vertical angle irons being placed on the joints and the dotted lines. B gives the bottom angle irons, the top set being similar, and all $3\frac{1}{2}" \times 3\frac{1}{2}" \times \frac{1}{2}"$. At C is shown the bottom flange, the top flange being the same, 12" wide $\frac{3}{8}$ " thick for 40 feet, and $\frac{1}{4}$ " thick for the strip which extends across the whole span. The cover plates for flanges are 26" long. The above dimensions are from an existing girder of 65 $\frac{3}{8}$ ft. span.

43. **Truss of Odd Number of Panels.** — Trusses which have a single system of bracing are usually designed with an even number of panels. But it is evident that an odd number of panels may be used if desired: indeed, as we shall see presently, trusses with a double system of bracing must have an odd number of divisions in one system. The truss of Fig. 16 has five panels and four loaded points, not including the points of support. The equilibrium polygon or the diagram for chord-stresses is drawn below the truss; and it is seen that the maximum ordinate C' O', as obtained by § 28, will not represent a chord-stress. But if we divide B' P into five equal parts, corresponding to the number of panels in the *whole* truss, or twice the customary number of parts, and connect alternate points of division with O' by O' Q and O' R, the ordinates I' K' and L' N' will be the chord-stresses; I' K' existing in E N and F I, and L' N' being found in I L and N O. The shears will be found as usual. The diagonals G I and F K cross the middle of the span, are not under stress for a complete load, and are properly counter-braces. The panel G F I K is sometimes known as a *cross-panel*.

44. Trapezoidal Truss.— Before closing this branch of the subject, we will allude to two or three trusses which should not be overlooked. The trapezoidal truss may be considered a truss of three panels, and will be made from Fig. 16 by removing two panels. Its analysis is, therefore, the same. It is also easily treated by a stress diagram, as applied in Part I., "Roofs;" and the reader is referred to Fig. 14 and § 25 of that part. As the only piece strained to a maximum by a moving load is the counter, or brace in the middle panel, two diagrams — one for a complete load, and one for moving load covering one joint — will suffice.

45. Fink Truss.— This type of truss, illustrated by Fig. 17, and invented by Albert Fink, does not properly belong to the class of this chapter, as it has no bottom chord. When the travelled way passes at a level with the feet of the main posts, there may apparently be a bottom chord by reason of the arrangement of the lower lateral bracing; but such a member is not concerned with the supporting power of the truss. An inspection of the figure will show that each short post will carry one panel weight; each post at the quarter-span, two panel weights; and the middle post, four panel weights. The stress in each tie must have as a vertical component one-half of the weight on the post which it supports. The compression in the top chord will be uniform from end to end. A moving load covering the entire span will cause the maximum stress in every piece. If a section is made at the middle of the span, the stress in the main or longest tie may be obtained by taking moments about the middle joint of the upper member; and, that stress being known, a section at the quarter-span will give the stress in the next tie. But moments taken about the bottom of the centre post will not give the compression in the upper member, as the stress from the ties which meet at the middle joint of the upper member must be added.

A modification of this truss has been used, which is shown in Fig. 18; and there is a bottom chord in this design. We have drawn a half-span, and will leave its analysis, which presents no special difficulty, as an exercise for the reader.

46. **Bollman Truss.**—The truss of Fig. 19, designed by Wendell Bollman, also requires no bottom chord. If the load is upon the lower line, the portion of load at each apex is distributed to each abutment directly by the two independent ties which run from each loaded point. The bridge being loaded throughout, the compression in the upper member will be the aggregate from the several triangular systems, and will be uniform throughout. The verticals will theoretically have no duty to perform: practically they keep the upper member from bending under its own weight and the thrust. If the load is on top, the posts will each carry a panel weight. The inventor added diagonals in each panel, giving a separate means of support for the foot of each post. Most of the ties run at too small an angle with the horizon, and their section will be large for the amount of vertical component which they convey to the abutments. The truss is not an economical type, therefore, and has been but seldom built.

47. **Wind Bracing.**—The wind exerts a thrust against the side of a truss, generally in a horizontal direction; and this thrust must be allowed for. Wooden bridges which are covered from the weather will expose a large surface to the wind; but, in any case, the total pressure will be obtained by multiplying the area of exposed vertical surface by the greatest intensity of the wind pressure per square foot, the latter being taken at from forty to fifty pounds. To the lateral surface of the truss add, for an open or a deck bridge, the area offered to the wind by a train of cars. The wind pressure on the bridge will be a uniformly distributed force, acting horizontally; and it will be resisted by a system of trussing between the two upper chords and also the two lower chords of the bridge, arranged upon any convenient plan. Usually the Howe Truss type of struts diagonal and ties perpendicular, or the Pratt type of ties diagonal and struts perpendicular, to the chords, is used. Both diagonals will be required in each panel of the lateral bracing, as the wind may blow from either side. The pieces of the bracing should, properly, increase in section as they approach

the abutments. Quite often they are made of one uniform section throughout the bridge.

As the cross-section of a bridge is a hollow rectangle, the thrust of the wind tends to rack it more or less. Vertical lateral bracing can be introduced in a deck bridge, should never be omitted when the truss is supported at the ends of the lower chord, and should meet the chords at the same points with the horizontal lateral bracing. In other cases, especially in through bridges, short knee-braces or gusset-pieces are used to give stiffness to the rectangle. Sometimes the verticals, if struts, are spread at the bottom transversely to the truss to aid in giving stability. The wind pressure has an overturning moment equal to the total pressure multiplied by the vertical distance of the point of application of the resultant wind pressure above the abutment; and this is resisted by the weight of the bridge multiplied by its half-width between centres of trusses, or by the tension of bolts which may fasten the truss to the masonry.

48. **Shear shown by the Moment Diagram.**—The following interpretation of the moment polygon may be interesting :

As, by § 7, the shear at any point, such as S, Fig. 4, of a loaded beam or truss, is equal to that part of the load line included by those two lines of the stress diagram which are parallel to the two sides, such as A' B' and D' E', of the moment polygon cut by a vertical dropped from that point—the shears at successive points of a beam are proportional to the inclinations of the two-sides of the moment polygon below the successive points to one another. When the side A' B' is horizontal, the shear may be said to vary as the tangent of the inclination of the side, such as D' E' of the moment polygon ; to be constant over each space covered by a side of the equilibrium polygon ; to change sign with the tangent ; and to pass through zero at the point of maximum bending moment. The *change* in the shear, or the amount of load from point to point, is proportional to the change in slope of the successive sides of the equilibrium polygon.

If $AS = x$, and $IK = y$, we may say that the bending moment varies as y , the shearing force as $\frac{dy}{dx}$, and the intensity of load as $\frac{d^2y}{dx^2}$.

The small parabolic segments between the equilibrium polygon for loads at joints of a truss and the full parabola for the same uniformly distributed load represent the bending moments on the stringers, if the latter are simple beams.

49. Skew Bridges.—Trusses are sometimes built with panels of unequal lengths, to adapt the bridge to some peculiarity of place. Trusses over a skew crossing not infrequently have unequal end panels, in order to bring the floor-beams at right angles to the axis of the bridge. As the structure is not symmetrical, it will be necessary to find the chord-stresses for all panels, and to pass the rolling load across the truss from either end, to complete the strain sheet.

Thus the bridge, one truss of which is shown in Fig. 23, span 105 feet, height 20 feet, has a skew of *five* feet in its width; hence the end panels may be made 10 feet and 20 feet, while the others are 15 feet each. For a load of uniform intensity per foot, the panel weights at B and O will be respectively less and greater than those from D to L. The vertices of the equilibrium polygon or chord-stress diagram will be inscribed in the usual parabola for a uniform load; but it will probably be easier to lay off a load-line and draw a polygon at once, as shown. When the truss is fully loaded from end to end, the two reactions, including the portions for the weights at A and R, will be equal, and, if the pole is taken opposite the middle of the load-line, the closing line will be horizontal. The chord-stresses will be found from the ordinates marked. As, with a full load, the shear is positive in panel F I and negative in panel I L, G I and I N are in action, and the ordinate K' I' gives the compression in G N.

The shear curves must be drawn for loads advancing from either end. The end ordinates will be $ab =$ half the total load, and $rs =$ half the weight of the truss. Divide the inclined lines bm and ms by verticals dropped from the loaded points and by points midway between. Draw 1-1, 2-2, &c. The compression in D E is de ; the tension in E F is ef . The stresses for loads advancing from the left are given by the lower curve.

50. Trusses on a Grade.—Trusses sometimes occur on more or less steep inclines. If the truss is horizontal and the floor-beams alone vary in elevation, the truss is unaffected, except so far as the work done in a brief time in raising a rapidly moving

train may increase the stresses. Since those web members which together connect two loaded points will have the same amount of vertical force, the stress in the lower part of a vertical which carries a floor-beam attached at such a point as P, Fig. 26c, will be given by the shear ordinate for the panel to the left, and the stress in the upper part by the ordinate for the panel to the right.

51. Inclined Trusses, supported on Loaded Chord.—If the truss is inclined, and the rolling load comes on the chord which is supported by the abutments, Fig. 26A, the effect of the inclination is to shorten the span of the equilibrium polygon and shear diagram. The moment ordinate may be multiplied by H and divided by the perpendicular height of the truss between chords; or the chord-stress diagram may be used, as usual. The ordinate which gives the stress in any chord-piece is drawn below the joint opposite the chord-piece, as marked.

The stresses in the pieces A C, I L, L O, and O R of the bottom chord will be found by extending the side of the equilibrium polygon below each one until it meets the ordinate under the opposite top chord joint B, N, Q, or S, and measuring the longer ordinate so found. For the right section of the truss through Q, for instance, cutting L O, leaves the weight at O on its right. This weight, therefore, should not be included in the bending moment on the left of O Q, and the side of the equilibrium polygon should be extended to O' to give the ordinate indicated for L O. The usual ordinate below B will give the tension in C D; the longer one, found by extending A' C' to B', will give the stress in A C. These added quantities are all the same, being due to the moment of a panel-weight with an arm equal to the horizontal projection of a post, as shown in § 10. The stresses in the bottom chord are not symmetrical, owing to the longitudinal components of the weights.

As the chords are inclined, the stresses in such pieces as D E and E F will be found by drawing the right-angled triangle def on the usual shear ordinate. Compare § 63. If $h e$ is the shear on a vertical section, $d e$ will be the shear on a right or normal

section and will be the compression in D E, while ef will be the tension in E F.

52. Inclined Trusses, supported on Unloaded Chord.—When the load is on the chord which is not supported by the abutments, Fig. 26B, the inclination of the truss throws all of the loads a certain distance to one side, so that the distribution resembles that of the skew bridge. Whether the first and last panel-weights, B and S, shall be different from or equal to the others, depends upon the way the approaches are made. Follow the construction for the skew-truss in the former case. In the latter case, represented by Fig. 26B, the reactions will be unequal, but the panel-weights will be alike. The loads, here seven in number, are laid off on the load-line, the polygon drawn, and the reactions at once found. The sides of the equilibrium polygon will be prolonged as before, in some cases. The ordinates and the respective chord-pieces to which they belong are marked.

Lay off the above reactions in the two end panels of the shear diagram, at *half a panel distance* from the loaded joints B and S, as shown, so that the ordinates will be equidistant, and complete the shear diagram as usual. The web stresses will be found from the shear ordinates, as in the previous case, and as indicated in this figure.

53. Shear Diagram for Concentrated Loads.—The amount and distribution of the loads to be used in designing a bridge are frequently prescribed specifically. When wheel weights must be used, the case will be similar to that shown in Fig. 29A. Conceive this train to be reversed, and the head of the engine to be at B, ready to move toward A. If the first wheel weight is supposed to roll across A B, from B to A, the shear at any point in advance of that wheel will, by § 31, Fig. 10, be given by the vertical ordinate to the straight line B b , measured at the point where the wheel then rests, if A b is the weight. When the first wheel has reached C, the second wheel enters at B. The successive shears due to the second wheel, and in advance of it, will be given by vertical ordinates between the lines B b and B c , where $b c$ is the second weight. In order to have these two shears

added together at the first wheel, the line Bc may be moved parallel to itself—that is, a line parallel to Bc is drawn through the point where Bb cuts the vertical through C . When the first wheel arrives at D , the third wheel enters the span, a line parallel to Bd will be drawn through D , and so on. A broken line is thus constructed whose ordinates will give the shear at the head of the train, as it advances from the right.

To construct this shear line most easily, set off the positions of the given engine and train weights from B , *in reverse* as shown, the first wheel being at B . Erect verticals at the several points, and at convenient intervals on the uniformly loaded portion. Set off the loads in the same order upward from A , Ab being the first engine weight. Draw BC , CD , DE , EF , &c., in succession, parallel to Bb , Bc , Bd , Be , &c. To the left of any point, such as G , the shear will be GI , when the first wheel of the engine stands at G and the engine and train cover GB . The several intercepts of GI by lines from B , C , D , &c., parallel to Bc , Bd , &c., would be the partial shears from the several wheel weights.

If AB represents the span of a beam or plate girder, the shear at any point, such as G , may be greater when the head of the train has advanced further to the left; for, while the weights then to the left of G must be subtracted from the ordinate at the head of the train, the movement of so many loads towards the left may have increased that ordinate to a greater amount. Draw at G the broken line GKN , whose vertical distances correspond to Ab , $b c$, &c., and horizontal distances to BC , CD , &c. When the first wheel is at G , the shear to the left of G is GI . When the first wheel is at J , the shear between J and G will be $JL-KJ$ or KL . When the first wheel is under N , the shear just to the left of G will be NO , and so on. It is found by measurement that KL is the maximum shear just to the left of G . By drawing GKN , with base GA , on a separate piece of paper, one can slip it along from point to point and find maximum shears with ease. Carefully ruled cross-section paper is very convenient for use.

In applying this shear diagram to any bridge-truss, as to the Pegram truss above it, when the head of the train advances beyond a panel-joint, we deduct from the ordinate at the head of the engine only so much of the load in the panel as is carried by the stringers to the panel-joint in advance. Since the stringer, for a span of one panel, acts like the beam $A B$, set off $A P$ to represent a panel length, and draw $P b$; make $P Q$ horizontally equal to $B C$, draw through Q a line parallel to $P c$, make $Q R$ horizontally equal to $C D$, and draw through R a line parallel to $P d$. Draw $P Q R$, &c., on a separate piece of paper. Place P at the several panel-joints in $A B$, and scale in each panel the longest ordinate, at P , Q , or R , as the case may be. From the relative rates of slope of $P Q R$ and $B F I O$ it will be seen that the shear will be maximum at the head of the train when near B , and that the point of maximum shear will shift successively to wheels further back as the train advances to A . If the panels are so short that the head of the train gets into the next panel before the maximum shear is found in a particular panel, the whole amount of wheel weights in that next panel is to be subtracted.

Shears from steady load must be found separately, by Fig. 29c, and added algebraically. Since they will exceed the live load shears on the extreme right, the latter need not be scaled, unless maximum and minimum stresses are wanted.

54. Maximum Bending Moments and Chord-Stresses from Concentrated Loads.—The truss above Fig. 29A represents a bridge of 200 feet span, and this bridge is to be subjected to a moving load of 3,000 pounds per foot, headed by two engines, as marked on the line $A B$. The first wheel is placed wherever convenient, as at B , and the train is supposed to extend indefinitely beyond A . Drop verticals from the loaded points and at convenient equidistant intervals from the portion covered by the uniform load. Lay off to a scale of tons or pounds, on the vertical line $A X$, the wheel loads in order, beginning at A with the wheel load which rests at B , and follow with the uniform load. Assume a pole S , and proceed to draw the equilib-

rium polygon $A' B'$, Fig. 29B, for this moving load only. The portion to the right of B' , in advance of the engine, will be a straight line. If the truss is placed above the train as shown in the figure, $A' B'$ will be the closing line, and ordinates under the several joints will give bending moments. From them we may obtain the chord-stresses for the given position of load; but these stresses will not be maximum ones. Space off some more panel distances at each end. As it is easier in the drawing to move the span $A'' B''$ than the moving load and its accompanying equilibrium polygon, imagine the bridge moved successive panel distances to the right and left. If the span is moved one panel to the right, $W U$ will be the closing line, and the first engine wheel will be at T' ; if the span is moved one panel to the left, $T V$ will be the closing line, and the first wheel of the tender will have nearly reached B'' . The other closing lines shown in the figure may now be added, all having a constant horizontal projection equal to the span, and cutting off ordinates proportional to bending moments at joints as the load traverses the bridge.

The ordinates shown cut these lines into panel-lengths. If the curve 1-1 is sketched through the points at one panel-length horizontally from their right extremities, the vertical ordinates between the curve 1-1 and the equilibrium polygon will give all the bending moments at joint T' as the train passes, and on a drawing of reasonable size it is easy to select the maximum value. It is noticeable that the bending moment remains very nearly constant for a considerable movement of load. The maximum ordinate comes under a weight, and may be found quickly by scale; it may be looked for under one of the drivers.

The curve 2-2 cuts the closing lines at points distant horizontally *two* panels from their right extremities, and from it is found the maximum ordinate for the second joint. Similarly the curve 3-3 is found for the joint *three* panels out from B'' .

The dotted curve, between 1-1 and 2-2, belonging to the top joint a little to the left of T' , will furnish the bending moment

ordinate for maximum stress in the second panel of the bottom chord.

When the uniform load is heavy, it may be well to see whether the moment at any joint is greater for that load than when the engines are on the bridge. Moments for uniform load may be found on the left portion of this polygon or by the figure to the right of 29c, which is intended to be used for the truss-weights.

Since this equilibrium polygon for engine-weights does not depend upon the truss, it may be carefully drawn in ink to a satisfactory scale; the closing lines and joint curves 1-1, 2-2, &c., can then be constructed in pencil for any proposed truss.

CHAPTER IV.

TRUSSES WITH HORIZONTAL CHORDS. — MULTIPLE SYSTEMS.

55. **Double Quadrangular Truss.**— When, from length of span and corresponding economical height of truss, with a single system of bracing at the usual inclination, panel joints come too far apart, it is frequently the practice to add another independent system of bracing. Such trusses will now be discussed. The truss of Fig. 20 is the first one to be taken up. Where the compression members of the web are vertical, and the ties inclined, there seems to be a general agreement that an inclination, for the latter members, of 45° , is the most economical: hence follows this type of truss when the height is much more than a desired panel length. The name of *double quadrangular* truss may be used to designate it, distinguishing it from the single quadrangular of a preceding section. Many apply the term *double intersection*, meaning that the tie crosses two panels: it is also known as the Whipple Truss, and is probably better entitled to that name than is the truss of Fig. 11, II, III. An inspection of the figure will show that there are two independent systems of bracing. The two trusses thus formed might, therefore, be analyzed separately, and the chord-stresses found in the complete truss by the addition of the two stresses which would co-exist in each panel. Thus one system would give a certain compression in B F, the other system a compression in D I: the addition of these stresses would give the stress in the piece D F.

It will be found, however, comparatively easy to treat the truss as a whole. The bending moments and shears being independ-

ent of the type of truss, the two diagrams for these quantities will be drawn as usual, and are seen below the truss. The ordinates in the first diagram may as well denote chord-stresses for a single truss of ten panels, as the bending moments will all be divided by the constant height A B.

When the truss is fully loaded, the counters, represented by dotted lines, will be free from stress, and the middle ordinate, N' O', will be the compression in the top chord from I to S. As I N carries half a panel-weight, the horizontal component due to that amount of load is to be subtracted from the stress in I L to give the tension in K N. The stress in G K will be less than that in F I by the amount of the horizontal component in F K; and so on. Then, let C = compression in any piece of the top chord, such as D F. Let T = tension in piece of the bottom chord E G, on the other side of a vertical section at E. Let M = bending moment at this joint, and h = height of truss. Taking moments about the point where the diagonal crosses E F,

$$C \cdot \frac{1}{2}h + T \cdot \frac{1}{2}h = M, \text{ or } \frac{1}{2}(C + T) = M + h.$$

As $M \div h$ = ordinate E' F', it is evident that T must fall short of E' F' by the amount that C exceeds it.

Hence follows the rule: Project O' horizontally to T', and lay off T' L', the intercepted portion of the vertical that is below L', at L' U'. K' U' will be the stress in K N, and also in F I. Project U' similarly to V'; lay off V' I' at I' W'; and so proceed as indicated in the figure.

It will now be found, from the properties of the parabola, that the distances so spaced off increase in arithmetical proportion from the middle to the end, and that the distance A' B' is the first subdivision of the end vertical of § 28. Hence: Draw a straight line from B', through N', to the other end of the figure; the ordinates from the vertices of the equilibrium polygon to this line will give the chord-stresses, those on the left of the middle belonging to the adjoining panels of the lower chord on the side next the middle, and those on the right belonging to the adjoining panels of the upper chord on the side next the abutment, as marked.

56. Stresses in Diagonals and Verticals.—Turning our attention to the shear diagram, we see that the shear in the first panel, AC , is equal to cf , the ordinate below the middle of this panel, and that this shear must be divided between the two ties in this panel, BC having a little the larger portion, as the set of trussing to which it belongs has one more loaded joint than the other system. When BC is strained to its maximum, the load must cover the whole truss, including the joint C : therefore the counters will not be in action, and IN will have a stress due to one-half the load at N . To this vertical component as it passes through GD will be added the load at G , the corresponding load at R being carried to the other abutment; then the load on GD must pass through BC , and, in addition to it, the load at C , making the vertical component of the tension in BC equal to two and one-half panel weights. Since there are ten panels in the span, this amount is one-quarter of the entire weight of the truss and load. As ab represents one-half of the same combined weight, divide ab at its middle point d , and project d horizontally to u ; then ad , db , or cu , will be the shear in BC , and uf must be left for BE .

When we pass to the next panel CE , the total shear will be ei , distributed in BE and DG when the load extends from T to E inclusive. But the removal of the load from C can produce no effect on BE : hence uf will still be the shear in BE ; and, if uf is laid off at ev , vi will be the shear in DG . Thus all the construction necessary is, bisect the end vertical ab , lay off by scale or parallel lines the upper half at the bottom of the next ordinate, then the upper portion of that ordinate at the bottom of the next, and so on, when the two portions of each ordinate will be the shears in the two diagonals in the panel whose middle it occupies, the lower portion belonging to the lower diagonal, and the upper portion to the one above. By drawing lines parallel to the ties, the stresses in them are obtained. The verticals carry the stresses of the diagonals which run to their upper or unloaded ends.

57. Inclined End-Posts.—Fig. 21 shows the usual modification of the ends of this truss. Such a change introduces a certain though generally trivial indetermination in the calculation of the stresses: it arises from the fact that the weight at W, conveyed to X by the tension rod W X, may be classed with the system of weights which affect X R, R S, S N, &c., or with X U, U V, V P, &c. If we use the shear diagram of Fig. 20, or imagine the truss of that figure to be modified at the ends, we see that the shear in T X will be cf . The total shear in the next panel U W will be ei . The rolling load previously at W has been removed for maximum shear in the panel U W, and hence there is a doubt whether its removal has affected X U, X R, or both. If it belongs with the weights at R, N, &c., the previous analysis for shear is correct, and ev will belong to X U, while vi will be the shear in X R. If, on the other hand, it belongs with the weights at U, P, &c., X R will have a greater stress, and X U a less, than just given. When the panel weight of rolling load w' was removed from W, the reaction at T was, for this example, diminished by $0.9 w'$, and at A by $0.1 w'$: therefore the shear in X U, on the second supposition, will be $ev - 0.1 w'$, and in X R will be $vi + 0.1 w'$. The alternate points v, w , &c., will therefore move down and up by this constant amount; and the indeterminateness lies within the amount $w' \div N$, where N = number of panels. The stress in T U will be given by the first ordinate, C' D'.

What portion of the shear in X U this change affects may be seen as follows: If w = panel weight of steady load, $bd = \frac{1}{2} ab = cu = \frac{1}{2} N (w + w')$, $cf = \frac{1}{2} (N - 1) (w + w')$, $ev = uf = cf - cu = (\frac{1}{2} N - \frac{1}{2}) (w + w')$; then the shear in X U may be altered to $(\frac{1}{2} N - \frac{1}{2}) (w + w') - \frac{w'}{N}$. The variation in amount will not usually be more than one or two tons, changing the cross-section an almost inappreciable amount.

58. Odd Number of Panels.—If this type of truss, Fig. 22, has an odd number of panels, the method of finding the stresses in the chords will not be changed; but it will be noticed that

K N, the middle panel of the lower chord, has the same amount of stress as exists in F V. By referring to § 43, Fig. 16, the change in the moment or chord-stress diagram will be seen, and the analysis may be conducted as usual. When we turn to the consideration of the stresses in the diagonals, we find that when the whole truss is loaded, and consequently the ties in the middle panel as well as the rest of the counters are not in action, each system of bracing has the same number of loaded joints. The shears in B C and B E in the first panel must then be equal, and each will be one-half of the ordinate for that panel. If the rolling load is removed from C, it first appears that the subtraction of the shear in B E from the ordinate for shear belonging to the second panel will determine the remainder to be carried by D G; for the system of weights E K N R has not apparently been disturbed. But the removal of the weight from C carries the centre of gravity of the *remaining* weights on the system C G P S to the right of the mid-span, and hence brings into action one of the dotted diagonals. As those diagonals which cross the middle connect loaded joints of alternate systems, the distribution of the shear in the two diagonals of any panel is rendered indeterminate between certain limits.

To illustrate: Remove the load from C, and suppose that the stress in B E is unchanged. The shear in B E will then be $2(w + w')$, if w = steady load, and w' = rolling load per joint. The shear in F K will be $w + w'$, and the load at K will then reduce the shear to zero, so that no counter is needed for this system. Passing on, we should have $2(w + w')$ in R U also. Again: as the reaction at A must be $4w + 3\frac{1}{2}w'$, the shear in B C will be $2w + 1\frac{1}{2}w'$, in D G $w + 1\frac{1}{2}w'$; and, on passing G, the shear is $\frac{1}{2}w'$, which must pass through I N: then we shall have $w + \frac{3}{8}w'$ in N V, and $2w + 1\frac{3}{8}w'$ in R U; which does not agree with the previous deduction. If, on the other hand, we start from T with a reaction of $4w + 3\frac{3}{8}w'$, and suppose that $2(w + w')$ passes through R U, $2w + 1\frac{3}{8}w'$ is left for U S. The diagonal W P will then carry $w + \frac{3}{8}w'$; and, on passing P,

the shear changes to $-\frac{1}{2} w'$, which must, therefore, go through P L. The shear in F K becomes $w + 1\frac{1}{2} w'$, and in B E $2w + 2\frac{1}{2} w'$, which is greater than before. We may, therefore, have the counter I N or L P in action as soon as a load is removed from C; and B E may, on one supposition, carry more than when C was loaded. The total shear in any panel will remain the same; but the distribution of a small portion between the two ties will be in doubt. An odd number of panels for a double-system truss is not desirable on this account. The two possible ways may be provided for by a sufficient cross-section of the pieces in question.

59. Double Triangular or Warren Truss.—If the truss of Fig. 24 is taken as a whole, it may be discussed as was the double quadrangular truss of § 55: an equally short method for finding chord-stresses can then be substituted for the detailed treatment. When the truss is fully loaded, the tie L N, which runs from the middle loaded joint N, will have a shear of one-half a panel weight. This shear will also be found in the strut L G; and, after passing the point G, it will be increased by the additional weight at that point, the shear in F G and F C being one panel weight and a half. In the other system the central trapezoid I K P S is in equilibrium under the weights at K and P; so that K O and O P have no stress, and the shear in I K and I E is one panel weight. In D E and D A there will be a shear of two weights. If we draw the usual polygon for chord-stresses, A' D' F' O', &c., we see, upon taking moments about N, that N' O' is the stress in L Q. If a section is made at L, and moments taken at that point, we also find, since K O has no stress, that the tension in K N is K' L'. The difference between K' L' and N' O', or O' U, must be the horizontal component of the stress in L N, or the horizontal component due to a shear of half a panel weight. As all the braces, struts or ties, slope at the same angle, the horizontal components of their stresses will be simple multiples of O' U. The difference between the stress in L O and that in I L will be due to the two horizontal components in L N and L G; likewise, in the

bottom chord, the stress in EG will be less than that in GK by the sum of the horizontal components in LG and FG . The compression in LO having been determined as $N'O'$, and the horizontal component of the stress in LN having proved to be $O'U$, subtract $2 O'U$ from $N'O'$, or make $L'L'' = O'U$, to obtain $K'L''$, the compression in IL . The stress in FI will again be diminished by reason of the action of IK and IE , or by $4 O'U$. From the well-known property of the parabola, if $O'U$ is the distance that L' is vertically above O' , I' is four times that distance above O' . As twice $O'U$ has been subtracted already from $N'O'$, to subtract $6 O'U$ it is necessary to make $I'I'' = 2 O'U$, and $G'I''$ will be the compression in FI . To get the stress in DF we must again subtract $6 O'U$, or $12 O'U$ from $O'N'$. From O' to F' vertically being $9 O'U$, make $F'F'' = 3 O'U$, and $E'F''$ will be the compression in DF . In the same way, $8 O'U$ must be again deducted upon passing the joint D , or $D'D'' = 4 O'U$.

For the bottom chord the stress in KN was found to be $K'L'$. The tension in GK will be less by the horizontal component of IK or $2 O'U$. As $O'U$ has been subtracted in going from O' vertically to L' , and as the whole vertical distance to I' is $4 O'U$, to subtract, in all, $8 O'U$ from $N'O'$, brings us to I''' , a distance $I'I''' = O'U$, below I' . Passing G , we must subtract $4 O'U$, or, in short, add $2 O'U$ to $E'F'$, making $E'F''' =$ tension in EG . Without elaborating further, we have the rule:—

Set off, inside the parabola, once, twice, thrice, &c., the distance which $O'N'$ exceeds $K'L'$ on the successive ordinates from the middle, at L'' , I'' , F'' , &c., and outside the parabola, at I''' , F''' , D''' , and B' , commencing on the second ordinate from the middle. The inside points will determine the upper chord-stresses, and the outside points the lower chord-stresses. The accuracy of the construction will be checked by $C'D''$ and $A'B'$, being respectively the horizontal components of the stresses in BC and AD . As in § 55, so here, a line drawn from N' , Fig. 24, to B' , the first point below A' , will cut off the

ordinates for upper chord-stresses, and one through P' , parallel to $B'N'$, will limit the ordinates for lower chord-stresses.

The construction of Fig. 24 may also be made as follows: Draw $L'U$, $I'V$, $F'W$, &c., horizontally across one panel; lay off $O'U$ at $L'L''$ and $I'I'''$; lay off $L''V$ at $I'I''$ and $F'F'''$; and so proceed to the close.

The distribution of the maximum shear between the two diagonals of each panel is easily obtained. One of the two systems into which the web members may be divided, having one more loaded joint than has the other system, will carry one-half the entire weight of one truss. For example, the truss of Fig. 24 has ten panels; the system $BCFGLN$, &c., has five loads: hence, when the truss is fully loaded, the shear in BC will be one-half of the *end* ordinate; and, if this half is projected upon the ordinate in the middle of the first panel, the remaining portion will be the shear in AD . If the load is withdrawn from C only, the shear in DE , which belongs to the system not connected with C , cannot be disturbed, and must be equal to the amount just obtained in AD . Deduct this amount from the ordinate for the panel CE , and the remainder will be the shear in CF . Thus the operation is the same as in finding the vertical components of the stresses in the diagonals of the double quadrangular truss, § 55. Some pieces in the middle portion of the span will be subjected to alternating stresses of tension and compression, as in the single triangular truss.

If the load is upon the upper chord, the chord-stresses will change from one chord to the other, and the diagonal stresses from one diagonal to the other, all in the same panel; the change being one of amount and distribution, not of kind. That the stresses are changed arises from the fact that a load on a joint, when shifted from one chord to the other, is also moved laterally in its own system half a panel: in other words, the two systems change places.

✓60. **Lattice Girders.** — Multiple systems are used in the triangular type of truss in riveted bridges, when the diagonals

are frequently riveted together at intersections, and a *lattice girder* is produced. It is probable that such connection of the diagonals causes more or less distribution of stress from one web member to another; and the compression members, being stayed at frequent intervals, may be much more slender than would otherwise be possible. Although the several systems of Fig. 25 may be distinguished, and the stresses carefully determined, riveting of the diagonals to one another destroys the accuracy of such discrimination; and hence the assumption which is commonly made — that the shear at any section will be equally distributed over the pieces of the web cut by the section, and that the chord-stress in any panel will be found by dividing the bending moment at the middle of that panel by the height of truss — does not materially err from the truth.

For while it will be noticed, that, in such a truss as the quadrangular, the stresses in the two opposite chord-pieces of the same panel differ materially, — in fact, by the amount of the horizontal components of the web members in the panel, — it will be seen, that in the double triangular truss, and also in the lattice girder, the horizontal components of the stresses in the strut and tie diagonals in any panel partially neutralize one another, thus bringing the opposite chord-stresses more nearly to an equality. Thus in Fig. 24 the compression in FI is $G'I''$, and the tension in EG is $E'F'''$. They do not differ greatly from one another, nor from the ordinate at the middle of the panel: hence, as the number of systems increases, we approach the plate girder, where the two flange-stresses are equal at a section, and the shear is uniformly distributed over the section of the web.

The verticals shown in the figure are introduced at more or less frequent intervals, partly to distribute the moving load between the upper and lower chords, but more particularly to form, with the struts of the *lateral* bracing at top and bottom, stiff frames to resist lateral distortion. Gusset-pieces also may be riveted in the interior angles between these verticals and the lateral struts.

61. Effect of Locomotive on Double System.—The diagrams for engine excess, which were used in Fig. 11, § 33, may be added without difficulty to those for trusses with double systems of bracing. The shear in any panel from this extra weight is properly considered as coming upon that diagonal which runs to the joint thus heavily loaded. When the panel joints are so near together that it seems more wise to place a portion of the engine excess on each of two adjoining ones, an inspection of Fig. 10 will show that the maximum ordinate at each joint in succession will belong to the diagonal which sustains that joint, although this maximum is somewhat less than the other.

When two locomotives are called for, at the head of the train, two excess loads may be used at the proper distance apart. It has been found, in applying this treatment to some double quadrangular trusses, that the two excess weights were approximately three panel lengths apart. As the weight thus influenced separate web systems, the curious result was obtained that two locomotives at the head of a train caused no larger web-stresses than did one.

The analysis of stresses arising from definite wheel loads applied to trusses with double or multiple web systems becomes very complicated and unsatisfactory. Such a distribution of given weights with given spacing is very frequently specified, and the difficulty of a satisfactory analysis, added to the measure of indeterminateness of web-stresses, already referred to in some of these trusses, has led to the adoption of other designs for long spans, such as Truss VII., Plate II., or similar designs of varying depth, like what is known as the Pettit Truss.

The method of finding bending moments and shears, and hence chord and web stresses, for definite concentrated loads on any truss, was explained in §§ 53 and 54. The analysis of trusses whose chords are not parallel will be taken up in the next chapter.

NOTES.—If the panels of the double Warren girder of Fig. 24 are long, and the web members are riveted or fastened together where they cross one another,

short intermediate verticals may be added to reduce the panel lengths, as was done in changing Truss III., Plate II., to Truss VII. The analysis of such a modification of the double Warren girder can be made under the principles already laid down.

In designing bridges of long span it is not uncommon to treat the horizontal trussing, which carries the wind pressures to the points of support, as if it were subjected to a moving load equal to this wind pressure, advancing across the span. Such an analysis is based on the view that the strongest wind pressures come in gusts, are more or less local or of limited extent, and that therefore they may not be distributed over the whole span. Such a treatment, as it gives larger shears in the panels of the horizontal trusses, errs, if at all, on the side of safety. Many trusses are deficient in strength in their wind and portal bracing.

Some specifications require that account shall be taken of the additional load thrown upon the leeward truss by the tendency of the pressure of the wind to tip the cars, and hence to make them press more heavily on one rail than the other. Usually such increase in the truss stresses is not computed.

In some rare cases bridge trusses have been placed in a plane inclined to the vertical. The weight which is applied to the truss will then represent the vertical component of the actual load in the plane of the truss at each joint. The corresponding horizontal component must be provided for by another truss; or, if small, it may be carried by the resistance of the loaded chord to lateral flexure. In the latter case the maximum intensity of stress in that chord, resulting from the combination of direct stress and flexure at any section, should not exceed the safe unit stress. For an example, see the Transactions of the American Society of Civil Engineers, December, 1890.

A railroad bridge is sometimes located on a curve. In such a case the track approaches one truss at the middle of the span and the other truss at its ends. The trusses must therefore be farther apart than is ordinarily the case, not only to clear the curve, but to allow for the cant of the cars. The several transverse beams of the floor will, at most panel joints, be loaded eccentrically and at different points. Hence the loads thrown on the panel joints of the truss will vary, even when the same weight is assumed to be on each floor-beam. If the effect of centrifugal force or of the oblique pull exerted on the cars is considered, the problem becomes more complicated.

The trusses of a highway bridge, supporting a roadway and two outside sidewalks, will each have the maximum loading when the roadway and adjacent sidewalk are loaded, and will have the minimum loading when the opposite sidewalk alone is loaded. The transverse floor-beams will have maximum positive bending moment when the roadway alone is loaded, and minimum positive bending moment when the load is on the sidewalks only. The maximum negative moment on the floor-beam occurs at its points of support whenever the adjacent sidewalk is loaded.

When a certain uniformly distributed load and an alternative concentrated

load are prescribed for a highway bridge, it will be found that the stringers, and often the floor-beams, must be proportioned to carry the concentrated load, but that the trusses will have the greatest stresses under the uniform load.

The outside or safety stringers of the floor of a railroad bridge cannot be considered to relieve the track stringers of any material portion of their load, unless the former are nearer the latter than half of the gauge. Practically, therefore, the track stringers should be designed as if the outside stringers were absent.

CHAPTER V.

TRUSSES WITH INCLINED CHORDS.

62. General Remarks ; Effect on Chord-Stresses.— While the determination of the stresses in a truss with inclined or curved chords is not quite so simple a matter as where the chords are parallel, no particular difficulty will be experienced, if, as with parallel chords, the members are considered to be jointed at each intersection. Take, for example, the truss represented by A B C D, Fig. 30. The span and the load are the same as those of the truss, Fig. 6, before described. The bending moment at each joint, and the shear in each panel, for the truss as a whole, will be unaltered, and the diagrams of Fig. 6, therefore, will apply here. In many practical examples one chord or the other is made straight throughout its entire length ; but both are inclined in this case for a general illustration.

The maximum bending moment at any joint will then be obtained by multiplying the proper ordinate of the equilibrium polygon, A' F' I' K' D', by H from the stress diagram ; and this moment must be equal, by the theorem of moments and by what has been previously said, to the stress in the proper chord-piece multiplied by the *perpendicular* from the origin of moments to that piece of the chord. As this perpendicular is not always a convenient quantity to use, and will not be the same for the two chord-pieces in the same panel, it probably will be better to substitute for the product just mentioned the equal product of the horizontal component of the stress in the chord multiplied by the height of the truss at the joint in question. That one product is equal to the other is easily seen, if

we notice that the forces and lines referred to are respectively base and hypotenuse of two right triangles whose sides are perpendicular to one another. From the horizontal component may be obtained the stress itself by multiplying the former by the ratio of the length of the inclined piece to its horizontal projection; that is, by multiplying by the length of the inclined chord in the panel, and dividing by the horizontal distance between panel joints. If we call the length of a panel horizontally a , and the difference of level of the two joints in the same chord b , we may multiply the horizontal component above by

$$\sqrt{\left(1 + \frac{b^2}{a^2}\right)},$$

and we shall obtain the direct stress in that piece of the chord: hence, *to find the stress in any piece of the chord*, multiply the proper ordinate under one end of the piece by H , divide by the height of the truss at that joint, and multiply by the above expression.

63. Effect on Stresses in Diagonals.—As one or both of the chords are inclined, such inclined members are able to and must convey a certain portion of the vertical force which exists at a section in any panel. Thus, suppose a vertical plane of section to be passed through the panel $RFGQ$ of Fig. 30. Neglecting one of the diagonals, as we know that in a good design both cannot be strained at once (see § 23, last part), the vertical shear at the section is distributed over the three pieces RQ , RG , and FG , all of which are inclined, and therefore capable of carrying a vertical component. As the portions of vertical force in RQ and FG will be just sufficient to cause, when combined with the horizontal components derived from bending moments, *direct* stresses along those chord-pieces, it will only be necessary to find those horizontal components for such a distribution of load as gives maximum shear in the panel, to easily deduce the amount of shear which passes through the chords: the balance of the shear will be left for the diagonal to carry.

For example: To find the maximum stress in any diagonal, such as $R G$, the rolling load will be moved so as to cover all joints from D to G inclusive (see § 69). The equilibrium polygon, reproduced from Fig. 6, is $D' K' I' F' A''$, and the shear ordinate is $q f$. By inspection of the enlarged sketch of this panel, shown on the right, we see that on one side of the plane of section, say the left hand, we have the vertical shearing force, acting upwards, the horizontal pull, obtained by taking moments at R , which acts through F , and the horizontal thrust, obtained by taking moments at G , which would act through Q . On the other side of the plane of section we find three inclined pieces, whose stresses must balance the rectangular components on the left side.

From the equilibrium polygon for the given position of the load take $G' Q'$, multiply by H , divide by $G Q$, and lay off the quotient, which is the horizontal force through Q , horizontally at $f y$, Diagram I.¹ In the same way, $F' R' \cdot H$, divided by $F R$, gives us the horizontal force through F , laid off at $f x$. The vertical shearing force, $f q$, is plotted vertically upwards at $f q$. Now draw $q r$ from q , parallel to $Q R$, and limit it by a vertical from y . Draw $f g$ from f , parallel to $F G$, till it meets a vertical through x . Draw the line $r g$, and it must be parallel to $R G$, or some error in construction has been made. Thus we have a check on the accuracy of our work. By dispensing with this check it is necessary to determine only one of the horizontal stresses, and then we may draw the other lines parallel to the respective pieces.

64. Stresses in the Verticals. — It remains to find the stress in $R F$, the vertical which joins the unloaded end of the diagonal. The vertical component in $G R$ will not pass unchanged in amount through $R F$; for $R S$ and $R F$ together will carry the vertical components of the stresses in $Q R$ and $G R$. The portion that will pass through $R S$ is definitely fixed by the consideration, that, when combined with the horizontal force at

¹ The scale of these diagrams is increased one-half for distinctness.

the joint R due to bending moment, the resultant must be a direct thrust along RS. It is, however, unnecessary to have recourse anew to the equilibrium polygon. Consider the joint R. This joint is in equilibrium under the action of four forces which meet at that point; and two of them, qr and rg , have just been determined. The two remaining sides of the polygon of force are readily drawn. QR and RG being taken in order, draw gt parallel to FR, and tq parallel to RS, to close on q . The required stress in RF is, therefore, the compression gt . The arrows on this quadrilateral proceed round the figure as usual, and show the directions of the stresses exerted on R by the several pieces.

65. **Remarks.**—It may not be amiss to call attention to the fact, that, since we may dispense with one of our data, that one may be the shear, and thus we may do without the shear diagram; for it will be noticed that the horizontal projection of rg is yx , the difference of the horizontal components in the two pieces of the chords in this panel: hence, by drawing a line from x parallel to RG, we may find the stress in RG very quickly. But, as we then have no check on the correctness of fx and fy , it is hardly advisable to take this course.

The arrows on qr , rg , and gf , show that the three stresses just determined will together balance the shear, since their vertical projection is equal and opposite to it; and that the horizontal projection of the stress in the top chord is exactly balanced by the horizontal projections of the tensions in the tie and lower chord. The horizontal projection of the stress in RS is the same as of that in FG; which result is to be expected, from the fact that these two chord-pieces lie between adjacent diagonals sloping the same way. The stresses qr , qt , and fg , are those existing, in the pieces to which they refer, for the present load; but, as they are not the greatest stresses which the chord-pieces must resist, they are of no special value here. The maximum stresses were determined in § 62.

66. **Construction for Horizontal Components.**—The por-

tion of the figure within the triangle $f d y$ gives a geometrical construction for finding the horizontal components $f x$ and $f y$. For example, since

$$f y = \frac{G' Q' \cdot H}{G Q},$$

draw $f d$ at any convenient angle with the horizontal line; make $f w = H$, $f b = G Q$, $f d = G' Q'$; draw $b w$, and, parallel to it, draw $d y$ through d . The desired component will be $f y$; for, from similar triangles,

$$f b : H = f d : f y = \frac{f d \cdot H}{f b}.$$

Similarly, $f a = F R$, $f c = F' R'$, and $c x$, drawn parallel to $a w$, determines $f x$. If $c x$ and $d y$ cut the horizontal line at favorable angles, the values of $f x$ and $f y$ may be thus obtained quite satisfactorily.

✓ 67. **Stresses when Shorter Segment is loaded.**—The stresses in $G Q$ and $Q I$, as well as in the panels to the left of the one just discussed, will be obtained by a figure similar to the one already described. After passing the middle of the span, for example, to the panel $P I J O$, the moving load now extending from D to J inclusive, we shall have Diagram II.; in which case we use ordinates $I' P'$ and $J' O'$. In this case, $i j$, the stress in the lower chord, runs below the horizontal line; but the method of construction is still the same. It is noticeable that the inclinations of the chord-pieces $I J$ and $P O$ will *increase* the vertical force transmitted by the diagonal $J P$ *towards* the abutment A , while the *resultant shear* is unaffected: hence it appears that the stress in any diagonal is *increased* by such an inclination of either chord as tends to make the *height* of the panel, on that side to which the diagonal conveys its load, greater, and *vice versa*.

After we pass the point where the curve which limits the ordinates for shear passes below the horizontal line $b c$, the vertical for shearing force must be drawn *below* the horizontal line of these minor stress diagrams. Thus, taking the panel

O J K N, we draw, in Diagram III., no , ok , and kj , exactly as before. Since no and kj cross, we still have tension in the diagonal O K, as shown by the arrows; and we may also find a compression in O J, in case the inclination of O P does not take all of the remaining vertical force, or, as seen by inspection of Diagram III., if the point v , on nv , does not fall at or below k . If it does fall below k , there will be tension in the vertical. If we advance one panel nearer abutment D, the moving load now resting on D and L only, the panel N K L M will give us Diagram IV. On attempting to find the stress on a diagonal from N to L, we find, by the necessary direction of the arrow on nl , that a piece N L would thrust against the plane of section. As the diagonals are in this example supposed to be ties, we have passed the limit where those sloping in this direction are required. A similar set, from D to F, will complete the truss. If the truss is deeper at the centre than at the ends, the effect of the inclination of the chords is to require more diagonals sloping one way than is necessary for a truss with parallel chords.

The several diagrams for stresses in the web members are readily combined into one, as is done in Fig. 31, the scale of which is also increased one-half: the vertical line contains the shear ordinates all laid off from one point b . One additional simplification, explained in the next section, condenses the analysis of trusses with inclined chords into a very brief construction.

68. Maximum Equilibrium Polygon Sufficient. — It is not necessary to draw the several equilibrium polygons for rolling loads covering different portions of the truss when we desire to find the stresses on the successive braces. It will be seen, that, when the stress upon the diagonal R G of Fig. 30 is sought, the ordinates $F'R'$ and $G'Q'$ are used, and that these ordinates, although measured to the polygon which terminates at A'' , have their lower extremities at F' and G' , points which belong as well to the polygon $D'I'A'$. Their upper ends are situated upon the line $A''D'$, which closes the polygon $D'I'A''$.

In the same way, for the panel P I J O, the ordinates I' P' and J' O' are included between D' A''' and the main polygon A' I' D'. As all the ordinates will be found to terminate on the maximum equilibrium polygon, we need draw no other, if we can locate D' A'', D' A''', &c., by finding the points where these lines cut the vertical dropped from A'.

By referring to § 10 we see that the length of an ordinate intercepted between the prolongations of any two lines of the equilibrium polygon will be proportional to the bending moment at that point due to the weights included between the prolonged sides. If, then, a weight is removed from E, Fig. 80, the polygon strikes the vertical below A at A₁. The change in the bending moment at any point of the span, due to the diminution of the supporting force at D, must be equal to the product of H by the ordinate between D' A' and D' A₁; and this decrease of moment is all that will be found until we pass the whole of the fully loaded portion. The change due to the removal of the load from E will be proportional to the ordinate between E' A' and E' A₁: therefore the intercept A' A₁ must be equal to the decrease of reaction at D, multiplied by the span, and divided by H. If w' = rolling load at one panel joint, N = whole number of panels, and l = span of truss, the decrease of reaction at D, due to the removal of w' from the first joint beyond A, is $\frac{w'}{N}$, and the moment of this force about A is

$\frac{w' l}{N}$: whence the intercept A' A₁ must be $\frac{w' l}{H \cdot N}$

If the rolling load is next removed from F, the reaction at D will be diminished again by twice the previous amount, and consequently the distance from A₁ to A'' will be $2 \frac{w' l}{H \cdot N}$; the next interval will be three times the above amount; and so on. Hence, if we calculate the above quantity, and lay off in succession, from A', once, twice, three times, &c., that amount, or measure from A' one, three, six, ten, &c., times $\frac{w' l}{H \cdot N}$, the points

so determined will be the extremities of the closing lines; and these lines, with the maximum equilibrium polygon, will supply all of the needed ordinates. They have been added to the polygon of Fig. 30, and the points to which ordinates are measured are marked by small circles. They go in pairs, the two on any line, on each side of a panel, being the ones applicable to that panel.

✓ 69. **Rolling Load to extend to the Panel for Maximum Stress in the Brace.**—The stress on any diagonal, such as $R\ G$, will be a maximum when the rolling load extends from the abutment D up to the panel, including the joint G , as has been previously proved for trusses with parallel chords, provided the two chord pieces in the panel, such as $R\ Q$ and $F\ G$, if produced towards the unloaded end, do not meet before reaching the abutment. Then, since moment divided by panel height gives chord stress, and since the equilibrium polygon for a single load is a triangle with the apex under the weight, if a load is added at F , the bending moment at that point will be increased more than will the moment at G , and hence fx will be lengthened more than the increase of fy : hence their difference xy will be diminished, and rg must be less. Again: if the load is withdrawn, so that G is uncovered, the bending moment at G will be decreased more than that at F ; so that fy will be diminished more than fx will be: hence xy will again be shortened.

70. **Strut Diagonals; Load on Top Chord.**—If the diagonals had been struts in place of ties, the diagrams would have been constructed with the same ease. In the panel $R\ F\ G\ Q$, $Q\ F$ would then be the member whose stress was desired: moments at Q would give the horizontal component of $F\ G$; and moments at F , the horizontal component of $R\ Q$. The vertical through x would limit qr , and that through y would intercept fg . The diagonal would therefore lie in the other direction, and give a compression. The magnitude of the stress would not necessarily be the same as rg . The tension in the vertical $G\ Q$, running to the unloaded end of $Q\ F$, would

then be found as usual for four forces in equilibrium at the joint Q.

If the load for this truss were on the top chord, instead of the bottom as here, in place of finding the tension and compression in R G and R F, we should find them in R G and G Q those two web members connecting two adjacent loaded points. The four pieces which meet at G would then give the desired closed quadrilateral, and the change in Diagram I. would be, that, in place of drawing qt and gt , we should prolong fg (shown by a dotted line) to meet the vertical through y , and the upper intercepted portion of this vertical would be the stress in G Q. It is well to notice, that, even with tension diagonals, some one or more of the verticals may occasionally prove to be under tension in some types of truss. Such pieces must be adapted to both kinds of stress. When the stresses on the web members are under investigation, it may be found convenient to use Bow's method of notation, explained in Part I., "Roofs."

71. Bowstring Girder. — While the general treatment of trusses with inclined chords, which has now been given, will enable one to analyze any single-span truss of this type, there is one form which deserves special treatment, both from the frequency of its occurrence, and from our ability to develop certain ways of shortening the analysis very materially. The type to which we refer is the bowstring girder. In practical construction the bow is sometimes, perhaps often, bent to an arc of a circle, or the upper ends of the verticals lie upon such a curve, while the several panel lengths of the upper member are straight; but in theoretical treatment, and in many structures, the bow is a parabola, or a polygon coinciding with the equilibrium polygon for a complete load. The parabolic girder will now be discussed, and the circular segment will be referred to later. It may not be amiss to say, that, when the rise of the circular arc does not exceed one-tenth of the span, no error of consequence is committed by assuming the curve to be a parabola. For results, see § 77.

72. Chord-Stresses.—The truss is represented by Fig. 32. The first step will be to find the chord-stresses under a full load. If we draw the equilibrium polygon for such a load, and remember, that, as explained in § 28, the vertices of this polygon all lie in a parabola, we shall see, that when we seek to find the horizontal force at any joint, by multiplying the ordinate to the equilibrium polygon by H , and dividing by the height of the truss at the joint, the ratio of the ordinate divided by the corresponding height of the truss just above it will be a constant quantity; that is, any ordinate divided by the height at the joint in question will be the same as the middle ordinate divided by the centre height. Hence *the horizontal stress will be constant in all the chord-pieces*, the lower chord will have a uniform tension throughout, and in the upper chord the direct stress will increase, from the middle toward each abutment, according to the inclination.

We may find the stresses in the chords graphically as follows: Conceive that the equilibrium polygon passes through the upper ends of the verticals of the truss; then the ratio just referred to is unity, and the stress in the lower chord is H of the stress diagram. As the *equilibrium curve* for a load of uniform intensity over the whole span is a parabola which will pass through the vertices of the equilibrium polygon for concentrated loads, and as the tangent at the springing point of this parabola will cut the middle ordinate at twice the height of the curve above the base, make $DJ = DC$, and draw AJ , which, being the tangent, is the direction of the force at the beginning of the equilibrium curve. Lay off half the weight of truss, and complete rolling load at 3-2; draw 2-0 parallel to AJ ; and 3-0 will be the desired value of H , the stress in the bottom chord, and the horizontal component of the stress in all parts of the bow.

The first side of the polygon for concentrated loads will run from A to E , and will be parallel to a line, which, starting from O , cuts off below 2 a distance equal to the half-load at A . If, then, 0-4, 0-5, 0-6, &c., are drawn parallel to AE , EF , FG .

G I, and I D, these lines will be the compressions in the respective parts of the top chord, as they will all have a horizontal component H.

The amount of H may be seen from the figure to be, if $2-3 = \frac{1}{2} W''$, $A B = l$, and $C D = k$,

$$2k : \frac{1}{2} l = \frac{1}{2} W'' : H = \frac{W'' l}{8k};$$

which agrees with the value deduced in § 28. If the pieces of the top chord are curved, they will be exposed to a small bending moment, equal at their middle points to the direct thrust multiplied by the perpendicular from the chord of the curved piece to its centre line.

73. No Stress in Diagonals for a Complete Load; Tension in Verticals.—Since the equilibrium polygon may be drawn to coincide with the bow, for a uniform load over the entire truss there can be no stress in the braces; for we have only to remember what the equilibrium polygon signifies, to see that the bow will require no bracing to keep it in place. Or, since the stress in the lower member and the horizontal component of the stress in any piece of the bow are equal, their difference, the horizontal component of the stress in any brace, is zero: hence the stress itself is zero. Or, since the stress in the lower member is constant, there is no increment from any brace, and therefore no stress in any brace. It is right, consequently, that all of the verticals should be adapted to convey a tensile stress to the bow equal to whatever amount of load may be placed at their lower ends. It is also apparent that the steady load, being always uniformly distributed, will exert no stress on the braces, and may be neglected in their analysis, but must be subtracted from the compression which the rolling load may cause in the verticals. The assumption of uniform distribution of steady load is not strictly accurate; but the increase in the weight of the bow per *horizontal* foot, as we approach the abutments, is partially offset by the increase of weight per horizontal foot for the web members as we go towards the

middle. One may, therefore, include the steady load, or neglect it, in treating the braces, with the same result.

We have proceeded upon the assumption that the diagonals are ties, it being the usual and most economical construction to put the compression members on the shortest lines; but, in case the diagonals are struts, the verticals will always be in tension.

74. Chord-Stresses when Verticals will not transmit Tension. — In case the truss has no verticals, or where the verticals are not fastened at their ends so as to transmit tension (a mode of construction not to be commended), the weights at the loaded points must strain the diagonals on each side; and the stresses in the chords, while not varying much from the amounts previously deduced, will not have strictly a constant horizontal component. The simplest way to show the effect of the absence of verticals is to draw the stress diagram of Fig. 33 by the method explained in Part I, "Roofs," when the results are apparent at a glance. The weight of the bow may properly be left out of consideration, as it is practically in equilibrium by itself, and is exactly so as far as it is of uniform weight per horizontal foot. The horizontal thrust due to the weight of the bow and the equal tension in the lower chord must then be added to the results of this diagram. While, in any case, where there are three members at one joint, each capable of carrying the same kind of stress (as, in the bowstring girder of Fig. 32, we have two diagonals and a vertical meeting at each lower joint, and all designed to resist a possible tension), the paths which the concentrated weight at any joint may take in going to either abutment are somewhat doubtful, depending upon the cross-section, resistance to extension, and rigidity of attachment of the respective pieces, it is most natural that the weight should pass up the vertical to the bow; for in this line it meets the most direct resistance or reaction. That is, if we imagine the weight to strain all three of the pieces at once, a slight yielding, or extension of the two diagonals, will lower the loaded point more than the same amount of extension in the

vertical, and hence the weight will be thrown more upon the vertical; and as the vertical, being the shortest member, will stretch least in total amount for the same stress per square inch, the weight will still more be carried by it. Hence the assumption, that the verticals carry the load when it is complete, cannot be far from the truth.

75. Maximum Stresses in Braces.— By deducing a formula for the braces, we shall be enabled to prove a very short construction for obtaining the desired stresses. In finding these stresses, we will avail ourselves of the fact that the horizontal projection of the stress in a brace must equal the difference of the horizontal forces in the two chord-pieces of the panel in which the brace is situated; and we will prove that this horizontal component is a constant quantity when the brace experiences the maximum stress. If the steady load is neglected at present, the polygon A E F G I D B may represent the equilibrium polygon for a complete moving load. If the sides F G, G I, &c., are prolonged until they intersect the vertical through A, the points K, L, M, &c., will be the extremities of the equilibrium polygons for partial loads, extending from B to the successive joints. The distance A K has been proved, in § 68, to be $\frac{w' l}{H \cdot N}$ when w' = travelling load on one joint. As H will, by § 72, equal $\frac{N w' l}{8 k}$, the value of

$$A K = \frac{w' l}{N} \cdot \frac{8 k}{N w' l} = \frac{8 k}{N^2},$$

a quantity evidently independent of w' , since it is obtained by construction from the bowstring girder itself.

If n represents the number of any panel and of any vertical from A, any distance A L or A M, intercepted on the vertical, will be seen, from § 68, to equal $\frac{n(n-1)}{2} \cdot \frac{8 k}{N^2}$. The height of any ordinate, P R, from the base line A B to the closing line B L of a particular polygon, will then be

$$\frac{n(n-1) 4 k}{N^2} \cdot \frac{(N-n)}{N}.$$

In the panel S G I R the stress in S R, if G R is the tie in action, will be $\frac{G Q}{G S} \cdot H$, and the horizontal force in G I will be $\frac{I P}{I R} \cdot H$. As the horizontal force in the piece of the bow exceeds that in the lower chord for the same panel, the desired difference of horizontal force will be

$$\left(\frac{I P}{I R} - \frac{G Q}{G S}\right) H = \left(\frac{I R - R P}{I R} - \frac{G S - S Q}{G S}\right) H = \left(\frac{S Q}{G S} - \frac{R P}{I R}\right) H.$$

As $D C = k$, any ordinate numbered n from $A = \frac{4k}{N^2} (N - n) n$:

I R is the n th ordinate, and G S is the $(n - 1)$ st ordinate, when S R is the n th panel. The above expression becomes, by substitution,

$$\left[\frac{n(n-1)(N-n+1)}{(N-n+1)(n-1)} - \frac{n(n-1)(N-n)}{(N-n)n}\right] \frac{4k}{N^2} \cdot \frac{N^2}{4k} \cdot H = \frac{H}{N} = \frac{w'l}{8k},$$

which is a constant quantity.

If, then, we construct a parabola divided into N panels, each $\frac{w'l}{8k}$ long, the entire span being H for rolling load, and the height proportional to the height of the original truss, or if, in other words, we draw the truss anew to this altered scale, the diagonal lines will be parallel to the diagonals of the truss, and, having the required constant horizontal projection, will be the desired tensions in those diagonals. The required middle ordinate of the parabola will be obtained by the proportion

$$l : k = \frac{N w' l}{8 k} : \frac{N w'}{8};$$

or it is one-eighth of the entire travelling load. All of these values may be obtained, but with more labor, by the general method.

This truss is drawn below the original truss in Fig. 32, and is lettered in small type to indicate the stresses in the respective diagonals. It will be seen that diagonals sloping each way are required in all panels; that the diagonals in the middle of the span have the most stress; and that all diagonals which have the same vertical height have the same maximum stress.

76. Stresses in the Verticals.—An inspection of the diagrams of Fig. 30 shows, that, since the bottom chord of the bowstring girder is horizontal, the vertical which transfers the stress from the diagonal to the first lightly-loaded joint carries all of the shear from that panel except what is taken by the upper chord in the next lightly-loaded panel. If, then, we find the horizontal component of the stress in that piece of the chord and the shear, we can draw our diagrams. In the first place, it is apparent that the horizontal component of the stress in F G is the same with that in S R. From the last section, the horizontal component in S R = $\frac{G Q}{G S} H = \left(1 - \frac{S Q}{G S}\right) H$. By inspection of the value for the ratio $\frac{S Q}{G S}$ written above, and cancelling factors, we get

$$\text{Horizontal component in F G} = \left(1 - \frac{n}{N}\right) H.$$

If we draw a horizontal line from F to meet G S, we see that

$$\text{Vert. comp. in F G : hor. comp. in F G} = G S - F T : \frac{l}{N}.$$

G S being the $(n-1)$ st ordinate, and F T the $(n-2)$ d ordinate,

$$\begin{aligned} G S - F T &= \frac{4k}{N^3} [(N-n+1)(n-1) - (N-n+2)(n-2)] \\ &= \frac{4k}{N^3} (N-2n+3): \end{aligned}$$

therefore

$$\begin{aligned} \text{Vert. comp. in F G} &= \left(1 - \frac{n}{N}\right) \frac{N w' l}{8k} \cdot \frac{4k}{N^3} (N-2n+3) \frac{N}{l} \\ &= \frac{(N-n)(N-2n+3)}{2N} w'. \end{aligned}$$

As the travelling load alone is now under consideration, the shear on the left of the loaded portion will be constant, and equal to the reaction at A, which is

$$\frac{(N-n) w' (N-n+1)}{2N}.$$

The amount of compression in the vertical G S will be the shear in the panel S R minus the vertical component in F G, or

$$\frac{N-n}{2N} w' (N-n+1-N+2n-3) = \frac{(N-n)(n-2)}{2N} w'.$$

As G S, by our notation, is the $(n-1)$ st vertical, the compression in the n th vertical will be obtained by writing $(n+1)$ for n , or

$$\begin{aligned} \text{Compression in } n\text{th vertical} &= \frac{(N-n-1)(n-1)}{2N} w' \\ &= \frac{(N-n)n}{2N} w' - \frac{N-1}{2N} w'. \end{aligned}$$

The height or rise of our parabola for stresses in diagonals was found to be $\frac{N w'}{8}$. The n th ordinate of this parabola will

be $\frac{N w'}{8} \cdot \frac{4}{N^2} (N-n)n = \frac{(N-n)n}{2N} w'$; and the first ordinate,

when $n=1$, is $\frac{N-1}{2N} w'$. We see, therefore, that, if a horizon-

tal line is drawn through e at the height of the first ordinate above the base, the remaining portion of each vertical above this line will be the compression from travelling load upon the corresponding vertical of the girder. Another horizontal line, $w'k'$, at an altitude above the one just drawn of the steady load w at the foot of each post, neglecting the weight of the bow, as before stated, will cut off the stress of tension due to the steady load, and the remainder will be the maximum compression in each vertical. The result is, that the first vertical is never compressed, but has a tension of $w + w'$; the second vertical will not be compressed unless w is less than the difference between the first two verticals of the stress parabola; and the remainder are compressed, except, again, the last two.

77. Recapitulation.—These constructions are all found in Fig. 32, and may be briefly summed up as follows: Having constructed the skeleton A D B of the truss, make D J = C D, and draw A J, the tangent to the parabola at the abutment. Lay off 2-3 vertically, equal to one-half of the weight of truss when

fully loaded, and divide it, at 4, 5, . . . 8, into panel weights beginning and ending with a half-load. Draw 2-0, parallel to J A, to meet the horizontal line 3-0: 4-0, 5-0, &c., are the stresses in the pieces A E, E F, &c., of the bow; and the stress in the bottom chord is 3-0. The value of 3-0 can be checked by the formula $\frac{W'' l}{8 k}$. Draw the truss anew to a different scale,

the panel length being $\frac{w' l}{8 k}$, or the span $= \frac{N w' l}{8 k}$, and the rise

$c d = \frac{N w'}{8}$. The diagonals will be the tensions in the corre-

sponding diagonals of the truss. Draw $u' k'$ horizontally at a distance $e u' =$ one panel weight of steady load minus one panel weight of bow: the ordinates from this line to the bow will be the stresses in the verticals, — compression when above the horizontal line, tension when below. If $k' l'$ is maximum load which can come on each lower joint, it equals possible tension in each vertical.

78. **Triangular Bracing.** — It is not uncommon to introduce the arrangement of bracing shown in Fig. 34 into the bow-string truss. The chord-stresses may then be most readily obtained by the method employed in Fig. 33, when, as there is but one system of braces, the small parallelograms $a b d c$, &c., due to the intersections A B D C, &c., will disappear. An adaptation of the construction for diagonal stresses in Fig. 32 will probably give the alternating compressions and tensions in the diagonals, so far as they are due to travelling load; and to these stresses may be added *algebraically* that fraction of the diagonal stresses previously obtained in the chord-stress diagram, which is properly due to steady load. It is hardly expedient to take up space with a detailed analysis: any reader who is specially interested in such a type of truss can, from the general methods already laid down, elaborate a set of diagrams for himself. If the bow is spaced off into pieces of equal length, the stresses on the various parts will be modified by the change. As the method of Fig. 30 applies to any of these trusses, it will

doubtless be satisfactory when required for occasional use. Any special method which is not so simple as to be readily recalled, but which must be reviewed whenever it is needed for use, will only be valuable to those, who, from frequent necessity, can keep it freshly in mind.

79. Bowstring Girder with Circular Bow.—The same general remarks apply to the girder with a bow in a circular arc. As the ordinates to a circular arc from the chord of the arc are not expressed in any simple terms of the span and rise of the truss, and as these latter quantities have no fixed ratio, it is doubtful whether any short construction can be devised for finding the web-stresses. The general method is perfectly applicable. Special care is necessary in drawing the lines parallel to the pieces of the chord in Fig. 31, because they are many of them long, and a slight inaccuracy would affect the ascertained magnitude of the stresses in the braces. Good results can be obtained with the girder of the present section, since the effective lines of all the pieces of the bow are perpendicular to radii drawn to their middle points, and hence their directions can be accurately ascertained. A similar precaution applied to the parabola, of drawing tangents or long chords, will give the direction of the pieces of the bow closely. When the girder with a circular bow has a comparatively large rise, the web stresses do not, as in the parabola, increase from the abutments to the middle. As the bow will not be in equilibrium under a load distributed uniformly horizontally, the steady load will affect the braces, and must be taken into account in that connection.

80. General Remarks.—In some cases these trusses are inverted, as in Fig. 35, making the suspended bowstring truss. And, where the locality permits, that form of truss offers some theoretical advantage: the compression members will occupy the shortest lines, the bow will resemble a suspension-bridge cable, and the truss, being in stable equilibrium, will require but moderate lateral bracing in a vertical plane. Occasionally a designer is tempted to draw normals to the curve of the bow

in place of verticals from the joints of the horizontal member: such an arrangement is of doubtful utility in the parabolic girder, which is in equilibrium under vertical forces distributed uniformly per horizontal foot. The circle being a curve of equilibrium under normal forces, of uniform intensity along the arc, such a construction may be more defensible for a circular bow. It may also be advantageous, for constructive reasons, to have all the pieces of the circular bow of one length. Verticals will then divide the chord into unequal parts, shortest near the abutments: normals will also divide the chord into unequal lengths, shortest at mid-span. The change thus produced in the magnitude of the load at each joint must be regarded. In a few instances both chords have been parabolas or circular arcs, with the convexity turned opposite ways. The analysis would present no difficulty.

All iron trusses should have rollers, or other means of movement, at one abutment, to allow for elongation and contraction under changes of temperature.

81. Extent of Continuous Load to produce Maximum Stress in a Diagonal of any Truss.—In determining the maximum stress in any brace of a truss, we have supposed that the rolling load extended from one abutment to the panel in question, and it was proved that such a disposition of load gave the greatest shear in the panel. It is manifestly impossible, however, to so dispose a *continuous* load that one panel joint shall sustain a full panel weight of moving load, while the next joint in advance carries none; for the load which comes on a joint must be thrown upon it by the floor-timbers or track-stringers on one or both sides; and, as soon as the head of the load advances beyond any joint, the next joint beyond begins to carry something: therefore no joint can be fully loaded until the load has covered the panel beyond. The shear in that panel, then, can never be quite so large as the usual assumption would make it: the results previously obtained will err a little on the side of safety, and the treatment is defensible on that account. Loads imposed by railroad-trains are not con-

tinuous, but are concentrated on the wheels. How far apart these may be, will depend upon the train: in one extreme we have coal-cars, and in the other palace-cars. In some cases the loaded points may be a panel length or less apart; in many cases they are distant more than a panel length. With continuous, uniform load the customary assumption is that of loads concentrated at panel joints, and the first loaded joint completely loaded, as we have treated it.

The treatment, where wheel-weights are prescribed, has been given in §§ 53, 54.

For a further discussion of uniform loading, see Appendix.

82. Construction for Web-Stresses.—If in Fig. 27 the forces T_1 , T_2 , T_3 , and F are in equilibrium, the resultant of T_1 and T_2 will act through their point of intersection B ; the resultant of T_3 and F will act through A , the point of intersection of the latter pair; the two resultants must then be equal and opposite, and act in the line AB . Therefore, if four forces, not parallel, are in equilibrium, the resultant of any two of the forces must be equal and opposite to the resultant of the other two, and must act in the line joining their respective intersections. If the lines of action of all of the forces are known, as well as the magnitude and direction of one of them, the kind and value of the others can be found. The above principle can be usefully applied for finding the web-stresses in trusses of variable depth, especially when the loading is concentrated at intervals.

Let $A D$, Fig. 28, represent a portion of a truss, such as that above Fig. 29A, known as the Pegram truss, with the engine in the position to give maximum shear in the panel BC , as determined by the process previously explained in § 53. The equilibrium polygon for the given loading, in the above position, will be $A' C' D'$, the horizontal distance from C' to A' being equal to AC , and $A' D'$ subtending a horizontal distance equal to the span. Draw a chord from C' to B' and prolong it to meet $A' D'$ extended at the point F' . The chord $C' B'$ is the side of the equilibrium polygon when the weights in the panel BC are distributed to B and C by the stringers acting as beams, § 48, and

the point F' lies in the action line of a resultant force which has the same moment about C , as actually exists there, § 10. The maximum shear to the left of C' may then be considered to act at F' , and the existing shear and bending moment will be provided for. Make CF equal to the horizontal distance from C' to F' , and lay off the maximum shear in BC at FI , in the direction proper to its sign.

To find the stress in EC , imagine a vertical plane of section passed through ED , EC , and BC . As ED and EC meet at E , and BC and the shear FI will intersect at F , EF will be the action line of the resultants of each pair of forces. Draw IK parallel to BC , giving FK as the resultant of FI and the force in BC . Make EL equal to FK . Prolong DE , and, to meet it, draw through L a line parallel to EC . Then will EN and NL be the components at E , and NL will be the desired tension in EC . The arrows denote the direction of action in the pieces cut by the section, on the right side of the same, to balance the upward force FI on the left.

For the piece BE , pass the imaginary vertical plane of section through that piece. BC cuts FI as before; BE and GE again meet at E . Prolong GE , if necessary, and draw through L a line parallel to BE . This line LP will be the compression in BE . Or, one may find the two unknown forces at E in GE and BE , after those in ED and EC have been found, by completing as usual the figure $ENLP$.

This construction, devised by the late Prof. C. A. Smith, attains the same results as does the one of § 63, but there is no contained check on the accuracy of the work. It will be seen that, by this employment of the equilibrium polygon, three unknown forces can be determined at any section.

Thus Figs. 29A and 29B may be used to find the stresses in any truss with single system of bracing. The maximum ordinates in Fig. 29B when divided by the height of truss at the proper joints give horizontal components of maximum chord-stress, and the polygons for partial loading locate the position of the maximum shear for use in the above construction. If the polygon is

drawn as in Fig. 29B, for convenience of using the same spacing of loads on the span as in Fig. 29A, it must be remembered that the train for the shear curve actually heads to the left, and that therefore the point corresponding to F, Fig. 28, will be found on the right of B in Fig. 29B, and must be set off to the left of A in Fig. 28.

CHAPTER VI.

FLEXURE AND DEFLECTION OF BEAMS.

83. Flexure of Beams, &c. — The bending moment at any section of a beam is opposed or balanced by the moment of resistance of the fibres at the section. Under the tensions and compressions to which the fibres are exposed, the particles of the beam are extended in one portion of the section, and compressed in the other; so that a curvature of the beam results, with a change in direction, in a vertical plane at that point of the centre line or axis of the beam, as represented and exaggerated in Fig. 39: therefore the change of direction or inclination at any point is proportional to the bending moment at that point, or to the product of H by the ordinate between the equilibrium polygon and the straight line. By reference to Fig. 39 we can see the change of inclination produced in the beam at A by the elongation of the upper fibres at that point, and the compression of the lower ones. We can also see the effect of successive changes of inclination at B , C , and D , in altering the direction of the remainder of the beam; and we note that the changes of inclination at E , F , &c., produced by bending moments in the opposite direction, tend to bring the beam back toward its original *direction*.

84. Change of Inclination. — The total change of inclination between any two points is proportional, therefore, to the *sum* of all the bending moments between those points, or to the *area* included between the equilibrium polygon, the closing line, and the two limiting ordinates under the points. If portions of the bending moments are of opposite kinds, the *algebraic* sum of

the ordinates is to be taken, or the difference of areas of contrary signs. As the angle of inclination is measured by the ratio of the vertical movement of the end of the beam to the horizontal distance from the apex of the angle, the numerical quantity to be obtained will be the tangent of the angle which the tangent to the bent beam at any point makes with its original horizontal position.

85. Modulus of Elasticity.—But it is manifest that the amount of flexure of a beam will be influenced by the material of which it is made. Let us imagine two bars—one of iron, the other of wood—of the same length and the same cross-section, firmly held horizontally by one end, and having equal weights attached at the free ends. The movement of the two bars below the original horizontal line, for similar points, will not be the same, nor will the changes of inclination. The bar which elongates and compresses most for a given stress on the square inch of section will bend the most as a beam under a given load; and therefore the *ratio of the force on a square inch of section to the elongation or compression produced in a piece an inch long* influences the flexure of any piece under a bending moment. This ratio, known technically as the *modulus of elasticity*, and denoted by the symbol **E**, is deduced from experiments upon flexure, or upon extension under a tensile stress, and varies with the material. As the change of length is the denominator of this ratio, the more rigid body, the one which bends less for a given bending moment, other things being equal, will have a higher modulus of elasticity: therefore the curvature, the change of inclination, and deflection of each point from its original position, will vary *inversely* as this modulus; its values will be found in tables of strength of materials.

86. Moment of Inertia.—Again: if we compare the action of two beams of the same length and material, of different but similar cross-sections, we know that they may not be equally affected by equal weights similarly placed; that is, by equal bending moments. If they are of equal depths, the broader

one will be the stiffer; and, if the breadths are the same, the deeper beam will be much the stiffer. Referring to Fig. 33 of Part I., "Roofs," with the explanation of that figure there given, we see that the moment of resistance is made up of the summation of the products of the stress on each particle, multiplied by its distance from the centre line, or neutral axis, where the stress changes from tension to compression. As the stress on each particle increases with the distance from the centre line, the same moment of resistance might be represented by the stress on a particle at a unit's distance multiplied by the summation of the products of the area of each particle by the *square* of its distance from the centre line. As the stress upon the extreme or most remote particle in a section will be less for a given moment of resistance the deeper the beam is, the flexure of the beam will vary inversely as the summation last spoken of. This summation of the area of each particle, multiplied by the square of its distance from the centre line, is known as the *moment of inertia* of the section, and is denoted by I .

It may be obtained from the moment of resistance, deduced graphically in Part I., "Roofs," § 89, by dividing by the stress on a square inch at a unit's distance, which is equal to the stress, f , on the extreme fibre per square inch, divided by the distance from that fibre to the neutral axis, usually one-half the depth of the beam. For example: The moment of resistance of a rectangular cross-section was shown to be $\frac{1}{8} f b h^2$, where b = breadth, and h = height of the section: the moment of inertia of a rectangle will, therefore, be

$$I = \frac{1}{8} f b h^2 \div \frac{2f}{h} = \frac{1}{12} b h^3.$$

87. Formula for Change of Inclination. — The *change* of inclination at any one point of a beam will, therefore, be

$$\frac{M}{EI} = \frac{H y}{EI},$$

where y equals the ordinate to the equilibrium polygon at the point; and the total change of inclination between any two

points will be, if i equal angle of inclination to the horizon, and Σ is the sign of summation,

$$\tan. i = \Sigma \frac{H y}{E I}$$

Of the three quantities involved, M , the bending moment, depends upon the load and its distribution, thus including the span of the beam: E , the modulus of elasticity, depends upon the material; and I , the moment of inertia, upon the form of cross-section. All the variables are thus included in the general expression.

88. Deflection; Area Moments.—Upon reference to Fig. 40 it is also evident that the vertical *deflection* of any point of a beam from the original horizontal line depends upon the several changes of inclination, and the distances of the points at which they occur from the above point. Thus, if an originally straight rod ag is bent at a , the point g will be carried to a point on the line al ; the distance through which it is displaced depending upon the angle at a and the distance ag . If another angle is made at b , the point g will be found on bi ; and, on a further bending at c , it will move to the direction ch . The changes of inclination at d , e , and f , in the contrary direction, will carry the point which was originally at g through dk , em , fg , finally back again to g . (The deflections of all beams and trusses are so small, that the curved line of a beam under a load is always considered practically equal in length to the horizontal distance between the two points of support.)

As the expression of the last section measures the angle at a , it will only be necessary to multiply it by the horizontal distance of g from a to find the vertical displacement of g by reason of the change of inclination at a . The modulus of elasticity and the moment of inertia will affect the deflection as they do the changes of inclination. It is then evident that the total deflection or displacement vertically of any point, such as d , from the straight line tangent to the curve of the beam at a , will be obtained by summing, from a to d , the products of the

bending moment at each point of the beam multiplied by the horizontal distance of each point from d , and dividing the sum by $E I$. If x denotes the distance of any point horizontally from d , the above deflection, v , may be written

$$v = \sum_d^a \frac{M x}{E I} = \sum_d^a \frac{H y x}{E I},$$

where the letters attached to the sign of summation Σ denote that the addition of products is to extend from a to d .

In the same way that the summation of the ordinates y to the equilibrium polygon gives an area, the summation of the products of each ordinate into its distance from d is equal, on the principle that the moment of a resultant equals the sum of the moments of the components, to the product of the area just referred to by the *distance of its centre of gravity horizontally* from d . To this last product we give the name of *area moment*. As a convenient aid in remembering in which direction the horizontal distances are to be measured, we may note, that, if we regard the angle made by the original line of the beam and its new direction, the measurement is to be made away from the vertex, towards the opening of the angle.

89. Mathematical Solution. — Another demonstration of the above theorems, which is brief, and which depends upon the usual expressions for the investigation of curvature, slope, and deflection of beams, by mathematical analysis, is as follows: —

Let M denote the bending moment at any point of a beam supported in any way. Let E denote the modulus of elasticity of the material; and I , the moment of inertia of the cross-section. Let the originally straight horizontal line of the beam be the axis of x , and let v be measured vertically. M will be a function of x . Let r equal radius of curvature of the axis or centre line of the bent beam at any point. Then we may write the well-known equation for the curvature,

$$\frac{1}{r} = \frac{d^2 v}{d x^2} = \frac{M}{E I}.$$

If we integrate this expression once, considering I constant, we have

$$\frac{d v}{d x} = \frac{1}{E I} \int M d x = \tan. \text{ inclination.}$$

If we determine and introduce the constant of integration, we find the incli-

nation of the beam at each point to the horizon; but if we integrate from 0 to x , the origin being taken at one of the points of support, we get a complete integral, — the area included between the equilibrium polygon and the closing line, — but one expressing only the change of inclination from the slope already existing at the origin, or the angle between the tangent at any point and the tangent at the origin.

Integrating again, we have,

$$v = \frac{1}{EI} \int \int M dx^2 = \text{deflection.}$$

This integral is a volume, and taken between limits, as before, is the summation of each area from 0 to x into a height dx , giving a cone with a base equal to $\int_0^x M dx$ and a height x : it is also equal to the area $\int M dx$ multiplied by the distance of its centre of gravity from the point whose abscissa is x . I is here considered constant, and may be so taken in most trusses. If I varies, it will be expressed in terms of x , and retained within the integral sign.

90. Applications. — The areas are readily measured by scaling equidistant ordinates, and multiplying by the constant distance between two ordinates, as is done in calculating the contents of any irregular area by offsets. For a continuous load the equilibrium polygon becomes a curve, and the included areas with their centres of gravity are easily obtained for all practical cases.

We will now proceed to find the inclination, or slope, and the deflection, of several beams loaded in different ways. As E and I are usually expressed in units of pounds and inches, these units must be employed in denoting the imposed weights, the weight and dimensions of the beam. In all cases, l will denote the length or span of the beam; W , a single concentrated load; and w , the intensity per inch of a distributed load. The reader will see, after an inspection of one or two cases, what quantities are successively multiplied together.

91. 1st, Beam fixed at One End, loaded at the Other. —

The beam built into a wall or otherwise fixed at one end, and carrying a weight W at the free end, will take the form of the dotted curve sketched in Fig. 41. If W is laid off on a vertical line, it will represent the load, and also the shear at any

point of the beam from this load. Drawing H from one end, and completing the stress diagram, we see that the equilibrium polygon is a right-angled triangle, the bending moment increasing simply as the distance from the free end of the beam. Let k denote the height of this triangle. Then, by proportion, we shall have

$$W : H = k : l; \text{ or } Hk = Wl = M \text{ (max.)}.$$

The area of the triangle $= k \cdot \frac{1}{2} l$. If the beam is of uniform cross-section, I is constant. We have, then, for the slope at the extreme end, where the weight is attached,

$$\tan. i = \sum_0^l \frac{H}{EI} y = \frac{H}{EI} \sum_0^l y = \frac{1}{EI} Hk \cdot \frac{1}{2} l = \frac{W l^2}{2 EI}.$$

The centre of gravity of the triangle being distant $\frac{2}{3} l$ from the apex horizontally, the deflection will be obtained by multiplying $\tan. i$ by this distance, or

$$v = \frac{W l^2}{2 EI} \cdot \frac{2}{3} l = \frac{W l^3}{3 EI}$$

which is the vertical distance that the point of attachment of the weight is below the tangent at the fixed end.

92. 2d, Beam fixed at One End, and uniformly loaded. —

If the uniform load over the whole extent of the beam is considered as concentrated at a series of equidistant points, the equilibrium polygon will be readily drawn; and the true curve, when the polygon's sides are increased in number indefinitely, will be seen to be a parabola, Fig. 42, with its vertex below the free end of the beam, and tangent to the closing line. As this curve is in equilibrium, the two tangents at the extremities must, by § 10, meet on the vertical through the centre of gravity of the load; that is, at $\frac{1}{2} l$ from one end. Therefore, upon drawing the tangent at the left extremity of the curve, which will be parallel to the most inclined line of the stress diagram, we get, by similar triangles,

$$wl : H = k : \frac{1}{2} l; \text{ or } Hk = \frac{1}{2} w l^2 = M \text{ (max.)}.$$

As the area of a parabolic segment is two-thirds of the enclosing rectangle, the area of this figure will be $\frac{1}{3} l$ multiplied

by its altitude k ; its centre of gravity lies at $\frac{1}{4}l$ from the right: hence, as before,

$$\tan. i = \frac{1}{EI} \cdot \frac{1}{4} w l^2 \cdot \frac{1}{4} l = \frac{w l^3}{6 EI};$$

$$v = \frac{w l^3}{6 EI} \cdot \frac{1}{4} l = \frac{w l^4}{8 EI}.$$

93. 3d, Beam fixed at One End, both uniformly loaded and loaded at Free End. — If the beam is supposed to carry a weight W at the free end in addition to its own distributed weight, we may combine the two preceding figures, and therefore add the above expressions for slope in the one case, and for deflection in the other. Thus we obtain

$$\tan. i = \frac{1}{EI} \left(\frac{w l^3}{6} + \frac{W l^2}{2} \right);$$

$$v = \frac{1}{EI} \left(\frac{w l^4}{8} + \frac{W l^3}{3} \right).$$

94. 4th, Beam supported at Both Ends, a Single Weight in the Middle. — The equilibrium polygon is drawn in Fig. 43, and the reaction at each abutment is $\frac{1}{2}W$. On account of symmetry of loading, the beam will be horizontal at the middle, the greatest slope will be found at either abutment, and the deflection at the middle will be equal to the elevation of the end of the beam above the horizontal tangent at the middle: hence

$$\frac{1}{2}W : H = k : \frac{1}{2}l; \text{ or } Hk = \frac{1}{2}Wl = M(\text{max}).$$

Reckoning the change of inclination from the middle to one end, we have for the area a triangle of area $k \cdot \frac{1}{2}l$, and

$$\tan. i = \frac{1}{EI} \cdot \frac{1}{2}Wl \cdot \frac{1}{2}l = \frac{W l^2}{16 EI}.$$

Remembering that the horizontal distance from the centre of gravity is to be measured towards the opening of the angle between the tangent and the beam, we find

$$v = \frac{W l^2}{16 EI} \cdot \frac{1}{2}l = \frac{W l^3}{48 EI}.$$

95. 5th, Beam supported at Both Ends, and uniformly loaded with wl . — The equilibrium curve, Fig. 44, will be the

well-known parabola. The tangent at one end, as explained in § 72, will cut the middle ordinate prolonged at $2k$ from the horizontal line: hence

$$\frac{1}{2} w l : H = 2k : \frac{1}{2} l; \text{ or } Hk = \frac{1}{4} w l^2 = M (\text{max.}),$$

as has previously been shown. Then from the parabolic area, as above explained, we get

$$\tan. i = \frac{1}{EI} \cdot \frac{1}{2} w l^2 \cdot \frac{2}{3} \cdot \frac{1}{2} l = \frac{w l^3}{24 EI}.$$

The centre of gravity of the semi-segment of the parabola is $\frac{5}{8}$ of $\frac{1}{2} l$ from the abutment: hence

$$v = \frac{w l^3}{24 EI} \cdot \frac{5}{8} l = \frac{5}{192} \frac{w l^4}{EI}.$$

It is worthy of notice, that with beams of the same length and the same *total* applied load, under Cases 1, 2, 4, and 5, the maximum bending moments are relatively as 1, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$; and the relative strengths are as the reciprocals, 1, 2, 4, and 8.

✓ 96. **6th, Beam supported at Both Ends, carrying a Single Weight, distant a from the Right.** — This case is represented by Fig. 45, and is given as a sample of the flexibility of the method. a is greater than $\frac{1}{2} l$. The reaction at the left point of support will be $\frac{W a}{l}$. Then by proportion, as usual,

$$\frac{W a}{l} : H = k : l - a; \text{ or } Hk = \frac{W a (l - a)}{l} = M (\text{max.}).$$

The point where the beam is horizontal is at present unknown; but at that point, which will not be at the weight, is manifestly the maximum deflection. Suppose that the point is C, distant x from B. The distance which C is below A will be equal to

$\frac{H}{EI}$ multiplied by the area moment of the area in the equilibrium polygon to the *left* of the dotted line below C. The area

moment to the *right* of the dotted line multiplied by the same quantity will be the deflection of C below B. As the tangent at C is horizontal, these two expressions must be equal: hence to find the point of maximum deflection resolves itself into so dividing the equilibrium polygon by a vertical line, that the area

moment on one side, about the abutment on that side, shall equal the similar moment on the other side about its abutment.

The quantity $\frac{H}{EI}$, being constant, will not affect x .

The dotted line will cut off a trapezoid from the triangle to the right of the weight. One of its parallel sides being k , the other will be given by the proportion

$$a : k = x : \frac{kx}{a},$$

and its area will be equal to one-half the sum of its two parallel sides multiplied by $a - x$, the perpendicular distance between them: hence the area of the trapezoid is

$$\frac{1}{2} \left(k + \frac{kx}{a} \right) (a - x) = \frac{1}{2} k \frac{a^2 - x^2}{a}.$$

By taking moments about F for the triangle FDI, we see that $DI \cdot F \cdot \frac{2}{3} a - EG \cdot F \cdot \frac{2}{3} x = DI \cdot GE$ multiplied by x' , the distance of centre of gravity from F, or, in symbols,

$$\frac{k a}{2} \cdot \frac{2}{3} a - \frac{x}{2} \cdot \frac{k x}{a} \cdot \frac{2}{3} x = \frac{1}{2} k \cdot \frac{a^2 - x^2}{a} \cdot x';$$

$$\frac{1}{2} k \frac{a^2 - x^2}{a} = \frac{1}{2} k \frac{a^2 - x^2}{a} \cdot x', \text{ or}$$

$$x' = \frac{2}{3} \frac{a^2 - x^2}{a^2 - x^2};$$

and the distance of the centre of gravity of the trapezoid from $A = l - x'$: hence, making the area moment of the small triangle plus the trapezoid about A equal to the moment of the remaining area about B, we have

$$\frac{1}{2} k (l - a) \cdot \frac{2}{3} (l - a) + \frac{1}{2} k \frac{a^2 - x^2}{a} \left(l - \frac{2}{3} \frac{a^2 - x^2}{a^2 - x^2} \right) = \frac{1}{2} k \frac{x^2}{a} \cdot \frac{2}{3} x. \quad (b)$$

Dropping common factors, we get

$$(l - a)^2 + \frac{a^2 - x^2}{a} \left(\frac{2}{3} l - \frac{a^2 - x^2}{a^2 - x^2} \right) = \frac{x^2}{a}$$

$$a (l - a)^2 + \frac{2}{3} l (a^2 - x^2) - a^2 + x^2 = x^2$$

$$\frac{2}{3} l x^2 = a l^2 - \frac{1}{3} a^2 l$$

$$x = \sqrt{\frac{1}{3} a (2 l - a)}.$$

Substituting this value of x in the second member of the deflection equation (b) from which it was deduced, we see that

$$v = \frac{H}{EI} \cdot \frac{1}{2} \frac{kx^2}{a} = \frac{W a (l-a)}{l k EI} \cdot \frac{1}{2} \frac{k}{a} \cdot \frac{1}{2} a (2l-a) \sqrt{\frac{1}{2} a (2l-a)}$$

$$= \frac{W (l-a) a (2l-a)}{9 EI l} \sqrt{\frac{1}{2} a (2l-a)};$$

which expression, when $a = \frac{1}{2} l$, reduces to $\frac{W l^2}{48 EI}$, as in Case 4.

The slope at B will be

$$\tan. i = \frac{H}{EI} \cdot \frac{kx}{a} \cdot \frac{1}{2} x = \frac{W (l-a) a}{6 EI l} (2l-a).$$

The slope at A will be obtained similarly.

✓ 97. **Beam on Two Supports, but overhanging.** — Before taking up additional cases of deflection of beams, it may be well to discuss the equilibrium polygon for a beam loaded in any manner, carried on two supports, and overhanging at one end. Let A I, Fig. 48, be the beam supported at A and B, and let the weight of the beam be considered as concentrated with the additional loads. Draw the stress diagram, 0 1 2, as in other cases. Commence at A', and draw A' C' D' E' F' G' I' parallel to the radiating lines of the stress diagram, the angles occurring on the verticals let fall from the *weights*. If the applied weights are upon the upper side of the beam, and the reaction at B on the lower side, the force through B will not be encountered as we first pass across the beam. There will be one line, parallel to 0-2, to be added; and we return, as it were, below the beam, drawing the line from I' to the vertical through B, in the reverse direction. Connect A' and B' by a closing line, and the equilibrium polygon is complete. A line through 0, parallel to A' B', will divide the load line into the two supporting forces, P₁ at A, and P₂ at B.

If we lay off al below A, equal to P₁, and then construct $l c d e$, &c., as was done in Fig. 4, we shall reach a point b under B; then, laying off P₂ upwards from b to b' , we proceed to $g i$, finally closing on the horizontal line when we subtract the last weight. Drawing the horizontal line marked H through 0, we find the bending moment at any point by multiplying H by the ordinate between the equilibrium polygon and the line A' B' or

$B'I'$ for that point. At K' , there being no ordinate, the product is zero: consequently the beam is not bent at K . As we pass from K' to B' and I' , the ordinate, being below the polygon, may be called negative. The bending moment is in the contrary direction over the portion KI from that existing over AK ; and it tends to produce convexity on the upper side of the beam, reversing the tension and compression in the fibres. The point K is called a *point of contra-flexure*. The curvature of the beam is shown to an exaggerated scale by the dotted line $ALBN$.

In case the beam overhangs sufficiently, or is heavily weighted on the portion BI , it may be found that the line from O , parallel to $B'A'$, cuts the load-line vertical above 1 . The supporting force at A will then be negative, and the beam must be fastened down at that point to prevent its rising. The reaction at A will, as a tension in that case, be laid off below an in place of above it. In case the beam overhangs both points of support, we may have two points of contra-flexure; but the overhanging portions may be sufficiently weighted to cause convexity upwards over all the intermediate portion, when the line corresponding to $A'B'$ will pass entirely below the curve, and there will be no points of contra-flexure. If the two points of support are brought together into one point, and the overhanging portions balance each other, we have a diagram for each portion akin to Fig. 42. It will now be seen why the moment diagrams of Figs. 41 and 42 were drawn on the upper side of the horizontal or closing line.

✓ 98. **7th, Beam supported and fixed in Direction at Both Ends, Weight W in Middle.**—The requirement that the beam shall be fixed in direction at its ends necessitates that it shall be so restrained, by being built into masonry or by the application of certain moments, that the tangent to the curve of the bent beam shall remain horizontal at the points of support. The equilibrium polygon for this case will be the two inclined lines of the moment diagram of Fig. 46. The beam will be horizontal at its middle, by reason of symmetry of loading;

and, referring to Fig. 39, we see, that, from the middle of the span to the abutment, the total change of inclination in one direction must balance that in the other; or, since the changes of inclination are proportional to the ordinates to the closing line, we must draw the closing line at such a height above the vertex of the equilibrium polygon, that, from the middle to one support, the area remaining within the polygon shall just equal the area thus formed without the polygon. Since the two halves of the beam are subject to like forces, the closing line will be horizontal; and, if k represents the maximum ordinate to the original polygon, it is plain that the line must be drawn horizontally, at a distance $\frac{1}{2}k$ from the vertex, to satisfy the prescribed conditions.

We then get, by proportion,

$$\frac{1}{2}W : H = \frac{1}{2}k : \frac{1}{2}l; \text{ or } \frac{1}{2}Hk = \frac{1}{2}Wl = M (\text{max.})$$

at either abutment and at the middle, but of contrary signs, as shown by the direction of $\frac{1}{2}k$. We have points of contra-flexure where the closing line cuts the polygon, or at one-fourth the span from either end. At these points will be the greatest slope, but in opposite directions. Hence

$$\tan. i = \frac{1}{EI} \cdot \frac{1}{2}Wl \cdot \frac{1}{2} \cdot \frac{1}{2}l = \frac{Wl^2}{64EI}$$

The deflection of the middle point below either abutment, or the height of the abutment above a tangent at the middle of the span, will be obtained by taking area moments about the abutment, remembering that one area is positive and the other negative, inducing deflections in opposite directions.

$$v = \frac{1}{EI} \left\{ \frac{Wl^2}{64} \left(\frac{1}{2}l + \frac{1}{2} \cdot \frac{1}{2}l \right) - \frac{Wl^2}{64} \cdot \frac{1}{2} \cdot \frac{1}{2}l \right\} = \frac{Wl^2}{64EI} \cdot \frac{1}{2}l \left(1 + \frac{1}{2} - \frac{1}{2} \right) = \frac{Wl^2}{192EI}$$

It will be seen that the bending moments on the middle portion of the beam are the same as would be found were points of support placed at the points of contra-flexure, and the outside portions of the beam removed.

99. 8th, Beam fixed at Both Ends, and carrying a Distributed Load. — If we draw the equilibrium curve, a parabola, Fig. 47, of depth $k = \frac{w l^2}{8H}$, as seen in Case 5, whose area is two-

thirds of the enclosing rectangle, and superimpose on it a rectangle whose depth is $\frac{2}{3}k$, it is evident, that, the portions which cover one another being neglected, the portion of the rectangle outside the parabola must be equal in area to the portion of the parabola outside of the rectangle: hence the closing line is to be drawn horizontally at $\frac{1}{3}k$ above the vertex of the parabola. The bending moment at the abutment is, therefore, twice that at the middle of the span; and the latter is one-third of the moment which would have existed had the beam been simply supported at the ends: hence

$$M \text{ at abutment} = -\frac{2}{3} \cdot \frac{1}{8} w l^2 = -\frac{1}{12} w l^2.$$

$$M \text{ at middle} = \frac{1}{3} \cdot \frac{1}{8} w l^2 = \frac{1}{24} w l^2.$$

To find the point of contra-flexure, we must find that abscissa of the parabola whose ordinate from a tangent at the vertex is $\frac{1}{3}k$. Calling the distance of the point of contra-flexure from the middle of the span x , we have

$$k : (\frac{1}{3}l)^2 = \frac{1}{3}k : x^2; \text{ or } x^2 = \frac{l^2}{12}$$

$$x = \frac{1}{2}l \sqrt{\frac{1}{3}}.$$

The point of maximum slope will be at the point of contra-flexure; for, beyond this point, the curve bends the other way. The value will then be

$$\tan. i = \frac{H}{EI} \cdot \frac{1}{3}k \cdot \frac{2}{3}x = \frac{w l^2}{24 EI} \cdot \frac{2}{3}x = \frac{w l^2}{72 EI \sqrt{3}}.$$

We may easily obtain the deflection by taking the area moments of the original semi-segment of the parabola and of the semi-rectangle, or

$$v = \frac{w l^2}{8 EI} \left(\frac{2}{3} \cdot \frac{1}{3}l \cdot \frac{2}{3} \cdot \frac{1}{3}l - \frac{2}{3} \cdot \frac{1}{3}l \cdot \frac{1}{3} \cdot \frac{1}{3}l \right) = \frac{w l^4}{384 EI}.$$

100. 9th, Beam fixed at One End, and supported at the Other, with a Single Weight distant a from Fixed End. — The equilibrium polygon for this weight being drawn, Fig.

49, and its depth denoted by k , we recall, from Case 6, that $Hk = \frac{W a (l-a)}{l}$. The position of the closing line is required.

As there will be no bending moment at the end which is simply supported, the closing line will start from that extremity of the equilibrium polygon, and meet the vertical dropped from the other extremity at a distance to be found, and here denoted by y_0 . As the tangent at the fixed end is horizontal, it will always pass through the supported end: hence the summation of the several small deflections from the fixed to the supported end must be zero. Or, as the triangle with y_0 for its altitude overlaps the polygon first drawn, we may say that its area moment about the supported end must equal the area moment of the original triangle. We have, then,

$$y_0 \cdot \frac{1}{2} l \cdot \frac{2}{3} l = \frac{W a (l-a)}{H l} \cdot \frac{1}{2} l \cdot \frac{2}{3} \frac{l+l-a}{2} = \frac{W a (l-a) (2l-a)}{6 H}$$

$$y_0 = \frac{W a}{2 H^2} (l-a) (2l-a).$$

The bending moment at the fixed end is, therefore,

$$H y_0 = -\frac{W a}{2 l^2} (2 l^2 - 3 a l + a^2).$$

To determine the point of contra-flexure, we have the condition that the ordinate to the original triangle at the distance x from the fixed end shall equal the ordinate to the closing line at the same point, or

$$\frac{k}{a} x = \frac{y_0}{l} (l-x); \text{ or } \left(\frac{k}{a} + \frac{y_0}{l} \right) x = y_0;$$

$$x = \frac{y_0 a l}{k l + a y_0};$$

a quantity easily computed when a definite example is attempted. Upon drawing a line in the stress diagram parallel to the closing line, the supporting forces will be determined. The maximum bending moments of opposite signs occur at the weight and the fixed end. The value of the deflection can now be calculated, the point where the beam is horizontal being found first.

101. 10th, Beam fixed at One End, supported at the Other, and uniformly loaded.—Here the curve being a parabola, Fig. 50, we superimpose a triangle whose area moment about the supported end shall equal that of the parabolic segment, in order that the beam may remain horizontal at the fixed end: therefore

$$y_o \frac{1}{2} l \cdot \frac{2}{3} l = \frac{1}{3} \frac{w l^3}{H} \cdot \frac{2}{3} l \cdot \frac{1}{2} l; \text{ or } y_o = \frac{w l^2}{8 H} = k.$$

The bending moment at the fixed end is, then, $M = -\frac{w l^2}{8}$.

The point of contra-flexure is obtained by (see 9th Case)

$$\frac{y_o}{l} (l-x) = \frac{k}{(\frac{1}{2} l)^2} x (l-x); \quad x = \frac{l y_o}{4 k} = \frac{1}{2} l.$$

Another point of maximum bending moment will be at the middle of the small parabolic segment; that is, at three-eighths of the span from the supported end: its value may be proved to be

$$M = \frac{9}{128} w l^2.$$

The beam will be horizontal at the point where the positive area cut off on the left of the ordinate equals the negative area. The deflection will be measured to such point. The reaction at the supported end will be $\frac{3}{8} w l$. It is not expedient to carry out all of these steps in detail here: the method of doing so has already been indicated.

102. Beam of Two Spans; Special Device.—The last two cases might have been treated as beams of two equal spans continuous over a pier; for an equal and symmetrical load on each span would have made the beam horizontal over the middle support. The treatment would have been exactly the same. A method of analyzing the last case is here submitted, however, as illustrative of a modified treatment. Suppose that a beam of span l carries a uniform load of w per unit of length: the deflection at the middle, by Case 5, will be $\frac{5}{384} \cdot \frac{w l^4}{EI}$. If a force from below is applied at the middle of the span, of

just the magnitude required to bring this point to its original position, on a level with the two ends, the beam will be transformed to one of two equal spans, each one-half of the original amount, and the applied force will be the reaction of the middle pier. By Case 4, the deflection from a weight W at the middle

of a span is $\frac{W l^3}{48 E I}$. Putting these two deflections equal to

one another, we at once obtain $W = \frac{4}{3} w l$; or, if the new span $l' = \frac{1}{2} l$, $W = \frac{4}{3} w l'$, and the end reactions will each be $\frac{2}{3} w l'$. If the load line be divided accordingly, the closing lines of the equilibrium polygon can be drawn, and all the values of Case 10 at once obtained.

CHAPTER VII.

CONTINUOUS TRUSS OF TWO SPANS.

108. General Principle of Continuity.—The fact which was brought out in the last chapter, that the *area moment* of the equilibrium polygon is proportional to the deflection of a beam or truss, will enable us to deal with continuous trusses easily. A truss extending over two spans will first be taken up. Suppose that we have a beam, represented in Fig. 51, supported on the two abutments A and C, and divided by the pier at B into two unequal parts: its own weight is uniformly distributed, and it carries, in addition, a uniformly distributed load of twice the intensity from C to D, and also from E to F. Divide the two spans into convenient parts, equal or unequal (here taken equal), and consider the load to be concentrated at the points of division. Draw verticals through these points, and having constructed the load line and stress diagram, 0 1 2, draw the equilibrium polygon between C' and A' as in previous examples. The loads at A and C are neglected, and are represented by the portions of the load line which project at 2 and 1. Since B carries a load, the vertical through B will determine one of the angles of the polygon in the same way as any other loaded point.

As no bending moment exists at A, the desired closing line A'B' must start from A', and similarly C'B' is drawn through C'. We know that B' must lie below the curve; for we have a negative moment of flexure over B; that is, a moment which makes the beam convex on the upper side. We usually have two points of contra-flexure where A'B' and C'B' cut the

curve. Some condition is necessary to limit the position of B' , since there is manifestly but one correct value for the moment of flexure over the pier for a given position of the load.

104. **Abutment Deflections.**—From the demonstration in regard to *deflection* which has gone before, it may be seen that the vertical displacement of a point D , Fig. 51, in a beam under flexure, in reference to some point, such as C , as origin, and from a tangent CL to the beam at that origin, depends upon the successive changes of inclination between the two points, and the distances from the point D at which they occur, regard being paid to the direction of the change of inclination. Then, as each change of inclination is proportional to the ordinate to the equilibrium polygon, the deflection of a point from a tangent through the origin is proportional to the summation of the products of each ordinate into its distance from the point in question. As the summation of these products is the same thing as the area between C' and D' multiplied by the distance of its centre of gravity horizontally from D' , which we have styled an *area moment*, the deflection of D from the tangent through C is proportional to the area $C'D'$ multiplied by the horizontal distance of its centre of gravity from D' . If, then, CL is the tangent through C , the deflections or vertical distances of the points B and A from this tangent, or BK and AL , will be proportional to the proper area moments, and the closing lines to B' must be so drawn as to satisfy this condition. From the similarity of triangles, BK and AL are proportional to BC and AC , two known quantities.

We may, with advantage, by drawing a tangent $M.B.N$ to the beam at the pier B , obtain a relation of area moments more easily remembered and used. It is evident that $NC : MA = BC : BA$. From the preceding reasoning, denoting the centres of gravity of the respective areas by a, b, c , and d , and taking the distances towards the opening of the angle, we write a proportion of area moments

$$\frac{\text{Area } C'P'D' \cdot dl - \text{area } K'D'B' \cdot ck}{\text{Area } K'B'G' \cdot bh - \text{area } A'F'G' \cdot ag} = \frac{CN}{AM} = \frac{BC}{AB}.$$

The deflection MA being on the opposite side of the tangent from NC , the similar areas in the above proportion are taken with the opposite signs; that is, $K'D'B'$ being reckoned as negative in the first term of the proportion, $K'B'G'$ is considered positive in the second term; and so of the others. Or we may consider the distances to the right of B plus, and those to the left minus. It is evident that there is but one position of B' which will satisfy the above condition: for, if B' is carried still farther below K' , the first term of the proportion is diminished, and the second term is increased; while, if B' is raised, the reverse takes place. It will be remembered that area moments do not give absolute deflections, but are only proportional to them: to obtain actual deflections, the area moments must be multiplied by H , and divided by EI . As these quantities are constant, they disappear from the equation just deduced. If the moment of inertia of the beam or truss is variable, its rate of variation being known, each ordinate must be changed in the due proportion before the areas are computed. Generally I is considered to be constant.

105. Areas and Centres of Gravity.—The areas in question are easily measured. If they are bounded by broken lines, as is the case when we deal with concentrated loads, the ordinates represented by the dotted lines can be scaled and multiplied by the horizontal distance between them, which is usually constant. As a continuous load gives a curve which passes through the vertices of the polygon described when the same load is concentrated on detached points and the abutments, and as uniformly loaded portions have parts of parabolas for their equilibrium curves, many of the areas are parabolic segments, triangles, and combinations of the two. The centre of gravity of a parabolic segment, such as $C'D'P'$, is at d , half way horizontally between C' and D' . The centre of gravity of any triangle whose base is vertical is on the ordinate drawn at one-third of the horizontal distance from the base to the vertex.

If we connect C' and K' by a straight line, the segmental area $C'P'K'$ and the triangle $C'K'B'$ will mutually overlap;

so that $C'P'D' \cdot dl - K'D'B' \cdot ck$ will be equal to $C'P'K'$ multiplied by the distance of its centre of gravity horizontally from C' , minus $C'K'B'$ multiplied by two-thirds of the span BC . If an area is partly bounded by portions of two different parabolas, the common point of the two parabolas where the intensity of the load changes may be connected with the extreme points of the area, and it will thus be divided into a triangle and two parabolic segments. It may be unnecessary to add that any area may be divided into a number of parts, the respective centres of gravity found, and then the area moments of these parts calculated and combined, with the same result as if the area had been treated as a whole. $C'P'D'$ is bounded by a broken line; but, as the angles of this line lie on a parabola, the centre of gravity of the area is still half way horizontally between C' and D' .

As, with concentrated loads, areas are made up of trapezoids, the following method of finding the centre of gravity of any four-sided figure may be convenient when great accuracy is desired. Draw the two diagonals ac and bd , Fig. 53; bisect one at m , and lay off the short segment ae of the other at cf . Connect f and m , and the centre of gravity of the quadrilateral will lie in the line fm , at one-third of its length from m , the middle point of the diagonal bisected. By reversing the process with the diagonals, bisecting ac , and laying off de from b , another line may be drawn: the centre of gravity will be at its intersection with fm . See note, p. 135.

106. **Value of the Pier Ordinate y_o .**—The distance $K'B'$ may be determined easily, without the necessity of making trials to ascertain its value. We will illustrate by a simple example: the proof made use of applies to any case. Let a beam of two spans, c and d , Fig. 52, have a single load on each span. The equilibrium polygon will be similar to $ACDB$, the one represented. Let us suppose for an instant that there is no bending moment over the pier. In that case, drawing AI and IB , we should complete our figure, and calling the area of the triangle $ACI = \mathbf{A}$, of $IDB = \mathbf{B}$, and the distances

of the centres of gravity of these triangles from the verticals through A and B respectively **a** and **b**, we ought to have, by § 104, if the ends of the beams are on the supports,

$$\frac{-\mathbf{A} \mathbf{a}}{\mathbf{B} \mathbf{b}} = \frac{c}{d}.$$

Since a bending moment over the pier does exist, this equation will not be true. Then change the lines A I and I B to A E and E B, moving on the vertical a distance I E = y_0 . The area moments on one side are proportioned to the area moments on the other as c to d ; but the area moments on the left are equivalent to

$$\mathbf{A} \mathbf{a} - \mathbf{A} \mathbf{I} \mathbf{E} \cdot \frac{2}{3} c,$$

$\frac{2}{3} c$ being the distance of the centre of gravity of A I E from the abutment vertical. The area of A I E is $\frac{1}{2} c y_0$. A similar relation exists on the right. Therefore we may state our proportion as follows:—

$$\frac{-\mathbf{A} \mathbf{a} + \frac{1}{2} c y_0 \cdot \frac{2}{3} c}{\mathbf{B} \mathbf{b} - \frac{1}{2} d y_0 \cdot \frac{2}{3} d} = \frac{c}{d}.$$

Every thing here being known except y_0 , we obtain the distance which in general, as in Fig. 51, B' should be below K', when c and d denote the spans, **A** and **B** the areas A' F' K' and K' P' C', and **a** and **b** the distances to the abutments from their respective centres of gravity, —

$$y_0 = \frac{3 (\mathbf{A} \mathbf{a} \cdot d + \mathbf{B} \mathbf{b} \cdot c)}{c d (c + d)} = \frac{3}{c + d} \left(\frac{\mathbf{A} \mathbf{a}}{c} + \frac{\mathbf{B} \mathbf{b}}{d} \right).$$

$$\text{If } c = d, \quad y_0 = \frac{3}{2c^2} (\mathbf{A} \mathbf{a} + \mathbf{B} \mathbf{b}).$$

There is no failure when the beam happens to be horizontal over the pier B; for then M A and N C are each zero, and therefore area C' P' D' . $d l$ = area K' B' D' . $c k$, or K' B' G' . $b h$ = A' F' G' . $a g$.

107. **Remarks.**— The equilibrium polygons of the two spans might be constructed separately, as we should do for detached spans; but in this case we must have the same value of H for both polygons, and they must meet at one point on the vertical

through the centre pier: therefore, in drawing the polygons, we may start from K' , or any other point in the pier vertical, and work each way. Indeed, the polygon might cross a horizontal line through C' , and can be transferred at any time, if desired, to that line, by measuring off ordinates either above or below it, so that $A'B'$ and $B'C'$ shall coincide with or become the horizontal line. The construction of the polygon for each span, by a separate stress diagram, will be shown a little later in an example of four continuous spans. With a symmetrical load on the two spans, we may introduce the above value of y_0 into, and solve, Cases 9 and 10, §§ 100, 101.

108. Shear Diagram. — As the bending moments at all points are thus determined in Fig. 51, it remains to discuss the shear diagram. Upon drawing from 0 two lines, 0-4 and 0-5, parallel to $B'C'$ and $A'B'$, we shall divide the load line into three portions, which are, 1-4 = P_3 , the supporting force at C, 4-5 = P_2 , the supporting force at B, and 5-2 = P_1 , the supporting force at A. Lay off $P_1 = 5-2$ at mn ; draw the inclined lines, as was done in Fig. 6, steeper where the load is more intense, striking the vertical under the pier at q ; make $qr = P_2 = 4-5$; and complete the diagram by reaching s at a distance $ts = -P_3$ below t . The ordinates between mt and the lines just drawn give the shearing forces at all points, on the left of a plane of section and positive when above mt .

109. Discussion. — If the load is shifted in position, we may draw new equilibrium polygons, and then complete the diagrams; but, as in the case of a single span, a few diagrams will suffice, as will be seen presently.

The beam of Fig. 51, as now loaded, has two points of contra-flexure, — at D and G. It may happen, that, when one of the spans is much more heavily loaded than the other, the point of contra-flexure G' , on the shorter span, moving towards the outer end, may finally pass off altogether. As G' moves towards A' , the point 5, where the line parallel to $B'A'$ cuts the load line, will approach 3; and when G' reaches A' , and disappears, 5 will pass beyond 3. There will still be some slight pressure on the

abutment, although the span AB will be convex upward throughout its whole extent; and the end of the beam will not rise from the abutment A until it is found necessary, in order to satisfy the condition of proportionality of area moments, to so draw A'B' that its parallel line 0-5 passes entirely outside of the end 2 of the load line. As soon as this occurs, unless the beam is fastened down, it must be treated as one resting on two supports, and overhanging at one end, § 97, Fig. 48.

110. Formula for Pier Moment for a Continuous Load.— If the beam is completely covered with a uniform load, the equilibrium curve will be a parabola, and the middle ordinate for one span will be, by § 95, $\frac{w l^2}{8 H}$. If l_1 and l_2 denote the two spans, w_1 and w_2 the weight of load on the respective spans per unit of length, the formula of § 106 becomes, when we remember that the area of the parabolic segment = $\frac{w l^2}{8 H} \cdot \frac{2}{3} l$,

$$y_o = \frac{3}{l_1 + l_2} \left(\frac{w_1 l_1^3}{12 H l_1} \cdot \frac{1}{3} l_1 + \frac{w_2 l_2^3}{12 H l_2} \cdot \frac{1}{3} l_2 \right);$$

or, if M_o = pier moment,

$$H y_o = M_o = \frac{w_1 l_1^3 + w_2 l_2^3}{8 (l_1 + l_2)}.$$

If $l_1 = l_2$ and $w_1 = w_2$, we get the result of § 101.

Since the load on a truss is concentrated at joints, the equilibrium polygon for such a complete load will be *inscribed* in the parabola for the same load continuously distributed as on a beam: hence the area of this polygon will be a little less than $\frac{w l^2}{12 H}$, the area given above. It follows that the bending moment over the pier for a continuous truss of two spans, loaded at joints, is slightly less than the bending moment by the above formula. The original formula for y_o , § 106, will give the correct ordinate.

111. Extent of Load to produce Maximum Moments.— In treating a truss of one span, we found, that, since a load at any point caused positive bending moments at all points of the

span, the maximum bending moment, and hence the maximum chord-stress at every point, would occur when all possible loads were placed on the bridge, or it was covered from end to end. An inspection of Fig. 51, just discussed, will show that positive and negative bending moments occur in different parts of the same span, and that while a load added at any point in the beam already subject to positive bending moment will increase the bending moment, another weight put on near and to the right of B will tend to diminish the negative moment at the point of application by causing the point of contra-flexure to move nearer K'.

Suppose that the beam of Fig. 55 carries a single weight only on the span B C = l_2 , at a distance a from C. The equilibrium polygon for the span B C will be a triangle, and there will be none for the other span. By § 96 we see that k will equal $\frac{W a (l_2 - a)}{H l_2}$, and that

$$B = \frac{W a (l_2 - a)}{2 H}; \quad b = \frac{1}{2} (l_2 + a).$$

Substituting in the equation for y_0 , § 106, we get

$$y_0 = \frac{3}{l_1 + l_2} \cdot \frac{B b}{l_2} = \frac{3}{l_1 + l_2} \cdot \frac{W a (l_2 - a)}{6 H l_2} = \frac{k}{2} \cdot \frac{l_2 + a}{l_1 + l_2}.$$

If, upon plotting this value of y_0 , and drawing the closing lines, we find that the weight at a distance a from C causes a *positive* moment over any portion of the beam which has a *negative* moment under a full load, it is evident that the removal of this weight will increase the negative moment by just the amount of the positive moment thus removed from the particular point. The use of this construction will be seen in the sequel.

112. Example.—We will make a practical application of this method to a truss of two spans, one of 80 feet, and the other of 100 feet, as represented in Fig. 54, continuous over the pier. Let each truss weigh $2\frac{1}{2}$ tons per panel of ten feet, or 500 pounds per foot; and let the rolling load be a thousand

pounds per foot of one truss, or one ton per foot for the bridge. A panel weight for one truss will be maximum $7\frac{1}{2}$ tons, minimum $2\frac{1}{2}$ tons.

Take $1-2 = 135$ tons, the weight of both spans when fully loaded; divide $1-2$ into portions of $7\frac{1}{2}$ tons, with end portions at 1 and 2 of $3\frac{1}{2}$ tons at A and C; assume a point 0, preferably opposite the middle of $1-2$, and at a distance in this figure of 50 tons. Since the truss is drawn as $12\frac{1}{2}$ feet high, it will simply be necessary to multiply the ordinates to the equilibrium polygons by *four* to obtain the stresses in the chords; and this can be done, without multiplication, by measuring the ordinates by the proper scale, thus converting the moment diagram into a chord-stress diagram at once. Leave out of consideration the end portions of $3\frac{1}{2}$ tons, which come directly upon the abutments, and, commencing at A', draw the polygon A' M C' parallel to the several lines connecting 0 with the points of division of the load line. Only the extreme lines radiating from 0 are drawn in the figure, as the remainder would confuse it, and tend to render the position of the point 0 uncertain. As we know that the middle ordinate of **A** must be $\frac{7\frac{1}{2} \times 10 \times 100}{8 \times 50} = 18\frac{3}{4}$ feet, the remaining ordinates will be easily

calculated, 18, $15\frac{1}{4}$, 12, and $6\frac{3}{4}$ feet: hence, summing all of the ordinates, and multiplying by the constant panel length 10 feet, we get **A** = 1237.5. The other area may be obtained similarly. Scaling from a diagram of reasonable size will answer as well.

Now calculate M B' by the formula for y_o , § 106, by which

$$M B' = \frac{3}{100 + 80} \left(\frac{1237.5 \times 50}{100} + \frac{630 \times 40}{80} \right) = \frac{1}{80} (618.75 + 315) = 15.56 \text{ ft.}$$

(The formula for a *continuous* load, deduced in § 110, would give

$$M B' = \frac{M}{50} = \frac{\frac{1}{2} \cdot 80^2 + \frac{1}{2} \cdot 100^2}{50 \cdot 8 (80 + 100)} = 15.75 \text{ ft.}$$

The previous value is the correct one.)

Laying off $M B' = 15.56$ feet, draw A' B' and B' C'. The figure A' M C' B' encloses the ordinates for bending moment

when both spans are fully loaded. Draw 0-3 and 0-4 parallel to $A'B'$ and $B'C'$; lay off 2-3 upwards at ad , and 4-1 downwards at ce ; draw df at an inclination of $7\frac{1}{2}$ tons to a panel; make fg equal to 3-4; and draw ge at the same inclination: it should close on the point e , just plotted. The figure $adfg e c b a$ encloses the ordinates for shearing force when both spans are fully loaded. These statements follow from the investigations of single spans.

113. Load on One Span only.—Remove all of the rolling load from BC , including in this figure the load on B : the load line will extend from 2 to 5, and the extreme radiating lines are again shown by full lines. Use the equilibrium polygon $A'M$, and add the part from M to C'' . As the load on BC is one-third of its former amount, B will now be 210, and the other quantities will be unchanged: hence the pier ordinate will be

$$MB'' = \frac{1}{60} (618.75 + 105) = 12.06 \text{ ft.}$$

Plot this value; draw $A'B''$ and $B''C''$. Find anew the supporting forces; lay off ah upwards; draw hi at an inclination of $7\frac{1}{2}$ tons to a panel; make il equal the pier reaction plus one-half panel weight of moving load, or $2\frac{1}{2}$ tons; draw lk at an inclination of $2\frac{1}{2}$ tons per panel, closing with kc for the abutment reaction at C . The half-panel weight of moving load is added at i , because the entire travelling load was removed from B , and the real diagram would have been made by a line, shown in the figure, at an inclination of $2\frac{1}{2}$ tons per panel, meeting il in the middle of the first panel from the pier. It is easier to draw the complete line hi ; and no error will arise, as the shear ordinates are measured in the middle of each panel. If the two abutment reactions are plotted first, we need pay no heed to the half-panel weight of rolling-load on B , as the two inclined lines for shear will intercept between them on the pier vertical the proper reaction plus the half-weight.

114. Load again shifted.—If the remaining portion of the rolling load is removed, uncovering the span AB , the load line will be reduced to 5-6. As the part $C''M$ of the equilib-

rium polygon applies to the second span, add the portion $M A''$, and, finding the point B''' , complete the figure: $M B'''$ will be in this case exactly one-third of $M B'$, because that is the ratio of the two loads. The lines limiting the ordinates in the diagram of shearing force will be $m n$ and $p q$. The polygon $A'' M C'$ will represent the case of the shorter span covered with the rolling load, including the point B , and the longer span unloaded; and, as the value of $M B'''$ will now be

$$y_o = \frac{1}{4} (206.25 + 315) = 8.69 \text{ ft.},$$

we draw $A'' B'''$ and $B''' C'$, find the supporting forces, and thence the lines $r s$ and $u t$. (See the closing remark of § 113.)

Each one of the equilibrium polygons might be drawn independently; but by the method here carried out, of passing them all through a common point M , two complete polygons suffice for four different positions of moving load; and, in any case of trusses with horizontal chords, but little need be added to these polygons to determine the maximum chord-stresses. The two polygons might have passed through M without coinciding for one panel, as in this figure; in which case B would probably have been relieved of one-half of its rolling load when one span was unloaded. Some readers may prefer such a treatment; but we should then require new divisions of the load line, in place of using the original divisions as here. It is evident that the presence of a load at B , or its absence from that point, can in no way affect the bending moments.

115. Discussion of Results. — A little study of the polygons for bending moment will show, that, for the given intensities of load, all possible polygons, if drawn to pass through M , will lie between $M A'$ and $M A''$ on one side, and $M C'$ and $M C''$ on the other; also that the maximum bending moment which tends to make the truss concave on the upper side occurs when *one span is fully loaded*, the other being at the same time without travelling load. This fact might be anticipated, since the addition of a load on one span will tend to increase the deflection of all points on that span, and to diminish the deflec-

tion of all points on the other span. The maximum bending moment over the pier will occur when *both* spans are completely loaded, for every increment of load adds to such negative moment; and its value will, therefore, be $H \cdot M B'$. This moment will give tension in the top chord, and compression in the bottom chord, at that section. Conversely, $M B''''$ must be the shortest ordinate, as the polygon to which it relates represents the lightest possible load. All values of y_0 will, therefore, lie between $M B'$ and $M B''''$.

As $A' M C'$ and $A'' M C''$ are the limiting polygons, they give the extreme deviations of the points of contra-flexure; and from the positions of these points can readily be determined the portion of each chord subject to tension alone, to compression alone, and the portion which must be adapted to withstand either stress as the load shifts its position. It will be noticed that the point of contra-flexure for the hundred-foot span shifts from the second panel from B, when only that span carries the moving load, to the fourth panel from B when only the other span is fully loaded; while, for the eighty-foot span, the point of contra-flexure shifts from the second panel from B to the seventh panel from B under similar variations of load. The influence of the longer span on the shorter is very marked, as, when the longer one is fully loaded, the unloaded span presses at C with only the weight represented by ck . A check on the accuracy of construction is found in the fact, that, when both spans are loaded and both unloaded, the points of contra-flexure occur at the same place.

116. Length of Chord under each Kind of Stress.— Since there is always a negative bending moment at the *first* joint on either side of the pier, we shall find, when we take moments at that joint of the bottom chord and again at the first joint in the top chord, that the first panel of the top chord will always be under tension, and that in the bottom chord the second panel from the pier will always be subject to compression. The fine line of the figure denotes a tension, and the heavy line a compression member. The double line on the adjoining portions

of either chord signifies that such pieces must be adapted to resist both tension and compression.

When the point of contra-flexure advances to the fourth panel from the pier in the hundred-foot truss, there will still be a positive bending moment at the joint beyond: hence in the bottom chord the fifth panel from the pier will be a tension member, and in the top chord the fourth panel from the pier will always be in compression. As, in the eighty-foot span, the point of contra-flexure may reach the second panel from C, the first joint from C will be the only one always under a positive bending moment. The supporting force will then be $k c$, and the shear line $k l$, the shear will be of opposite signs in the first and second panels, the ties which meet at the common joint of the bottom chord for these two panels will be in action, and therefore two panels of the top chord will be in compression. There will be no corresponding stress in the lower member, as the two tie-braces react against one another. The first panel of the bottom chord, of course, has no stress.

117. Partial Load on One Span.—The polygon $M D'$ on the left applies to the case where the span from A to D has upon it no moving load. If the polygon $C'' M E D'$ is taken, the moving load extends from B to D only; if $C' M E D'$ is used, the load covers the span B C also. In the first case, the required point on the pier vertical is found to be just below B''' , as seen in the figure; and, in the second case, a little below B'' . The dotted lines from C'' , C' , and D' to M, and from E to M and D' , show the different areas used in finding the values of y_0 . The areas may be scaled or computed, as thought best. Upon finding the reactions, we construct $y v w$ and the dotted line between $q p$ and $k l$, as the lines limiting the ordinates for shear when a moving load extends over B D alone; while $x z o$ and the dotted line between $t u$ and $e g$ will determine the shear ordinates for a moving load over C D. The inclinations of these lines correspond to the intensities of the loads, and the inclination changes in the middle of the panel at the head of the load.

118. Completion of Shear Diagram; Analysis. — The greatest pressure on the abutment A occurs when A B is fully loaded, and B C carries no moving load. The supporting force is then $a h$, and the shear in the two spans will be given by ordinates at the middle of each panel to the lines $h i$ and $l k$. For the reason why ordinates should be measured in the middle of each panel, see § 17. If the load extends over both spans, the supporting force at A falls to $a d$, and we get the shear by drawing $d f$ and $g e$. If neither span has any rolling load upon it, we find the pressure at A to be $a m$, and then draw $m n$ and $p q$. Finally, if B C alone carries the rolling load, the pressure at A diminishes to $a r$, and the shear diagram will be completed by the lines $r s$ and $u t$.

Again: if the rolling load, at first extending entirely from A to C, moves off from the portion A D, the supporting force at A diminishes from $a d$ to $a x$, and the shear will be given in the span A B, as lately stated, by ordinates to the lines $x z o$; the point z occurring in the last lightly-loaded panel D. If the rolling load still covers B D, but the load on B C is supposed to be removed, the supporting force at A will immediately increase to $a y$, and the bounding lines for the ordinates will now be $y v w$. In the span B C the inclined dotted line of the greater inclination will give the shear for the first arrangement of load, and the dotted line of less inclination will limit the shear ordinates for the second arrangement of load. As it is manifest that $h i$ and $m n$ are the limiting lines for the extreme cases of load over A B alone and load upon neither span, and as the load may cover any number of panels from one to ten, from B towards A, the sets of lines, of which $y v w$ is one, will, for different positions of the load, shift between $h i$ and $m n$, and the point v , at the head of the load, will move on a curved line from the middle of one end panel to the middle of the other, in the span A B. At the same time, the line for the unloaded span moves parallel to and between $k l$ and $p q$.

If a load, having covered B C, should then extend from B

towards A, we should obtain in a similar manner a curve, traced by the point z , between the lines df and rs , from middle to middle of end panels. As the ordinates to this curve above the line ab are less than those to hvn , the latter line only need be drawn. Therefore, for a load advancing from B, the ordinates to hvn at the middle of each panel determine the maximum shear of this kind. Although this curve is not exactly a parabola, the construction of § 20 will give a curve which comes very close to the actual one, agreeing at the ends, and giving shear ordinates slightly in excess of the truth at the middle portion of the span. As the shear curve extends between the middle ordinates of the first and last panels of the span, set off the half-panel from h and n , and then divide the remaining portion of each tangent, up to their common intersection, into a number of equal parts one less than the number of panels in the span. The divisions are marked in this figure from p to t . While the small error is on the side of safety, the amount which the constructed curve passes outside of the true point v will indicate the magnitude of the greatest deviation; and the curve can then be corrected by setting in, towards its tangents, as is usual in corrections, according to the square of the distance. The allowance can be easily made by the eye, and is seldom large enough to be of practical importance. In any case, by placing a load on half of the span, and determining a point similar to v , the curve can be located with all desired accuracy.

If a rolling load advances from A, the span BC being unloaded, the pressure at A increases from am to ah , and the different limiting lines for shear will move between hi and mn ; but as the inclinations of the lines which correspond to yv and vw will be reversed, since AD now is loaded, and DB is not, the point j at the angle will, like v , trace a curve from the middle of the panel near m to the one near i of the opposite curvature. If, again, BC is loaded, and a load advances from A towards B, the curve drawn in the figure from r to f will be described. As the latter curve includes the former, it

is the only one required. While the load is shifted as just described, the line for the other span vibrates between the parallels kl and pq in the one case, or tu and eg in the other.

The shear-curves hvn and rjf limit all the ordinates for shearing force in the span AB for every position of moving load; the portion of the curve hvn , which lies above ab , giving the maximum shearing stress upwards on the left of any section, and calling for diagonals all sloping one way from A to the panel over v . The truss of the figure has ties. The part of rjf which lies below ab gives the maximum downward shear on the left of a section, and requires braces from B to the third panel from A , as shown, with an inclination in the opposite direction. A similar construction supplies the required shear-curves for the span BC ; and, because the spans are dissimilar, all four curves must be drawn, and all possible movements of the travelling load are then provided for. While some may think that a rolling load will never take all of the positions assumed above, we repeat that the worst possible combinations are provided for. It will be seen how the counter-bracing is shifted from the middle of the spans towards the free ends. The stresses in the diagonals can now be obtained by drawing lines parallel to those pieces, as in Fig. 11.

119. Remarks.—The desire to have the diagrams clear, while they are on so small a scale, forbids the drawing of many equilibrium polygons and lines of shear for various positions of the moving load; but if the reader will construct a diagram for himself, with the load shifted, panel by panel, for a few panels, he will be able thoroughly to assure himself of the truth of the statements here made as to the limiting values of bending moment and shearing force. As the truss has parallel chords, it is just as easy to have the ordinates to the equilibrium polygon represent chord-stresses as bending moments. It will only be necessary that the middle ordinate at E , for instance, to the dotted line MA' , shall equal $\frac{W'' l}{8 k}$, as explained in § 28, and similarly for the other span.

120. Checks on the Accuracy of Diagrams.—There are several tests for proving the accuracy of a set of diagrams. The shear for a particular load should always change its sign at the point where the bending moment for that load is greatest: thus, above the point where $h i$ cuts the horizontal line $a b$, will be found, if no error has been made, the maximum ordinate of the corresponding moment diagram. The two lines $m n$ and $d f$ must intersect on $a b$, as they apply to the cases of uniform loads over both spans; and $p q$ intersects $g e$ on $b c$ for the same reason. Points of contra-flexure for similar distributions of load must agree. The intersection of $u t$ with $p q$ must be vertically under the intersection of $g e$ with $k l$, or $k q$ must equal $e t$; and similarly $h d$ must equal $m r$; for a certain weight on one span will diminish the reaction at the farther abutment of the other span a certain definite amount, no matter whether the second span be loaded or unloaded: hence, putting the rolling load over the span B C will diminish the reaction at A from h to d , or an equal amount from m to r , depending upon whether A B is loaded or unloaded. This last check is an excellent one.

121. Maximum Negative Moments.—Since the maximum negative moments at different joints of these spans occur, with one exception, as seen in Fig. 54, when the rolling load is removed from the span in which the joint is situated, there is only one joint to which the method of § 111, Fig. 55, need be applied. At the first joint to the left of pier B it will be seen that the maximum negative moment of Fig. 54 is caused by a complete load on both of the spans A B and B C. The removal of such weights of moving load as cause positive moments at the joint in question will increase the negative moment by the amount of the positive moments removed. In applying the formula of § 111, $W = 5$ tons, $H = 50$ tons, $l_2 = 100$ ft., $l_1 = 80$ ft., and $a = 90$ ft., 80 ft., &c., for successive joints measured from A. For the first joint on the left of B we therefore have, if the load is on that joint,

$$k = \frac{W a (l_2 - a)}{H l_1} = \frac{5 \times 90 \times 10}{50 \times 100} = 0.9 \text{ ft.}$$

$$y_0 = \frac{1}{2} k \frac{l_2 + a}{l_1 + l_2} = \frac{0.9}{2} \cdot \frac{190}{180} = 0.475 \text{ ft.}$$

The vertical distance to the closing line at the first joint will be $\frac{1}{10} k \times 0.475 = 0.4275$ feet, and the remaining ordinate is $k - 0.427$ feet = 0.473 feet.

If the load is on the second joint from B,

$$k = \frac{5 \times 80 \times 20}{50 \times 100} = 1.6 \text{ ft., and } y_0 = \frac{1.6}{2} \cdot \frac{180}{180} = 0.800 \text{ ft.}$$

At the first joint the original ordinate will be $\frac{1}{2} k = 0.8$ feet, and $\frac{1}{10} y_0 = 0.72$ feet. The remaining ordinate = $0.8 - 0.72 = 0.08$ feet.

For a load on the third joint,

$$k = \frac{5 \times 70 \times 30}{50 \times 100} = 2.1 \text{ ft., and } y_0 = \frac{2.1}{2} \cdot \frac{170}{180} = 0.992 \text{ ft.}$$

At the first joint, original ordinate = 0.7 feet, and $\frac{1}{10} y_0 = 0.893$ feet. The remaining ordinate = $0.7 - 0.893 = -0.193$ feet.

It will be unnecessary to proceed any further since this load causes a negative moment at the first joint. Adding the previous ordinates for the first joint, and multiplying by H, we get

$$(0.473 + .08) 50 = 27.65 \text{ ft. tons ;}$$

which is the amount of negative bending moment to be added to the quantity indicated at the first joint to the left of B, Fig. 54. If preferred, the diagrams may be plotted, as at the bottom of the figure, to a large scale.

As the total ordinate just obtained is about one-half foot, it is evident that no removal of weights from the span B C will increase the ordinate given in the figure at the first joint on the right of B for a complete load on both spans sufficiently to make it greater than the one indicated for steady load alone on B C.

122. Closing Remarks.—While the full lines in this figure

show all of the polygons and diagrams required for a complete discussion of a two-span continuous girder with horizontal chords, a truss with curved or inclined chords cannot be analyzed without an equilibrium polygon for each position of the load, to be combined with the shear diagram, as explained in § 63. Continuous girders are, however, almost always designed with a constant height. Where the height varies, it is proper that the rate at which such a variation in height changes the moment of inertia of the cross-section of the truss should be introduced in the process of equating *area moments*. As such a process is simply finding deflections from a tangent at one of the points of support, I was dropped from the denominator as constant. If its rate of variation is assumed, the successive ordinates must be lengthened or shortened, as the case may be, at that rate, before the area is computed, and its centre of gravity found.

If the two spans are equal, the number of lines becomes less as B'' and B''' will coincide, and the shear diagram need be drawn for one span only.

The Area Moment of any area in a moment polygon, where loads are concentrated at equidistant panel joints, may be readily obtained by the following method, no matter how irregular the area may be :

Let the strips of equal breadth into which the area is divided by the verticals let fall from the joints be conceived to be divided by diagonals into pairs of equal triangles, as seen in the sketch below Fig. 29c, Plate III. Each pair of triangles, a, a, b, b , &c., having an ordinate for their common base, will have their common centre of gravity on that ordinate. Hence, if n is the number of panel lengths any ordinate y is distant from the point about which the area moment is to be taken, and p is a panel length, the area moment of each pair of triangles will be $y p \cdot n p$, and the whole area moment $= p^2 \sum n y$.

Therefore, multiply each ordinate by the number which represents its distance in panel lengths from the point about which the area moment is to be taken, sum the products, and multiply this sum by the square of a panel length. [Suggested by C. G. Wrentmore.]

CHAPTER VIII.

CONTINUOUS TRUSS OF MANY SPANS.

123. Truss of Four Spans; General Treatment.—From the example presented in Fig. 56 we can deduce such expressions for pier ordinates as will be applicable to any number of spans. This figure shows four spans, of eighty feet, one hundred feet, fifty feet, and forty feet successively, loaded, as represented, with a travelling load, from L to M and N to P, of five tons per panel, and steady load throughout, from A to E, of two tons and a half per panel. To prevent the load line and stress diagram from occupying too much space, as well as to show the perfect practicability of drawing separate moment polygons for each span, as we do for single trusses, we have laid off the load lines of the longer spans independently, but have taken the same value for H in all of the diagrams; and this must always be done. The stress diagram for the eighty-foot span is 0 1 2: the marks of division show the weights on the respective panel joints.

Starting from A', draw the moment polygon for this span terminating at B'; from B' draw the moment polygon for the hundred-foot span by lines parallel to those which would complete the stress diagram 0' 3 4, 3-4 being the load on this span, and the polygon ending at C'. As the last two spans are short, draw 5-6 equal to the load on both spans, and then construct C' D' E', as has been done before. Now draw A' B', B' C', C' D', and D' E'. Compute the areas between each of these straight lines and the respective moment polygons, and determine the centre of gravity of each area. Areas belonging

to spans which are partially loaded may be divided, as explained in § 105. Let the spans, commencing with A B on the left, be l_a, l_b, l_c, l_d . Let the areas be represented by **A, B, C, and D**. Let the distance of the centre of gravity of **A** horizontally from A be a , and from B be a' ; let b and b', c and c', d and d' , denote the similar distances for **B, C, and D**, from the pier verticals on their left and right.

124. Pier Ordinates.—We now desire to find, in the same way as we determined y_o for two spans, the distances y_1, y_2 , and y_3 , or B' F, C' G, and D' I, the ordinates at the piers, required to complete the diagram for bending moments. In the first place imagine that y_3 , or D' I, is plotted below D', so that the pier ordinates are all of one sign. If a tangent is drawn at B to the curve which the straight line A B C will assume under the given load, we know by §§ 104, 106, that the deflections at A and C from the tangent at B are proportional to the spans, or

$$\frac{A a - \frac{1}{2} y_1 \cdot \frac{2}{3} l_a^2}{B' F C' \cdot \frac{2}{3} l_b + F C' G \cdot \frac{1}{3} l_b - B b'} \text{ or } \frac{A a - \frac{1}{2} y_1 l_a^2}{\frac{2 y_1 + y_2}{6} l_b^2 - B b'} = \frac{l_a}{l_b}.$$

Similarly for the spans B C and C D when a tangent is drawn through C, and for C D and D E in reference to a tangent at D,

$$\frac{B b - \frac{1}{2} y_1 \cdot \frac{1}{3} l_b^2 - \frac{1}{2} y_2 \cdot \frac{2}{3} l_b^2}{\frac{1}{3} y_2 \cdot \frac{2}{3} l_c^2 + \frac{1}{3} y_3 \cdot \frac{1}{3} l_c^2 - C c'} \text{ or } \frac{B b - \frac{y_1 + 2 y_2}{6} l_b^2}{\frac{2 y_2 + y_3}{6} l_c^2 - C c'} = \frac{l_b}{l_c};$$

$$\frac{C c - \frac{1}{2} y_2 \cdot \frac{1}{3} l_c^2 - \frac{1}{2} y_3 \cdot \frac{2}{3} l_c^2}{\frac{1}{3} y_3 \cdot \frac{2}{3} l_d^2 - D d'} \text{ or } \frac{C c - \frac{y_2 + 2 y_3}{6} l_c^2}{\frac{1}{3} y_3 l_d^2 - D d'} = \frac{l_c}{l_d}.$$

Upon reducing these equations, by bringing the unknown quantities together, we obtain

$$2 (l_a + l_b) y_1 + l_b y_2 = 6 \left(\frac{A a}{l_a} + \frac{B b'}{l_b} \right); \quad (1)$$

$$l_b y_1 + 2 (l_b + l_c) y_2 + l_c y_3 = 6 \left(\frac{B b}{l_b} + \frac{C c'}{l_c} \right); \quad (2)$$

$$l_c y_2 + 2 (l_c + l_d) y_3 = 6 \left(\frac{C c}{l_c} + \frac{D d'}{l_d} \right). \quad (3)$$

It is evident that (2) is the general equation which applies to any pier of a continuous girder, it having been derived for the pier at C, and the spans B C and C D being restrained at B and D by moments caused by other spans beyond. If, in (2), y_1 equal zero, we have the form of (1); if y_3 equal zero, we get (3); if y_1 and y_3 each equal zero, we get the equation of § 106 for y_2 for two spans only. Hence, by beginning with an equation like (1), closing with one like (3), and writing equations similar to (2) a sufficient number of times to make in all one less equation than the number of spans, the required equations for the unknown pier moments for any number of continuous spans will be obtained. If $l_a = l_b = l_c$, &c., the equations simplify a little. The simplicity and symmetry of the general equation (2) is worth noting.

125. Solution of Equations.—To show that no difficulty exists in the solution of these equations, let us apply them to the above case, and, since the second members are known quantities, denote them by P, Q, and R. Then

$$\begin{array}{rcl}
 360 y_1 + 100 y_3 & = & P. \\
 100 y_1 + 300 y_2 + 50 y_3 & = & Q. \\
 50 y_2 + 180 y_3 & = & R. \quad (\text{Multiply 2d equation by 3.6.}) \\
 \hline
 360 y_1 + 1080 y_2 + 180 y_3 & = & 3.6 Q. \quad (\text{Subtract 1st and 3d equations.}) \\
 930 y_2 & = & 3.6 Q - P - R. \\
 y_2 & = & \frac{3.6 Q - P - R}{930}.
 \end{array}$$

Substitute in first equation, and find y_1 ; then in third, and find y_3 .

126. Positive Pier Moment.—If the reader will take the trouble to make a numerical solution of this example with the given intensities and distribution of the load as shown in Fig. 56, he will meet with one peculiar result,—the value of y_2 will be found to be *minus*; that is, D' I must be measured off from D' upwards, and denotes a positive bending moment, the truss being concave upwards. There is, therefore, no point of contraflexure in the span D E, and no negative moment over the pier D. The truss presses on the pier D, however, to an amount to

be found presently. That this pressure is small arises from the fact that the load on the hundred-foot span has a tendency to lift the truss from D, and it will be seen that almost all of C D is under a negative bending moment. If the point I had fallen on the straight line joining G with E', the bridge would have been lifted entirely clear of the pier D; and, if I had come above a line from G to E', another solution would have been required, with the pier D considered as removed, making C E one span; or else the truss would have required bolting down at D.

✓ 127. **Shear Diagram.**—Having drawn A' F, F' G, G' I, and I E', draw in the respective stress diagrams lines 0-7, 0'-8, 0''-9, and 0'''-10, parallel to them, cutting off the supporting forces at the several piers arising from each truss, and then complete the diagrams for shearing force which are drawn below the truss. We see that by drawing the stress diagram for each span by itself, as has here been done, we get the supporting force at any pier common to two spans in two portions, one belonging to each span. Thus 7-1 = ap , and 2-7 = fb ; 8-3 gives bm , and 4-8 gives gc . The pressure at B is $mb + bf$. The pressure at D is $-id + dk$ or ik . The shear diagram is reduced in scale to save room.

128. **Clapeyron's Formula for Uniform Loads.**—If a beam of uniform cross-section is continuous over several spans, and each span is loaded throughout its extent with a load of uniform intensity, such intensity differing on different spans, the area of the equilibrium polygon for any one span, if it is unconnected with the others, will be, by § 95, $\frac{2}{3} l k = \frac{w l^3}{12 H}$ and the horizontal distance of the centre of gravity from either support will be $\frac{1}{2} l$. If, then, these values of B b and C c' are substituted in (2), § 124, for such a case we get

$$l_b y_1 + 2 (l_b + l_c) y_2 + l_c y_3 = 6 \left(\frac{w_b l_b^3}{24 H} + \frac{w_c l_c^3}{24 H} \right).$$

Multiplying by H, and remembering that H y = moment at pier = M, we have

$$M_1 l_b + 2 M_2 (l_b + l_c) + M_3 l_c = \frac{1}{4} (w_b l_b^3 + w_c l_c^3).$$

The above equation is known as *Clapeyron's Formula*, or the *Three-Mo-*

ment Theorem, as applied to uniform loads on a continuous girder of uniform cross-section. From it may be readily deduced the formula for two spans; and the remark that concentrated loads give slightly less moments over the piers than does the same amount of distributed load applies here also.

129. Three-Moment Theorem for Single Weight.—Referring again to the general equation (2), § 124, we see that it is concerned with two spans only, l_b and l_c . If, then, a single load is placed on one of these spans, it will be interesting to see the form which the equation will assume. Suppose that a weight W is placed on l_b at a distance $n l_b$ from the left pier, where n denotes some fraction less than unity. Turning to § 96, or looking at Fig. 45, we see, that, for an independent span, the supporting force at the right will be $n W$; and hence the altitude k of the triangle which represents the area B , and whose base is l_b , will be found by the proportion

$$H : n W = (1 - n) l_b : k; \therefore k = \frac{(1 - n) n W l_b}{H}.$$

The centre of gravity of this triangle being found at two-thirds of the distance from the vertex on the line which runs thence to the middle point of the opposite side, the horizontal distance $b = \frac{2}{3} \cdot \frac{1}{2} (1 + n) l_b$. Substituting in (2), multiplying by H , and remembering that $C = 0$, we find that

$$M_1 l_b + 2 M_2 (l_b + l_c) + M_3 l_c = n (1 - n) (1 + n) W l_b^3 = (n - n^3) W l_b^3. \quad (1)$$

If, on the other hand, the weight is placed on l_c at a distance $n l_c$ from the left, B will be zero, C will have the same form as B had for the other span; but c' , being measured in the other direction, is

$$c' = \frac{2}{3} \cdot \frac{1}{2} \{l_c + (1 - n) l_c\} = \frac{1}{3} (2 - n) l_c;$$

hence we now get

$$M_1 l_b + 2 M_2 (l_b + l_c) + M_3 l_c = n (1 - n) (2 - n) W l_c^3 = (2n - 3n^2 + n^3) W l_c^3. \quad (2)$$

Quite a practicable way to analyze a continuous girder, although rather a long process, is by putting a weight at one joint of one span, finding the pier moments and reactions at all of the supporting points, then shifting the weight to the next joint, repeating the operations, and so on. The worst combinations of load for each piece can then be selected from a table giving the effect in detail of each weight. The equations for any number of spans will become

$$M_1 l_1 (=0) + 2 M_2 (l_1 + l_2) + M_3 l_2 = 0$$

$$M_2 l_2 + 2 M_3 (l_2 + l_3) + M_4 l_3 = 0$$

$$M_m l_m + 2 M_{m+1} (l_m + l_{m+1}) + M_{m+2} l_{m+1} = (2n - 3n^2 + n^3) W l_{m+1}^3$$

$$M_{m+1} l_{m+1} + 2 M_{m+2} (l_{m+1} + l_{m+2}) + M_{m+3} l_{m+2} = (n - n^3) W l_{m+2}^3$$

$$M_r l_r + 2 M_{r+1} (l_r + l_{r+1}) + M_{r+2} l_{r+1} (=0) = 0.$$

The equations for the two piers which carry the loaded span will have a term in W : all the others will equal zero, the end moments will be zero, and there will be one less equation than the number of spans.

For a continuous girder of a large number of spans, the solution of the equations become tedious: the method by *undetermined multipliers* will, perhaps, be the easiest. (We should prefer equations for complete loads, which will give all necessary pier moments.)

130. Piers not on the Same Horizontal Line.— Taking up anew the original equation (2), § 124, and multiplying by H , we have

$$M_1 l_b + 2 M_2 (l_b + l_c) + M_3 l_c = 6 H \left(\frac{B b}{l_b} + \frac{C c'}{l_c} \right).$$

It was shown in §§ 88, 104, that $B b$ was proportional to the deflection of one of the points of support of a span from a tangent to the beam drawn through the other point of support, and that the absolute deflection in inches would be $\frac{B b \cdot H}{E I}$. If,

then, one pier is an amount v vertically below a horizontal line through the top of the other pier at which the tangent is drawn, so much deflection will not be needed to produce the required result when the tangent passes below the horizontal line at that pier, and more deflection will be required if the tangent runs above the horizontal line: that is, imagining the span to first coincide with the inclined tangent, the truss need not be bent so much to place the free end on the other pier, if both the pier and the end of the span are on the same side of the horizontal line, as will be the case when both piers are on a level, or when the tangent and the top of the pier are on opposite sides of the horizontal line.

As we must, however, reckon from the horizontal line when the deflections are made proportional to the spans, the quantity v must be added to the numerator or denominator of the second equation of § 124, according as one pier or the other is taken as the origin. To make v commensurable with $B b$ and $C c'$, it must, as seen above, be multiplied by $\frac{E I}{H}$: hence, when intro-

duced in (2), § 124, the equation with which the present section opens becomes

$$M_1 l_b + 2 M_2 (l_b + l_c) + M_3 l_c = 0 \left\{ H \left(\frac{B b}{l_b} + \frac{C c}{l_c} \right) + H I \left(\frac{v_b}{l_b} + \frac{v_c}{l_c} \right) \right\},$$

where v_b and v_c denote the distances the piers at the ends of the spans l_b and l_c are *below* the middle pier. The above equation is the most general form of the Three-Moment Theorem for a girder of constant cross-section, on supports at any elevations, and loaded in any manner. Generally the piers are assumed to be on a level, or rather it is presumed that the wall-plates will remain at those elevations at which they were when the spans on the false works were first made continuous. A small settling of one support will make a serious disturbance of the moments, points of contra-flexure, and reactions. This point will be investigated later (§ 137).

131. Example: Three Spans.—It will probably be satisfactory to take up a special example, and go through with the necessary constructions. Let the iron truss, Fig. 57, of three continuous spans, be intended for a railroad bridge: its total length is 624 feet from centre to centre of end pins, and the spans are 192 feet, 240 feet, and 192 feet successively. The dead load is assumed as 1,500 pounds per foot of bridge, and the live load as 2,500 pounds per running foot. The panels are 12 feet long, giving 16 panels for the end spans, and 20 panels for the middle span. On one joint of one truss the dead load is $4\frac{1}{2}$ tons, the live load $7\frac{1}{2}$ tons; total, 12 tons. The trusses are drawn to a scale of 72 feet to an inch, and the diagrams to a scale of 72 tons to an inch.

The load lines for the middle span and for one end span having been drawn as usual, the polygons for bending moment are so drawn as to have their terminal points in the same horizontal line, by which construction we shall diminish errors due to instrumental work, keep the drawing from spreading over the sheet, and make two sets of curves suffice for the discussion. The equilibrium polygons of greater depth are drawn for live and dead load over the whole span; those of less depth,

for dead load only. As a check on the drawing, apply the customary formula, middle ordinate $= \frac{W l}{8 H}$. Here $H = 50$ tons: hence

First span, middle ordinates $= 34.56$ and 92.16 ft.

Second span, middle ordinates $= 54$ and 144 ft.

The two middle ordinates in each span will have the ratio $4\frac{1}{2}$ to 12. The polygons can be constructed by § 28, if desired; and all the ordinates can be easily calculated by the property that they are proportional to the product of the two segments into which each divides the span. Denoting the areas for full load by **A**, **B**, and **C**, and those for light load by **A'**, **B'**, and **C'** we have by measurement or calculation (see page 149),

$$\begin{array}{lll} \mathbf{A} = 11,736 \text{ sq. ft.} & \mathbf{A}' = 4,401 \text{ sq. ft.} & \mathbf{A} \text{ to } \mathbf{A}' \text{ as } 12 \text{ to } 4\frac{1}{2}. \\ \mathbf{B} = 22,992 \text{ "} & \mathbf{B}' = 8,622 \text{ "} & \\ \mathbf{C} = 11,736 \text{ "} & \mathbf{C}' = 4,401 \text{ "} & \end{array}$$

Since the centres of gravity of the areas are in the verticals at the middle of each span, the formulæ for pier ordinates become

$$\begin{aligned} 2(l_a + l_b) y_1 + l_b y_2 &= 3(\mathbf{A} + \mathbf{B}) = 864 y_1 + 240 y_2 \\ l_b y_1 + 2(l_b + l_c) y_2 &= 3(\mathbf{B} + \mathbf{C}) = 240 y_1 + 864 y_2 \end{aligned}$$

whence, by adding and subtracting,

$$\begin{aligned} \frac{1}{4}(y_1 + y_2) &= \frac{1}{7\frac{1}{2}}(\mathbf{A} + 2\mathbf{B} + \mathbf{C}) \quad (1) \\ \frac{1}{4}(y_1 - y_2) &= \frac{1}{1\frac{1}{2}}(\mathbf{A} - \mathbf{C}). \quad (2) \end{aligned}$$

After substituting the proper values of these areas, the sum of the last two equations will determine y_1 : their difference will give y_2 .

For finding maximum moments we may have six cases, as follows:—

1°. All spans loaded. — $\mathbf{A} = 11736 = \mathbf{C}$; $\mathbf{B} = 22992$: therefore

$$y_1 = y_2 = 94.37 \text{ ft.}$$

2°. All spans unloaded. —

$$y_1 = y_2 = \frac{1}{4} \times 94.37 = 35.89 \text{ ft.}$$

3° Middle span loaded, end spans unloaded. — $A' = 4401 = C'$; $B = 22992$.

$$y_1 = y_2 = 74.44 \text{ ft.}$$

4° End spans loaded, middle span unloaded. — $A = 11736 = C$; $B = 8622$.

$$y_1 = y_2 = 55.32 \text{ ft. Or } 94.87 + 35.89 - 74.44 = 55.32.$$

5° First span loaded, second and third spans unloaded. —

$$A = 11736, B' = 8622, C' = 4401. \quad \frac{1}{2}(y_1 + y_2) = 45.36; \quad \frac{1}{2}(y_1 - y_2) = 17.63, \\ y_1 = 62.99 \text{ ft.}; \quad y_2 = 27.73 \text{ ft.}$$

6° First and second spans loaded, third span unloaded. —

$$A = 11736, B = 22992, C' = 4401. \quad \frac{1}{2}(y_1 + y_2) = 84.40; \quad \frac{1}{2}(y_1 - y_2) = 17.63. \\ y_1 = 102.03 \text{ ft.}; \quad y_2 = 66.77 \text{ ft.}$$

These ordinates are plotted below the respective piers, their extremities are connected by straight lines with the ends of the polygons below the abutments, and with each other, as seen in Fig. 57, when the moment diagrams for the given distribution of load are complete. For convenience of reference, the arrangement of loads is marked on each set of lines. By drawing lines in the stress diagrams parallel to the above-mentioned lines we determine the supporting forces, or the abutment and pier maximum and minimum ordinates for the shear diagram.

132. Points of Contra-flexure; Chord-Stresses. — The points of contra-flexure are marked by small circles. For the end spans the third case of loading carries these points nearest the abutments, and the fourth case brings them nearest to the piers. For the middle span the fourth case removes the points of contra-flexure altogether, putting the entire top chord into tension, and the bottom chord into compression; while the sixth case carries one point of contra-flexure nearest a pier, thus giving the greatest range of positive moment. It is to be remembered, that, the end spans being alike, what is true of one is true of the other, and that a reversal of the sixth case will carry the points of contra-flexure in the middle span to the same distances on the other side of the centre. We can now, as in Fig. 54, show the portions of either chord which are liable

to one or both stresses. The parallelism of pairs of closing lines which cross the middle span is noticeable.

Those portions of the verticals let fall from the panel joints, which are intercepted between the respective equilibrium polygons and their closing lines, are the distances which alone are significant as giving, or being proportional to, the chord-stresses, and only the longest one of each kind for each joint is valuable. These maximum ordinates have been drawn with full lines, for convenience of reference. It will be seen that the greatest positive ordinates in the first span are given by A F and the greater polygon. The greatest negative ordinate at the pier is B L; and the greatest negative moments for one joint on the left and two joints on the right are given by lines from L, and by the greater polygons. The remaining negative moments for the first span are given by A I and the smaller polygon. In the second span, when it alone carries a complete live load, we have the greatest positive ordinates for the middle joint and five other joints each way from the middle. The sixth and seventh joints from the middle on the right will have greater ordinates to L Q; and by symmetry, when the second and third spans are loaded, the corresponding joints on the other side of the middle will have the same ordinates. The third and fourth joints from the pier B will have maximum negative ordinates between the smaller polygon and G N: for the remaining joints to the middle the ordinates will be measured to F P.

133. **Joints requiring Special Treatment.**—As previously stated, one joint to the left of B, and two joints to the right of B, have maximum negative moments when loads are on the first and second spans: according to §§ 111, 121, since rolling load is upon these spans at the time, so many joints must be unloaded as will thereby influence favorably the increase of negative moments at the above joints. The original formulæ are

$$2(l_a + l_b)y_1 + l_b y_2 = 6\left(\frac{A a}{l_a} + \frac{B b'}{l_b}\right),$$

$$l_b y_1 + 2(l_b + l_c)y_2 = 6\left(\frac{B b}{l_b} + \frac{C c'}{l_c}\right);$$

and the calculations may be made from these formulæ. In that case, as the loaded point is in but one span, two of the areas equal zero. By reference to § 121, the other quantities are easily obtained; but we may more conveniently avail ourselves of equations (1) and (2) of § 129, since they have already been deduced. They must, of course, be divided by H to give y_1 and y_2 . For a weight of $7\frac{1}{2}$ tons in the first span of our example, they become

$$\begin{aligned} 864 y_1 + 240 y_2 &= (n - n^3) \frac{W}{H} (192)^2 = 5529.6 (n - n^3) \\ 240 y_1 + 864 y_2 &= 0. \end{aligned}$$

Then

$$\begin{aligned} \frac{1}{3} (y_1 + y_2) &= 2.5 (n - n^3) \\ \frac{1}{3} (y_1 - y_2) &= 4.43 (n - n^3) \\ y_1 &= 6.93 (n - n^3). \end{aligned}$$

The value of y_2 is not needed for the weight on the first span.

For a weight of $7\frac{1}{2}$ tons on the second span we also get

$$\begin{aligned} 864 y_1 + 240 y_2 &= (2n - 3n^2 + n^3) \frac{W}{H} (240)^2 = 8640 (2n - 3n^2 + n^3) \\ 240 y_1 + 864 y_2 &= (n - n^3) \frac{W}{H} (240)^2 = 8640 (n - n^3) \\ \frac{1}{3} (y_1 + y_2) &= 11.74 (n - n^3) \\ \frac{1}{3} (y_1 - y_2) &= 6.92 (n - 3n^2 + 2n^3) \\ y_1 &= 18.66 (n - n^3) - 13.84 (n^2 - n^3); \quad y_2 = 4.82 (n - n^3) + 13.84 (n^2 - n^3). \end{aligned}$$

In the first span, for values of $n = \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$, and $\frac{1}{2}$, y_1 becomes 0.77 feet, 1.39 feet, 1.87 feet, and 2.22 feet. Drawing the several triangles for the weight on these successive joints, which are given at the bottom of the plate to a much enlarged scale, and whose vertices lie in a parabola, as indicated in § 30, we see that weights at the first and second joints will give positive moments at the first joint, and that these moments must, therefore, be added to the negative moment already obtained. The calculation of § 121 can be renewed here, if desired. Loads on five joints from the pier will have positive moments at the second joint; and their moments, added to the negative

moment at that joint due to complete load over the first and second spans, as shown by the ordinate from U, will make a resultant moment greater than the one for load on second span only: hence the former moment is the significant one.

Passing next to the two joints on the right of the first pier, we proceed in the same way. For loads on successive joints from the pier, or $n = \frac{1}{20}, \frac{2}{20}, \frac{3}{20}$, and $\frac{4}{20}$, $y_1 = 0.85$ feet, 1.56 feet, 2.11 feet, and 2.54 feet; while $y_2 = 0.30$ feet, 0.56 feet, 0.88 feet, and 1.21 feet. It will then be seen by the figure, or by calculation, that the removal of loads from the first two joints will increase the negative moment at the first joint, and that the removal of four loads to the right of pier B will increase the negative moment at the second joint of the second span. A load at the fifth joint will be found to give a negative moment at the second joint; and hence this investigation proceeds no further. All the moments are now obtained, and their amounts can be scaled.

It will be seen that the longest portion of the first span under positive bending moment at any time is about a hundred and sixty feet; and of the middle span, about a hundred and seventy feet. As the portion between two points of no bending moment acts, so far as chord-stresses are concerned, as if it were an independent span supported at these points, the height of the truss may be made twenty feet, nearly corresponding to one-eighth the span for a single-span truss. By varying the relative spans in any example, the distance in each span which shall be under positive moment can be brought to an equality, or nearly so; and hence a constant height for the whole girder may be chosen which shall give chord sections approximately alike in all spans. The negative moments at the middle of the centre span in this example are quite insignificant. Parts exposed to alternating stresses of opposite signs should be of heavier section than when called upon to resist compression only. As the ratio of H to the height of the truss is $2\frac{1}{2}$, the chord-stresses may be obtained from our diagram by multiplying the ordinates by this quantity; that is, changing the scale.

134. Shear Diagram. — The shear diagram can now be drawn by laying off the abutment and pier reactions as usual, equal to the segments of the load lines cut off by lines parallel to the closing lines, and connecting the extremities of the ordinates so obtained by the inclined lines seen in the figure. By a similar line of reasoning to that employed in constructing the diagrams for a two-span truss, the parabolas for maximum shear will be plotted upon the lines just referred to, and will answer to the combinations of load marked upon them. Some of the arrangements may not be probable ones; as, for instance, that a train shall cover A B, and another advance from C upon the middle span: but such combinations of parts of a train may occur as to render most of these positions possible in some localities. Enough shear curves are drawn to enable the stresses on all of the verticals and diagonals to be scaled, as the remaining web members will be put in by symmetry. The middle span will be symmetrical: the counters in the end spans will be found to move towards the abutments.

It has been stated that the shear parabola for a continuous girder as constructed on the two inclined lines gave, over the middle portion of the span, a stress a little in excess of the actual stress as obtained by separate constructions for the several partial loads. By drawing one moment diagram for each span for a rolling load covering one-half the span, and then plotting the special shear diagram for that case, the error of the middle ordinate can be found, and the whole error practically corrected by moving the curve slightly towards the tangents. The ends of the parabolas in the middle of each first panel are right. One such construction is carried out in the figure, for the middle span, as shown by the dotted moment polygon; and the slight error of the usual curve is seen, amounting to about a ton and a half at the maximum point, — a comparatively insignificant quantity.

135. Extent of Chord subject to One Stress. — To determine how far the stress in a chord will extend when a point of contra-flexure occurs in a particular panel, and there are two

diagonals in that panel, we must see which diagonal will be under stress at that time. Thus, in the end span, the point of contra-flexure reaches its extreme limit, T, when the middle span alone is completely covered with travelling load. The shear diagram for the end span will then be given by *ef*, and all the ties which slant down to the left will be in action: hence, taking moments at the joint to the left of T, we have tension in the seventh panel of the bottom chord from A, and compression thence to B, and compression in the seventh panel of the top chord, counting from the angle, with tension to the right of it.

136. Deflection of a Continuous Girder.—From the investigations of Chap. VI., it will be apparent that the deflection of a continuous girder under a load may be obtained by the method of area moments, as in more simple cases. It will be necessary to find the point where the tangent to the curve of the beam or truss is horizontal by dividing the moment area into two portions by an ordinate so placed that the area moment on one side, taken about its abutment, shall equal the moment of the area on the other side about that abutment. Either of these area moments divided by EI will give the maximum deflection. By making use of the original parabola and the negative area whose end ordinates are y_1 and y_2 , the desired quantities are easily obtained.

NOTE TO § 131.—Areas may be easily calculated by the formula, Area = $\frac{2}{3} k l \left(1 - \frac{1}{N^2}\right)$; where N = No. of panels in span, and k = middle ordinate

CHAPTER IX.

PARTIALLY-CONTINUOUS TRUSS.

137. Settlement of Point of Support.—The investigation has been founded thus far upon the assumption that the piers remain at the same elevations at which they were when the spans were joined on the false works, or what would be understood in an analytical investigation as a horizontal line. The trusses would then, if without weight, be without any strain. It is possible that one or more of the piers may settle a little, sooner or later, and the moments lately found will be disturbed by such a change. As the pressures on the points of support will be unlike, there may be more *compression* of one pier than of another. It is well to see how much the stresses may be altered by the compression of the foundation, or the settlement of a pier.

Let it be assumed that the bearing on the first pier B, Fig. 57, is lowered *one-fourth of an inch*. As the formula of § 130, which takes account of difference of level in supports, has a term involving **E I**, it will be necessary to find the value of these quantities. From the results previously ascertained, the average section of either chord in Fig. 57 is taken at 40 square inches. The depth of the truss from centre to centre of chords is 20 feet = 240 inches. The value of **I** will then be practically $2 \times 40 \times 120^2 = 1,152,000$. **E** may be taken as 26,000,000. As the units of **E** and **I** are pounds and inches, the areas **A**, **B**, and **C**, which have heretofore been expressed in square feet, may be reduced to square inches by multiplying by 144. When the tangent is drawn at B, A and C will be above

the horizontal line through B; when the tangent is drawn at C, B will be below C; while D is on a level with C. $H = 50$ tons = 100,000 lbs. The two equations of condition of § 131, modified by the formula of § 130, and divided by H , become

$$864 y_1 + 240 y_2 = 3 (A + B) - \frac{6 EI}{H} \left(\frac{\frac{1}{2}}{192 \times 12} + \frac{\frac{1}{2}}{240 \times 12} \right)$$

$$240 y_1 + 864 y_2 = 3 (B + C) + \frac{6 EI}{H} \cdot \frac{\frac{1}{2}}{240 \times 12}$$

In order to retain the original areas, and to obtain y_1 and y_2 in feet as before, it will only be necessary to divide the last term of each equation by 144 to make them commensurable with the others. The last term of the first equation is, therefore,

$$\frac{6 \times 26,000,000 \times 1,152,000}{100,000 \times 144} \left(\frac{1}{9,216} + \frac{1}{11,520} \right) = 2,437.5.$$

The last term of the second equation becomes

$$\frac{6 \times 26,000,000 \times 1,152,000}{100,000 \times 144} \times \frac{1}{11,520} = 1,033.3.$$

Hence we obtain, by addition and subtraction,

$$\frac{1}{2} (y_1 + y_2) = \frac{1}{736} (A + 2 B + C) - \left(\frac{1,354.2}{2,208} = 0.613 \right)$$

$$\frac{1}{2} (y_1 - y_2) = \frac{1}{416} (A - C) - \left(\frac{3,520.8}{1,248} = 2.821 \right).$$

It will now be seen that y_1 is diminished 3.43 feet, and y_2 is increased 2.21 feet, in every case, from the values previously obtained by a settlement at B of one-fourth of an inch.

How seriously the chord-stresses are changed by this small displacement of the pier-bearing is easily seen. A few of the changes are shown in Fig. 57 by the dotted closing lines, marked with accented letters. The points of contra-flexure are moved considerably also. As the height of the truss is twenty feet, the chord-stresses are increased or diminished two tons and a half for every alteration of one foot in the vertical ordinate below the particular joint. The supporting forces being altered, the shears, and hence the web-stresses, are somewhat changed also. If there is no reason for expecting a

settlement of one pier more than another, the possibility of a movement of either one should be considered. On account of the influence of change of level of piers upon the pier moments and reactions of a continuous girder, such a system seems best adapted to plate girders or heavy lattice riveted girders of moderate span.

138. Partially-Continuous Girders.—The evil effects of settlement may be avoided in two ways,—by the use of what we call partially-continuous girders, or by fixing the points of contra-flexure by *hinges*. As to the first method,—

Let the several spans of a bridge be erected independently, and swung clear of the false works; then let the upper and lower chords of successive spans be connected over the piers, the former by bolts, and the latter by compression-blocks or keys. The spans, when free from rolling load, will be subject to bending moments as single spans, none existing at points of support; and, when further loaded, the moments from rolling load only, computed as for a continuous girder, will be modified by the moments from steady load previously existing. If such a method is applied to the example of three spans just treated, the results in the two cases of continuous and partially-continuous girders can be readily compared. It is evident that the equations for pier moments must be applied to rolling-load polygons alone, the steady load being excluded entirely; and, as the *areas* for these spans have been carefully computed, it will be convenient to use **A — A'**, **B — B'**, and **C — C'**, in place of **A**, **B**, and **C**, for the areas due to rolling load.

But, as this example has been already worked out, if we note that the pier moments for steady load have been taken away by our device (which moments, by Case 2, § 131, are 35.39 feet), we may subtract this distance from all the values of y_1 and y_2 of § 131, and find the new quantities. Fig. 58 shows the results: the range of the points of contra-flexure is indicated, and the maximum ordinates are shown. The portions of either chord subjected to alternating stresses are very much reduced: the point of contra-flexure shifts but three panels from the

piers in the middle span, and five panels in the end spans. It is apparent that the positive moments near the middle of each span will be increased by the taking away of the pier moment due to steady load, and that the pier moments will all be diminished by this amount. The shear diagram for the centre span is unchanged; but the shear in the end spans is somewhat altered. The maximum *negative* moments must be investigated in the light of § 133.

There is one point to be especially noticed in finding the pier ordinates. The same letters refer to corresponding lines of Figs. 57 and 58. In subtracting the ordinate B E or C O of Fig. 57 from the remaining pier ordinates, the point N will be carried above C, as shown by the dotted line G N of Fig. 58. As this position indicates the existence of a positive moment at C, we must inquire whether the chords at that pier are adapted to resist that moment: if, as is probable, they are not, the ordinate at the pier B must be calculated anew for a girder of two spans, A B loaded and B C unloaded, remembering that the *travelling load alone* produces the moment at B. Thus was obtained B G, giving the closing lines A G and G C, drawn in full lines. By adapting the lengths of span to the intensity of travelling load it is possible to make the maximum positive and negative moments equal, or in any desired ratio.

Thus far the effect of settlement of piers has not been eliminated, and the arrangement here suggested would offer no advantages. If, however, it is suspected at any time that there has been a settlement of any one of the bearings, or to provide against such an occurrence, it is only necessary to loosen the connections over the piers when the bridge is empty, and to bring those pieces again just to a bearing or junction: the bridge is then restored to the condition under which it was designed. Whether it is practicable to apply such an expedient depends upon the supervision the completed structure will receive.

139. Fixing Points of Contra-flexure by Hinges.—If certain points in a girder extending across several spans are

linged, so that no bending moment can exist at such points, the closing lines of the moment polygons must always pass through the hinges; and the stresses are at once made definite, whether the points of support settle or not, provided that enough hinges are supplied to locate the closing lines. A notable example of the introduction of this expedient occurs in the case of the Kentucky-river Bridge on the Cincinnati Southern Railway, — a bridge of three spans, of 375 feet each, shown in Fig. 59. As the gorge which this structure crosses is 275 feet deep, the ordinary false works for erection were out of the question; and the spans were built out from each cliff as projecting trusses, anchored back to the rock. The details of construction are not in place here; but suffice it to say, that, while the bridge was necessarily a continuous girder during erection, the fact that the two piers were built of iron, and might rise and fall some two inches from variations of temperature, while the natural rock abutments were unchangeable in level, required some expedient to obviate the great changes in stress which would otherwise occur. After the structure was completed, the connections in the lower chord at E and G were severed; and since the ties are concentrated at D, E, F, and G, the trusses were thus hinged on the upper chord-pins at D and F. A D is, therefore, an independent truss of 300 feet, supported at D by the truss D F, which overhangs its span B C 75 feet at each end. The moments and shears in the middle span will be influenced by a load on an end span: but the moments and shears in the end spans will depend entirely upon the load on those spans; that is, the stresses in A D will be those of an independent girder. E B is sometimes known as a cantilever. See p. 157.

The shear diagram for this bridge is not drawn; but the moment polygons are shown, — one for steady load, and the other for a complete live and steady load. As D', E', F', and G' are known points, the possible combinations of closing lines are at once drawn, and the points of contra-flexure shown by small circles determined. Under the supposition that trains

may at once cover any or all portions of the bridge, we shall have, for bridge empty, the closing lines $A'ILF'$, with points of contra-flexure Q and R . For first span loaded we get $AKLF'$ when the entire middle span is under negative moments; for middle span alone loaded the points P and S are determined by $A'ILF'$; the condition all spans loaded fixes V and W by $A'KNG'$; and so on. If the bending moment at the middle of a single span of 375 feet for a travelling load which equals the steady load (an equality which is practically true for this bridge) be called 100, the maximum positive moment at 150 feet from A is about 63, the negative moment $B'K$ is 80, and the moment at the middle of BC is 60. By reason of the length of the bridge, and the fact that but one train will be on the bridge at one time, it is probable that some of these combinations will not occur. Of course only the possible moments will be considered in working out a design.

140. Element of Indetermination in Multiple Systems of Bracing.—The amount of shear which is found at any section of a continuous girder differs from that which would exist at the same section were the particular span independent, by reason of the influence of weight in one span upon reactions at other points. As the amount of vertical force which is thus added to or subtracted from the reaction at any point may pass across a span which has two or more systems of bracing by a choice of paths, the distribution of this portion between the different systems is indeterminate. Thus, in Fig. 57, the imposition of a complete load on the middle span, after the first and third spans are loaded, increases the positive shear in the third span by the constant amount gh , and diminishes the reaction at D by that amount. If there had been two systems of bracing, how much of this shear passed through either system would be unknown. The amount thus in doubt is not of serious consequence, the uncertainty being guarded against by a slight increase of section. A truss in which the stresses are perfectly determinate, exactly as when there are no superfluous pieces, is, however, the most satisfactory.

141. Weighing the Reactions. — It has been suggested that the girder may be designed for certain specific supporting forces, — either those which theoretically exist at the piers for a given load, or such other reactions as may make the maximum chord and web stresses what are desired; and then, upon putting the truss in position, that these reactions shall be weighed off to the proper amount by a lever or system of levers, and the bearing points shall then be blocked up until the index of the weighing machine returns to zero. For such a method of designing it will simply be necessary to lay off the reactions on the load line, draw lines from the points of division to the pole, and make the respective closing lines of the moment diagram parallel to the lines just drawn.

142. Conclusion. — The investigations which have preceded have not been conducted with a view to decide what type of bridge is the most economical, but to give the applications of this graphical method of analysis to such a variety of types, that the reader may be able, without difficulty, to find the stresses in any bridge-truss graphically. The advocates of the greater economy of continuous girders over single-span trusses rest their belief largely upon the reduction in absolute magnitude of the chord-stresses. The web-stresses are considerably increased. The weight of the structure can only be arrived at after the cross-sections of the pieces have been worked out, properly adapted to the stresses to which they are exposed. From the fact that a piece subject to reversal of stress requires, for equal safety, a larger cross-section than a piece exposed to but one kind of stress, or, in other words, a reduction of the safe intensity of stress per square inch, and that, on account of so large a portion of the two chords being liable to these alternating forces, special work must be put into the construction of such parts, a mere comparison of the numerical value of the stresses in two bridges, one continuous and the other discontinuous, will afford no good criterion of their relative cost. A fraction of a cent more per pound in the cost of executing the iron-work of a truss may cause it to be less economical in

first cost than a heavier bridge of a more simple design. Plate girders of moderate span on very stable supports, and with a comparatively large steady load, may well be built as continuous girders, for the full strength of the flanges and web may then be better developed; but, for skeleton trusses as usually built in America, we do not think the principle of perfect continuity over the piers is well adapted.

The investigation of continuous girders is especially useful as introductory to the following chapter.

The analysis of cantilever bridges agrees with § 139. Cantilevers may be introduced in different ways, and are used to facilitate erection without false works and to make long spans more practicable. They frequently vary in depth, to render chord-stresses more nearly uniform. Not more than two points of contraflexure or hinges can be located in one span, and the total number should be one less than the number of spans.

One-half of the Forth bridge is shown on Plate X., as also three other cantilever bridges, which latter may be compared with reference to type of web, outline, and lengths of cantilevers and suspended spans, since these bridges differ but little in length. Superfluous pieces are omitted on the right half of each.

Four spans of the Lachine bridge, the two channel spans with their next, or flanking, spans, were made continuous. Two of them are shown on Plate X. The cantilevers, having been built out to C, were raised or lowered by means of the adjustable tie C F and an adjustable bed at A, until one-half of the panel weight of dead load at C passed over C D, and one-half over C F. The section E F of the top chord was then riveted in place. The four spans then acted as a continuous girder for live load. The closing line of the equilibrium polygon for dead load will be tangent to the parabola below C, as drawn. The reactions and shears follow easily.

In case of unequal settlement of piers, vertical adjustments can be made at A and at the extreme end of the other flanking span. Observations on ties C D and C F will show if adjustment is required.

CHAPTER X.

PIVOT OR DRAW SPANS.

✓
148. **Draw-Spans.**—Following the treatment of the fixed spans of a bridge, either continuous or discontinuous over the piers, naturally comes the discussion of what is called the *draw-span*. While there are several simple ways of opening a small portion of a bridge to permit the passage of vessels, the only type of draw which requires special investigation is the one most commonly built at the present time, often of very large proportions, in which the span is capable of being revolved horizontally on a pivot or a wheel-ring at its middle, so as to open two channels, one on either side of the centre pier.

If the two parallel trusses of the draw-span are carried on the pivot by one cross-bearer, so as to be supported at a single point at the pier, the draw, when open, acts as two beams or trusses, each fixed and horizontal at one end, with a uniform load, arising from its own weight, over its whole extent. The two portions are usually, though not necessarily, equal in length, and they balance on the pivot; while the stability is assured by the wheels of a wheel-circle, which arrest any tendency to tip or cant. A certain amount of play permits the whole weight to be carried on the pivot. When the draw is closed, it may be elevated at the extremities by cams or wedges, so as to bring a greater or less pressure upon the piers; or it may simply swing into place over its wall-plates or seats, without practically pressing upon the piers until a travelling load comes upon it; or, finally, it may be so secured by horizontal locking-bolts, that, while these bolts do not bring the ends of the draw any more

closely in contact with the wall-plates, they prevent one end from rising from its seat when the travelling load first comes upon the other end.

✓ 144. **Draw as Two Single Spans.**—As the truss directly supported on the pivot is the simplest design for treatment, we will begin by referring to several modifications of this general type.

If the ends of the draw-span when it is closed are raised by mechanical means to such a height that the top chord over the centre pier is entirely relieved from tension, the two halves of the draw will then be independent spans. By making the top chord link with an elongated pin-hole, one can readily ascertain when no stress is transmitted through the member. So long as there is sufficient play or slack in the piece in question, the travelling load will also be carried by the two halves of the draw as if they were separate spans: hence the bending moments and shears for the closed draw will be ascertained as in Chap. II.; while the stresses caused by steady load in the open draw will be the same as in other cases, which will be treated more in detail presently. If the bending moment over the pier due to the steady load was just removed, and the outer ends of the draw elevated no farther, the travelling load would cause bending moments over the centre pier, and the draw would, when closed, come into the class of a partially-continuous girder of two equal spans, discussed in § 138.

✓ 145. **Draw as a Two-Span Continuous Girder.**—If the pivot-span is, on the other hand, when closed, supported at its ends at such an elevation that the moment over the centre pier is the same as exists in a continuous girder of two equal spans (in which case it is necessary, as seen by §§ 101, 102, that the supporting force supplied at each end by cams, hydraulic jacks, or otherwise, shall be, when there is no travelling load on the draw, *three-eighths* of the weight of *one span*), the draw when closed is circumstanced precisely like a continuous girder of two equal spans. If, then, we draw our diagrams for two equal, continuous spans, as described before, we have only to add the

moment polygon and diagram for shearing force for the draw open. A consideration of the method given for a beam overhanging at one end will show the construction in this case. Here the beam overhangs at both ends, and the two supports are united in one, the beam balancing upon it.

Construct, if it has not already been done, the equilibrium polygon for the truss loaded with its own weight. Remember that the *end joints* each carry one-half of a panel weight: therefore, from the points where the polygon cuts the verticals from the extreme ends of the span, draw parallels to lines in the stress diagram from 0 to the *extreme* ends of the load line, and these lines must cut the vertical under the centre pier in the same point. By referring to Fig. 60 we can see that the draw, when open, is in the condition of the beam $A B C$, if the abutments A and C are removed. The equilibrium polygon will be $A' E' C'$, and $A' B'$ and $B' C'$ would be tangents to the parabola drawn through the vertices of the polygon. The ordinates to the polygon thus intercepted will be, when multiplied by H , the bending moments at the joints for the open draw.

For the shear diagram draw lines from the extremities a and c of base line for shearing force to points b and f , below and above the base line a distance equal to the weight of one span, or one-half of the load line, $b f$ being the reaction of the centre pier, and equal to the entire weight of the unloaded draw. The ordinates at the middle of the panels will show the shearing force on the left of a section when the draw is open.

The diagrams for this case are thus completed; that is, these lines just described are to be added to the diagrams required for two equal continuous spans. When the draw is open, there will be tension throughout the upper chord, and compression throughout the lower chord, and braces will all slope in one direction from the centre pier to either end. The stresses when the draw is closed, and the direction of the braces required, will be as stated in Chap. VII. If the supporting forces at the ends exceed or fall short of the amount before stated, three-eighths of the weight of one span, the case

requires a different treatment. From the power applied to the cam, or jack, the actual amount of reaction may be approximated to, and the new state of affairs readily be classed under a case soon to be noticed.

med 146. **Draw balanced on Pivot when closed.** — More commonly a draw is screwed up at the centre by shortening the top chord link, or raising the pivot, until almost or quite the entire weight and bearing is upon the pivot, and the ends of the truss scarcely do more than touch the bearings on which they close, as is shown by the immediate tilting up of one end of a draw when a train enters the other end. The draw may then practically be considered as poised on the centre when shut and unloaded, with the same stresses as when open. Let A B C, Fig. 60, represent the draw closed, balanced on the pivot B, and barely in contact with the abutments at A and C. The curvature in the figure is exaggerated; but every draw is theoretically curved in this way, when supported in the middle, and deflected under its own weight, even when it is actually straight and horizontal, as is proved by the existing tension in the top chord, and compression in the bottom chord; and the points A and C can be brought to a level with B only by a *reversed camber* previously produced in the truss.

147. **Action of a Rolling Load.** — When a rolling load comes on at D, in the sketch below A C, the *farther* end F rises, the truss is carried by two supports, with one end overhanging, and the construction of the diagrams will follow the method described in § 97. As recently stated in § 145, A' E' C' B' is the diagram for bending moment of A B C, the draw open, and also for the draw closed, but not raised at the ends by cams. By drawing T' D' as the new portion of the polygon required by the load T D, we have D' T' E' C' as the new equilibrium polygon. Any load on D E will not alter the bending moments in E F: hence C' B' is still one of the closing lines, and B' D' must be the other required to complete this diagram. The point of contra-flexure is near T'. A line in the stress diagram drawn parallel to B' D' will give us the reaction at D, equal to ad ;

and the lines dte with fc will determine the ordinates for shearing force.

The farther end of the draw will rise still higher as the load advances over the first span, and finally reaches the centre pier. The progress of the load beyond this point, while it at the same time covers the first span, will cause F to move down again but the bridge will not touch the bearing on G until the train or other moving load has advanced a certain, often a considerable, distance over the free span, — sometimes more than one-fourth of the space from the centre pier. As, when the draw touches three points, another method of analysis must be applied, the important position to be determined is that of S , the front of the rolling load on IKL , when the end L of the beam is just forced down to its abutment.

✓ 148. **Condition that Draw shall rest on Three Points.** — Draw through K a tangent QKN to the beam at that point. When the beam rests on the abutments, the vertical deflections at the ends from the tangent at K are NL and QI . The points of support I and L are below the horizontal line RKO a distance RI or OL . If we add IR to QI , and subtract its equal OL from NL , we have a similar proportion to the one deduced for a two-span continuous truss; namely, —

$$\text{Deflection } NO : \text{deflection } QR = KO : KR = 1, \text{ or } NO = QR.$$

Now NL is proportional to the *area moment* of the span KL , as explained in § 104; QI is proportional to the area moment of the span KI ; and OL or IR is proportional to the original area moment of either span before any rolling load has come upon it, or to the area $C'B'E'$ multiplied by the distance of its centre of gravity horizontally from the vertical through L . Call this area moment Cc . The polygon $I'E'L'$ being drawn, the condition that the end L shall rest upon the abutment is therefore satisfied when the area moment to the left of $E'K'$ plus Cc equals the area moment to the right of $E'K'$ minus Cc . This equation will determine the position of K' , as in former cases.

To find the distance $E'K'$, or y_0 , for any position of the head of the rolling load between S and L : Draw a straight line from I' to E' , and one from E' to L' ; call the respective areas between these lines and the moment polygon A and B , A being the area belonging to the side which is entirely loaded; and the respective distances of their centres of gravity horizontally from I' and L' , a and b ; then by the same course of reasoning given in § 106, and from the relation stated above, we have, letting l equal either span,

$$Aa - \frac{1}{2}y_0 l^2 + Cc = \frac{1}{2}y_0 l^2 - Bb - Cc,$$

$$y_0 = \frac{3}{2l} (Aa + Bb + 2Cc).$$

In the special case now under discussion it must be noted, that when we find a point K' , satisfying the above condition, so placed that a line parallel to the last radiating line in the stress diagram to the *extremity* of the line of loads cuts $E'K'$ above K' , the span has risen from the abutment; for it is necessary that a parallel to $K'L'$ shall cut off some portion of the load line to give any supporting pressure on L . When one end of the span is off the abutment, the head of the load being to the left of S , see § 147. If the polygons are drawn as usual in these pages, $I'E'$ being common to all of the polygons for movements of the head of the load from S towards L , we have Aa and $2Cc$ constant, as well as the factor outside of the parenthesis; so that for each new position of the load we have only to calculate the quantity $\frac{3Bb}{2l}$, and add to the previous constant quantity, to obtain $y_0 = E'K'$.

149. Shear Diagram, and Points of Contra-flexure.—The shear is then readily obtained. A load from I to S gives the diagram *inlsc*: when the load extends to L we have the symmetrical curve $I'E'V'$ and the shear diagram *auwkvc*. The increase of rolling load from S to L diminishes the supporting force at I from ai to au , and manifestly the supporting force at I will be a maximum when the span IK alone is cov-

ered by the travelling load. Parabolas drawn on the limiting lines of the shear diagram will give the maximum ordinates. An example will be given in detail soon.

The points of contra-flexure will be found nearer the ends for given loads than in a continuous girder of two spans, as might be expected, since the load has first to overcome or neutralize the initial negative moments of flexure: consequently the bending moment over the centre pier is greater than in the case of a continuous girder. Most trusses for draws have a greater depth at the centre pier than at the ends, thus diminishing the chord-stresses near that point below the amounts which would exist in a truss with parallel chords and with an average height of this draw. If the variation of height is expected, by its effect on the moment of inertia at successive cross-sections, to seriously influence the deflection, the ordinates which make up the moment areas must be changed in the ratio of the change of I before the area moments are computed for the preceding equation for y_0 . In ordinary cases, the assumption of a constant moment of inertia will lead to no serious error.

150. Draw with Locked Ends.—When the ends of the draw are prevented by locking bolts from rising from the abutments, the action will be similar to that produced by hanging an additional but variable weight at F , just sufficient to bring F in contact with G . The greatest force will be required at F when the rolling load extends from D to E . It is readily seen, that, if F must always remain on G , the condition required for the previous case, § 148, instead of being limited to loads which give a pressure on L , must be satisfied for every position of the load from D to F ; that is, we must always have

$$y_0 = \frac{3}{2l^2} (Aa + Bb + 2Cc),$$

thus determining K' . If, then, the line to be drawn in the stress diagram, parallel to $K'L'$, passes beyond the end of the load line, the additional length of load line required to meet it will be the upward pull on the bolt at L ; and this pull will

increase, commencing with zero for the head of the load at S, until the rolling load retires to K, and will then diminish to zero again when the load entirely moves off at I. This modification makes the only difference in the treatment of these two cases. In the shearing diagram, the amount of stress on the locking bolt, being opposed in direction to a supporting pressure, is to be laid off at *c* upwards or at *a* downwards, and from its extremity will then be drawn a line parallel to and taking the place of *cf* or *ab*. This pull affects the stresses in all portions of the draw, and shifts the points of contra-flexure.

151. Draw with Ends partially lifted.—Suppose that the draw, in place of being circumstanced as in § 146, is raised at the ends by cams, or jacks, but that the supporting forces do not equal those required by § 145, where A and C are on the same level with B. Find, from the known power applied to the cams, the amount of supporting force at each end when the draw is unloaded; lay off these amounts, each on the proper end of the load line, and draw lines from the two points of division so obtained to the pole of the stress diagram, usually marked 0; construct the moment polygon for the unloaded spans; and then draw lines from the extremities of the polygon to the centre vertical parallel to the lines just drawn in the stress diagram. The ordinates so cut off will be the ones required to determine the bending moments on the closed and unloaded draw, and the area moment on one side (or the moments on each side, if not the same, owing to different supporting forces at the ends) will represent the quantity to be used, instead of **Cc**, in the equation for y_0 .

The ends of the draw may be raised so as to give a pressure of more than three-eighths the weight of one span on each abutment. It will then be necessary to determine the amount, and proceed by the steps just described.

152. Remarks on the Preceding Cases.—It is evident that the quantity **Cc** is proportional to the difference of level between the centre pier and the end pier, and hence is related to the quantity v of the formulæ in §§ 91, 180. From the

peculiarity of the draw-span, that it may swing over the abutments without pressing upon them when unloaded, this area moment is readily ascertained; and, being of the same form as **A a**, &c., it makes the equation very simple. A general case might be made of the difference of level just referred to. If **A** and **C** are above **B**, we shall have the last case of § 151. If **A** and **C** are sufficiently above **B**, we shall have no pressure on **B**, and hence one single span, unless a tension can be exerted at **B**. If **A** and **C** are level with **B**, **C c** will equal zero, and we have the case of § 145. If **A** and **C** are below **B**, we have the cases of §§ 146 and 151. An equation formed from that of § 106, by introducing two terms in **C c**, one for each span, will apply to this general discussion, and will give us, when the spans are equal, the equation of § 148.

153. Example.—To illustrate the principles thus far laid down, let us take a draw-span represented in Fig. 61, and find the stresses on the different parts under the case of § 150. The draw is 240 feet long, making two spans of 120 feet each ($= l$) divided into twelve panels of 20 feet each. The height at the centre is 25 feet, and at the ends 20 feet. The dead weight is assumed at 10 tons per panel of one truss, and the live load is also 10 tons per panel. Simple data are taken for brevity, and to keep a small figure distinct. To have the moment curves well separated, let $H = 60$ tons. The load line 1-2 is 240 tons, the maximum load; and 0 is opposite its middle. **A** and **C** carry 10 tons each, the other joints 20 tons each, when loaded. **A' M C'** is the polygon for the fully-loaded draw. Through **M** draw **A'' M C''** for the draw unloaded: the side of the first polygon at **M**, which is parallel to 0-7, being common to both, construct this polygon each way from **M**, with ten tons per joint, terminating at **C''** and **A''** with lines parallel to 0-8 and 0-9. As five tons, when the draw is free or open, are carried at **A** and **C**, draw from **A''** and **C''** the lines **A'' B''** and **C'' B''** parallel to 0-5 and 0-6. The ordinates between **A'' M C''** and **A'' B'' C''** are proportional to the bending moments at different points of the draw open, or closed and unloaded.

Calculate the area $A''M B'' = C$; find the distance of its centre of gravity from the vertical through A, and denote it by c . (See § 105. As the angles of $A''M$ lie in a parabola, c may be taken as $\frac{1}{4} AB$. By subtracting a parabolic segment from a triangle, the centre of gravity of a parabolic spandrel is easily obtained, as above. The small portions by which the polygon differs from the curve balance about the centre of gravity of the segment; and hence, if the area Δ , bounded by the polygon and the straight line connecting its extremities, is computed, the exact position of the centre of gravity of the area C , found by subtracting this first area from the triangle, is given.)

Draw a straight line from A' to M , and find Δ , the included area between it and the polygon $A'M$; $a = \frac{1}{2} AB$ is the distance of its centre of gravity from A : then, since $Bb = \Delta a$ for a full load, the formula of § 150 becomes simply

$$y_0 = \frac{3}{7} \left(\frac{1}{2} \Delta + \frac{1}{4} C \right) = \frac{3}{47} (2 \Delta + 3 C).$$

Lay this value off at MB' ; draw $A'B'$ and $B'C'$, and thus determine the bending moments for the draw closed and loaded. Draw a straight line from C'' to M ; compute B for this side; and, using Δ as before, find

$$y_0 = \frac{3}{47} (\Delta + B + 3 C) = MB''.$$

$A'B'''$ and $C''B'''$, as well as $A''B'''$ and $C'B'''$, will give the bending moments when one span is loaded, and the other unloaded.

Draw from 0 lines parallel to those which meet at B' , B'' , and B''' , thus finding the supporting forces, and plot them at a , b , and c . These reactions will be $a e$, $f o$, and $d c$, when both spans are loaded; $a k$, $i x$, and $n c$, when AB is loaded, and BC unloaded; $a l$, $m s$, and $g c$, when the load is upon BC alone. The maximum pressure at A must be $a k$, and the maximum shear on the locking bolt or tension on the abutment is $a l$. The lines $a h$ and $l m$ are inclined at 10 tons per panel run, and $e f$ and $k i$ at 20 tons per panel. The distance $a l$ must equal $e k$ by

§ 120. Parabolas drawn on these tangents as usual, beginning in the middle of the first panel from each point of support, will complete all *necessary* lines in the shear diagram. As the two halves of the draw are alike, the diagram for one-half is sufficient. To construct these parabolas, see § 118.

154. **Discussion.** — If the draw had parallel chords, the diagrams would now be completed; but, owing to the changes of depth, other moment polygons are needed. Those on the left span are drawn for loads extending from A to the joint whose letter stands at their left extremity; those on the right, for loads extending from B to the joint whose letter is placed at their right extremity: by combining these polygons with a full load polygon, or an unloaded polygon on the other side, or with one another, any variation of loading is obtained. As a load on B does not alter the bending moment, the polygon for a load which includes P becomes the one from A'. Quite a number of pier ordinates have been plotted, after calculating **A** and **B** as usual; and enough are given here to show the movement of the lines. The shears for partial loads can thus be obtained directly, if desired; and the maximum moments can be scaled. The statement affixed to the shear parabolas indicates the loaded portion for the maximum shear in each panel.

As the load advances from A to B, then, the span B C being fully loaded, we find that the ordinates for the maximum shears in the successive panels occur immediately in front of the load, and terminate in the points *u*, *v*, *w*, &c., of a parabola described on the lines *lp* and *pf*, and ending in the middle of the first and last panels. If B C were *unloaded*, these ordinates would terminate in a parabola drawn on *aq* and *qi*; but, as the former parabola includes the latter, the former alone is wanted. As a load on B A alone will cause a greater reaction at A than when B C also is loaded, therefore the increments of load from B towards A, while B C is unloaded, will give us points on a parabola drawn on *kq* and *qh*. Similar curves might be drawn on *tr* and *rg*, or *r* and *rn*.

The point of contra-flexure, advancing from the outer end as

a load enters upon the draw, will be seen to move no nearer the centre pier than the third panel from the end: consequently K R will always be in tension, G B will always be compressed, while D K and A G must be designed to withstand either stress. The maximum ordinate at any joint being readily selected from the figure A' M C' C'' B' A'', and multiplied by H, if we divide by the height of the truss at that joint we shall have the *horizontal* stress on that side of the joint not touched by the diagonal in action at the time. The horizontal component in the inclined chord must be increased in the ratio of the actual length of that portion of the chord to the length of a panel horizontally, in order to obtain the direct stress. A curved piece may be treated as straight between the two joints for finding the direct stress: its curvature introduces a separate bending moment on the piece itself.

155. **Web-Stresses.**—Finally, to find the stresses in the diagonals and verticals, compare § 63, &c. Take, for example, the pieces L Q and P Q. The maximum shear in the panel L P will be the ordinate from *ab* to *w*: lay off this ordinate from *w* to *o* in the lowest figure. This shear will be caused by a rolling load from A to L inclusive, together with one from B to C: as it is negative, it is plotted downwards. The moment polygon is L' M C', and the ordinates for bending moment under that load at L and P will be L'' N' and P' Q'. Multiply these ordinates by H, and divide by L N and P Q respectively; thus obtaining the *horizontal* stress towards N and the stress in P L. Lay off *w* L equal to the latter, and, drawing *o* Q parallel to N Q, make the horizontal distance of Q from *o* equal to the former. Negative shear and negative bending moments turn this diagram around, and bring the top chord-stress at the bottom of the diagram, or the reverse of Fig. 31. If no error has been made, Q L, when drawn, will be parallel to Q L of the truss, and will give the amount of existing tension; while the line marked P Q, drawn vertically from L, will be the stress in the vertical P Q. The remainder of the figure applies to the other pieces of the web, as shown by the letters. B R will

undergo double the compression shown in the diagram, as it resists the action of the inclined pieces in both halves of the draw.

156. Remarks.—A study of the diagrams of Fig. 80, and the explanations therewith, will show what modifications would be introduced in Fig. 61 by the inclination of the bottom chord, the substitution of struts for ties, or of a load on the top chord for one on the bottom chord. On account of the existing shears and moments, the inclination of the chord, opposed to that of Fig. 80, is favorable to the ties. In some examples with more numerous panels, joints near the pier may be subjected to maximum negative moments by partial loads, as explained in § 111. If all the polygons are drawn as here, the maximum moments can be selected at sight.

157. Changes by Omission of Bolt.—Without drawing other diagrams, we can determine, by inspection of Fig. 61, what changes would be effected in the draw by the removal of the locking bolts, bringing it under the case of § 146. Both when the draw is entirely loaded, and when the rolling load is altogether removed from the draw, there is no force exerted on the bolts. The absence of the bolt will, therefore, make no change in the position of B' and B'' . The point B''' , found when one span only is fully loaded, will be situated a little nearer M , owing to the omission of the moment over B caused by the pull on the bolt multiplied by the span BC . The point of contra-flexure will, therefore, come somewhat nearer the centre pier; and, if it gets into the next panel, the extent of chord subject to but one kind of stress will be diminished. All the values of y_0 which would give any pull on the bolt, and which correspond to loads on any portion of RS , Fig. 60, will be slightly less.

As $luvf$ is the curve for a load advancing from A while the other span is fully loaded, and as, under these circumstances, the holding-down bolts cannot be in action, this parabola will not be changed. When AB , and it alone, is covered with a rolling load, the supporting force at A is diminished by

the amount of the *pull* nc on the abutment at the other end. Remove the bolt at C, and the point k will now be found to have moved farther from e by the amount nc or al : that is, to find k , lay off lk from a . This increase of supporting force at A will affect the shear at all points of the span, when AB is loaded, by just this amount; and therefore ki will move parallel to itself to the new distance from a , just stated. The parabola kh will rise to its new lines, increasing the shear in panels near A.

158. Draw of Three Spans.—It is more common to design a draw-span with a middle panel, and to carry the trusses on two *transverse* beams or girders which are placed directly beneath the joints of this panel. In this construction the open draw is supported at two points equidistant from its middle; and, when closed and loaded, the draw may be carried by the abutments also, being thus divided into three spans,—two usually equal end spans, and one short middle span of the length of the middle panel. The transverse beams will rest at four equidistant points upon a deep circular girder, which, in turn, is either carried by the pivot, or rests directly upon the live ring of wheels below the girder. In some cases the attempt is made to distribute the weight between the ring and the pivot in some desired proportion. We propose to treat in this section the case where the draw rests directly upon the wheels.

We draw, as before, the moment polygon for the unloaded draw open, or closed, but not bearing upon the abutments. This construction is seen at $A'D'T'S'$, Fig. 62. In any determination of the pier ordinates for a partial or complete load, it is necessary to know the area moment which indicates the difference of level between A and B. Let the span $AB = l_1 = CD$; the span $BC = l_2$. The draw, when open, has a horizontal tangent at E, its middle point. If Cc is the area moment of $A'B'S'$, and Dd the area moment of $B'S'U'E'$, both about A, we have the

deflection of A below E = AS is proportional to $Cc + Dd$,

“ B “ E = ST “ “ $D(d - l_1)$:

“ A “ B = TA “ “ $Cc + Dd$.

159. **Values of Pier Ordinates.** — When a load comes upon the draw, and the ends are in contact with the abutments, either exerting a pressure there, or locked down, we shall have the condition of things represented by the lower figure. Upon drawing the tangents to the curve of the beam (L N at G, and O P at I), and noting that $Q F = R K = T A$, we may write two equations for y_1 and y_2 , the ordinates at G' and I', referring to §§ 104 and 124 for the general form, and denoting by $\mathbf{A} \mathbf{a}$ and $\mathbf{B} \mathbf{b}$ the area moments of each span l_1 cut off by F' G' and K' I' as before: —

$$\frac{L Q}{N I} = \frac{L F + F Q}{N I} = \frac{\mathbf{A} \mathbf{a} - \frac{1}{2} y_1 l_1^2 + \mathbf{C} \mathbf{c} + \mathbf{D} l_1}{\frac{2 y_1 + y_2}{6} l_1^2} = \frac{l_1}{l_1^2}$$

$$\frac{P R}{O G} = \frac{P K + K R}{O G} = \frac{\mathbf{B} \mathbf{b} - \frac{1}{2} y_2 l_1^2 + \mathbf{C} \mathbf{c} + \mathbf{D} l_1}{\frac{y_1 + 2 y_2}{6} l_1^2} = \frac{l_1}{l_1^2}$$

Clearing of fractions, transposing and factoring, we easily deduce

$$2 (l_1 + l_2) y_1 + l_2 y_2 = 6 \left(\frac{\mathbf{A} \mathbf{a}}{l_1} + \frac{\mathbf{C} \mathbf{c}}{l_1} + \mathbf{D} \right), \quad (1)$$

$$\frac{1}{2} y_1 + 2 (l_1 + l_2) y_2 = 6 \left(\frac{\mathbf{B} \mathbf{b}}{l_1} + \frac{\mathbf{C} \mathbf{c}}{l_1} + \mathbf{D} \right); \quad (2)$$

which equations are very readily solved for any given case. As \mathbf{a} , \mathbf{b} , and \mathbf{c} are fractions of l_1 , the second members are simple. By adding and subtracting, if more convenient, we at once obtain the half-sum and half-difference of y_1 and y_2 ,

$$\frac{1}{2} (y_1 + y_2) = \frac{3}{2 l_1 + 3 l_2} \left(\frac{\mathbf{A} \mathbf{a}}{l_1} + \frac{\mathbf{B} \mathbf{b}}{l_1} + \frac{2 \mathbf{C} \mathbf{c}}{l_1} + 2 \mathbf{D} \right), \quad (3)$$

$$\frac{1}{2} (y_1 - y_2) = \frac{3}{2 l_1 + l_2} \left(\frac{\mathbf{A} \mathbf{a}}{l_1} - \frac{\mathbf{B} \mathbf{b}}{l_1} \right). \quad (4)$$

G' O' and I' N' being thus obtained, the lines F' O' and K' N' will determine the moments, the reactions, &c. The polygon for the unloaded draw has been added to this diagram.

160. **Special Treatment.** — In case the end of the span, for instance at K, is not locked down, it may rise when a load enters at F. This will be indicated by obtaining a tension in

place of a pressure for the reaction at K: then $K'N'$ must be drawn parallel to the line from the extreme end of the load line to the pole, giving an independent value of y_2 , which is then to be substituted in equation (1) above, and the value of y_1 at G thus obtained.

It may rarely happen in some designs, under a very heavy travelling load, that, when there is no locking bolt to prevent, a load on the span FG may raise the draw, not only from K, but also from I. This occurrence will reduce the draw to a truss of the span FG, with the portion GK overhanging, § 97. In that case the straight line $K'N'$, referred to in the first paragraph of this section, will be continued to the vertical through G' ; and any value of y_1 , obtained as above, which is greater than the ordinate intercepted at G' by this line, cannot exist, for tension would then be necessary at K.

Again: it is plain, that, if $I'N'$ or y_2 is small enough, $O'N'$ and $N'K'$ may become one straight line when the lines which cut off the reaction at I from the load line coincide, giving a pressure of zero. It may be possible, therefore, when locking bolts are used, and sometimes when they are not employed, that a sufficient load on one span, as FG, may reduce the pressure at I to zero, lifting the draw and circular girder from the wheels on that side: hence if N' , as found by the above equations, falls nearer I than a straight line from O' to K' would locate it, tension would be necessary at I to keep the girder on the wheels. As this force cannot be supplied, the draw for that given distribution of load must be treated as a draw of two unequal spans, $FG = l_1$ and $GK = l_1 + l_2$. The equation of § 148 then becomes, upon substituting the proper quantities,

$$\frac{Aa - \frac{1}{2}y_1 l_1^2 + Cc + D l_1}{\frac{1}{2}y_1 (l_1 + l_2)^2 - B'b' - Cc - D l_1} = \frac{l_1}{l_1 + l_2},$$

in which B' is the area cut off by a line from K' to G' , and b' is the distance of its centre of gravity from K. From this equation is deduced

$$y_1 = \frac{3}{2l_1 + l_2} \left(\frac{Aa}{l_1} + \frac{Cc}{l_1} + D + \frac{B'b'}{l_1 + l_2} + \frac{Cc}{l_1 + l_2} + \frac{D l_1}{l_1 + l_2} \right).$$

This ordinate is to be plotted at $G' O'$, and lines drawn from O' to F' and K' . The preceding statements indicate when one and when another of these equations is applicable. The shear diagram can then be constructed, and the case completely solved. If the ends are raised by cams, &c., $C c$ is to be found from the known reaction, as in § 151.

161. Circular Girder carried by Pivot; Draw of Two Spans with Tipper.—Where the weight on the circular girder is transferred directly to the pivot, the wheel-ring is placed below the circular girder as usual; but sufficient clearance is allowed between the lower flange of the girder and the wheels to prevent any weight from resting upon them when the draw is closed. When the draw has swung off from the abutments in opening, the wheels will check any extreme oscillation of the draw. So far as the truss itself is concerned, it is supported, when open, upon two points a centre panel length apart, as in the previous case; and, when closed, upon the abutments, and on two points, which, under an unsymmetrical load, change their elevations by the rocking or tipping of the circular girder on the pivot. These two central points are, consequently, not at a constant elevation above the abutments. If Fig. 63 represents this case, it will be seen that the only point which does not change its position on the supposition that the points B and C in contact with the beam, in shifting to K and L , move in a vertical line, is the point T , midway between and in the straight line joining B and C , or K and L . All deflections will be referred to this point. (The above supposition is the customary one in investigating flexure of beams, that the span is unchanged, or the curved line is the same in length as the original straight one.) In the beam $A D$, therefore, we propose to use the deflection of A from a horizontal line through B , that is, $G A$ or $F A - F G$, and then to write the usual proportion for the beam $I N$, referred to a horizontal line $P X$ through T .

162. First Condition for Pier Ordinates.—When the tipper $B C$, which rocks on E , is level, the draw being open, or closed

and unloaded, the distance GA will, by the notation of the previous section, be proportional to $Cc + Dd =$ deflection of abutment below B when BC is horizontal. These areas are marked in the figure, $A'F'B' = C$ and $F'G'E'B' = D$. In endeavoring to determine the two moments or pier ordinates at K and L for a partial or complete load, we must establish two conditions involving the two unknown quantities. One condition is especially simple. As the pivot or fulcrum of the lever KL is midway between K and L , it follows that the *reactions* at these two points must be equal to one another. If we suppose, for the present, that $I'K'L'N'$ are the desired closing lines, let us prolong $K'L'$ until it meets the verticals from I' and N' at Q' and U' . Upon drawing lines 0-3, 0-4, and 0-5, in the stress diagram, parallel to $I'K'$, $K'L'$, and $L'N'$, the two intercepted parts of the load line cut off by these three lines must be equal. If, then, the vertical sides of the two triangles just constructed in the stress diagram are equal, and the side 0-4 parallel to $K'L'$ is common to both, the vertical sides of the similar triangles $I'Q'K'$ and $L'U'N'$, which have $Q'K' = L'U'$, must be equal, or $I'Q' = N'U'$: hence $K'L'$ will always be drawn parallel to the original closing line $I'N'$; and this condition must be satisfied for a draw of two equal spans. If the spans are unequal, it follows that $I'Q' : N'U' = IK : LN$. As this condition fixes the direction of $K'L'$, it will only be necessary to find one ordinate to it at the most convenient point to completely solve the problem.

163. Second Condition for Pier Ordinates.—If $IKLN$ represents the loaded draw, K and L the rocking supports carried by O , S the middle point of the *beam*, and T the point below it in the straight line KL , this point T is above I a distance PI , equal to GA , the original deflection of A below B , the measure of which was given in the last section. Draw the moment polygon $I'S'N'$; cut off the area **A** by the straight line $I'R'$, and the area **B** by the straight line $N'V'$. Let $I'K'L'N'$ be the desired closing lines; draw $I'S'$ and $S'N'$; also draw the vertical $S'T'$. Let area $R'I'S' = K$; $S'V'N' =$

\mathbf{N} ; $\mathbf{R}'\mathbf{K}'\mathbf{T}'\mathbf{S}' = \mathbf{E}$; $\mathbf{T}'\mathbf{L}'\mathbf{V}'\mathbf{S}' = \mathbf{F}$; and denote the distances of their respective centres of gravity horizontally from \mathbf{I}' and \mathbf{N}' by the usual small letters.

Drawing a tangent $\mathbf{Q}\mathbf{U}$ to the bent beam at \mathbf{S} , and a line $\mathbf{R}\mathbf{V}$ parallel to it through \mathbf{T} , we have, if $\mathbf{P}\mathbf{X}$ is the horizontal line through \mathbf{T} ,

$$\mathbf{P}\mathbf{R} = \mathbf{V}\mathbf{X}, \text{ or } \mathbf{Q}\mathbf{I} + \mathbf{I}\mathbf{P} + \mathbf{Q}\mathbf{R} = \mathbf{U}\mathbf{N} - \mathbf{N}\mathbf{X} - \mathbf{U}\mathbf{V}.$$

Now, $\mathbf{I}\mathbf{P} = \mathbf{N}\mathbf{X} = \mathbf{G}\mathbf{A}$ has been proved proportional to $\mathbf{C}\mathbf{c} + \mathbf{D}\mathbf{l}_1$; $\mathbf{Q}\mathbf{R} = \mathbf{U}\mathbf{V}$, being the deflection of \mathbf{K} below a tangent through \mathbf{S} , is proportional to $\mathbf{E}(\mathbf{e} - \mathbf{l}_1)$; $\mathbf{Q}\mathbf{I}$ is measured by the area moment to the left of $\mathbf{S}'\mathbf{T}'$, or $\mathbf{A}\mathbf{a} + \mathbf{K}\mathbf{k} - (\mathbf{K}\mathbf{k} + \frac{1}{2}\mathbf{R}'\mathbf{K}'\cdot\mathbf{l}_1^2 + \mathbf{E}\mathbf{e})$. A similar expression on the right of $\mathbf{S}'\mathbf{T}'$ will measure $\mathbf{U}\mathbf{N}$. Substituting these values in the above equation, noticing that the areas \mathbf{K} and \mathbf{N} cancel out, and remembering to write moments on opposite sides of the tangent with opposite signs, we get

$$\begin{aligned} &\mathbf{A}\mathbf{a} - \frac{1}{2}\mathbf{R}'\mathbf{K}'\cdot\mathbf{l}_1^2 - \mathbf{E}\mathbf{e} + \mathbf{C}\mathbf{c} + \mathbf{D}\mathbf{l}_1 + \mathbf{E}(\mathbf{e} - \mathbf{l}_1) = \\ &-\mathbf{B}\mathbf{b} + \frac{1}{2}\mathbf{V}'\mathbf{L}'\cdot\mathbf{l}_2^2 + \mathbf{F}\mathbf{f} - \mathbf{C}\mathbf{c} - \mathbf{D}\mathbf{l}_1 - \mathbf{F}(\mathbf{f} - \mathbf{l}_2); \text{ or} \\ &\frac{1}{2}(\mathbf{R}'\mathbf{K}' + \mathbf{V}'\mathbf{L}')\mathbf{l}_1^2 + (\mathbf{E} + \mathbf{F})\mathbf{l}_1 = \mathbf{A}\mathbf{a} + \mathbf{B}\mathbf{b} + 2\mathbf{C}\mathbf{c} + 2\mathbf{D}\mathbf{l}_1. \end{aligned}$$

From the figure we see, that, since $\mathbf{R}'\mathbf{V}'$ is a straight line, $\mathbf{E} + \mathbf{F} = \frac{1}{2}(\mathbf{R}'\mathbf{K}' + \mathbf{V}'\mathbf{L}')\mathbf{l}_2 = \mathbf{S}'\mathbf{T}'\cdot\mathbf{l}_2 = y_0\mathbf{l}_2$; so that the above equation may be written

$$\begin{aligned} &\frac{1}{2}y_0\mathbf{l}_1^2 + y_0\mathbf{l}_1\mathbf{l}_2 = \mathbf{A}\mathbf{a} + \mathbf{B}\mathbf{b} + 2\mathbf{C}\mathbf{c} + 2\mathbf{D}\mathbf{l}_1, \text{ or} \\ &y_0 = \frac{3}{2\mathbf{l}_1 + 3\mathbf{l}_2} \left(\frac{\mathbf{A}\mathbf{a}}{\mathbf{l}_1} + \frac{\mathbf{B}\mathbf{b}}{\mathbf{l}_1} + \frac{2\mathbf{C}\mathbf{c}}{\mathbf{l}_1} + 2\mathbf{D} \right), \end{aligned}$$

which is identical with (3) § 159.

The construction with the tipper or rocking circular girder then becomes simply, Make $\mathbf{S}'\mathbf{T}'$ equal to y_0 , draw $\mathbf{K}'\mathbf{L}'$ parallel to $\mathbf{I}'\mathbf{N}'$, and complete the figure with $\mathbf{I}'\mathbf{K}'$ and $\mathbf{L}'\mathbf{N}'$. The shear diagram for this load is seen below.

164. **Special Treatment.**—When the end of the draw is not locked down, all loads on one half of the draw, and even a portion extending upon the other half, will cause the unloaded end to rise from the abutment. It will then only be necessary

to draw a line from that end of the polygon which belongs to the free end, parallel to the line from that *extreme* end of the load line to the pole; and from the point where it cuts the first centre pier vertical draw the usual line $K'L'$ parallel to the original closing line $I'N'$. See the dotted lines of this diagram. Such a construction becomes necessary, if a bolt is not employed, as soon as $N'L'$, located by the formula of the last section, determines a tension at N , or, in short, when it falls below what would be a tangent at the point N' to a curve drawn through the vertices of the moment polygon.

165. **Remarks.**—In case the ends of the unloaded draw are lifted by known reactions, the value of C is readily ascertained, and the investigation will then proceed as before. Some draws have been designed with the intention that a certain portion of the weight on the centre pier should be carried by the pivot, and the remainder by the wheel circle. Although it is doubtful whether the desired adjustment, if obtained at first, is permanent, the last two cases, of a three-span draw and a draw with tipper, can be combined. The safer way will be to provide for the maximum stresses under either contingency, with the probability that the wheel circle will finally carry the greater part of the weight. The moment diagrams of the two cases can be readily superimposed, as the closing lines for any particular load coincide at T' , Fig. 63.

A careful analytical investigation of draw-spans may be found in "Continuous, Revolving Drawbridges," by Clemens Herschel, Boston, 1875, and an excellent paper on "Turn-Tables for Draws," by C. Shaler Smith, in Transactions Amer. Soc. of C. E., August, 1874, vol. iii., No. 4, xcii. It is probably unnecessary to amplify further, or to contrive other methods of support or construction. The flexibility of this method of area moments has been shown, and any one who understands what precedes will have no difficulty in making the application to any special modification. The diagrams will resemble in most particulars those already given in Fig. 61, slightly modified by the double pier moments.

APPENDIX.

Bending Moments on Pins.—If the forces which act *at any one time*, in the pieces assembled upon a pin at a given joint, are decomposed into horizontal and vertical, or rectangular components, one set of components may be imagined, revolved through a right angle, and they may then be plotted on two load lines with the same H , and moment polygons may be drawn on the two sides of a horizontal line which represents the pin. Then can the resultant moment at any section, and the maximum moment, be found by constructing a right-angled triangle, whose legs are the two moments at the section; the hypotenuse, when multiplied by H , will measure the desired moment. The best arrangement of pieces on the pin, and the utility of introducing a middle bearing, if the moment would otherwise be large, can then be studied.

A convenient rule, to be followed as much as possible, is to place those pieces in juxtaposition whose stresses tend to balance one another. Thus a diagonal should come between the vertical and the chord piece which resists its stress. Chord bars in adjacent panels should alternate on the pin.

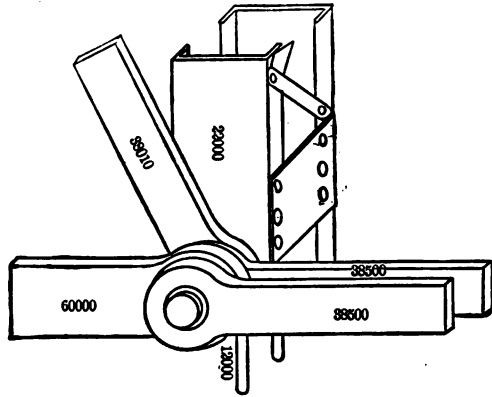
The following construction illustrates a graphical method for finding the desired resultant moments, and hence the maximum moment.

As the pieces acting on any well-designed joint are symmetrically arranged, it is unnecessary to consider more than one-half of their number, and the sketch of the portion of a joint shown on p. 179 illustrates the disposition of the pieces. The sketch, it is hoped, will also enable the reader to readily catch the idea of the relative position of the two bending moment diagrams. The pieces are parallel to the plane of the paper, and the pin is perpendicular to the same, but drawn in *perspective*, as it were, at an angle of 45° with the horizontal and vertical lines.

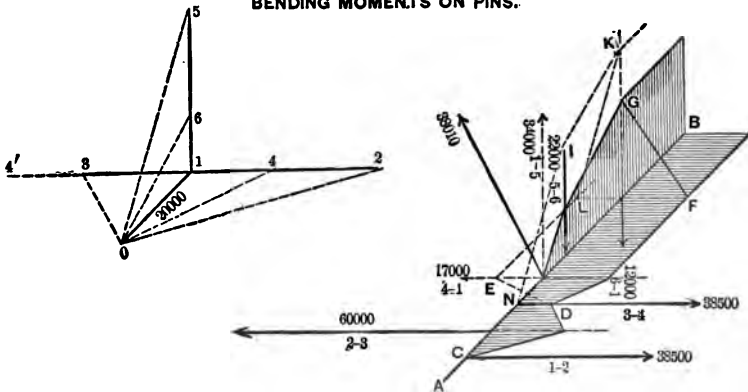
Then will AB represent the axis of the pin, and the arrows the

directions of the forces in the bars and post, their points of application along AB being laid off by scale at the centre lines of the respective pieces. The bars are here assumed to be each one inch thick and the channel's web to be one-half inch. The oblique force is decomposed graphically into two rectangular components.

In the stress diagram the horizontal forces are laid off in the order in which they occur, and in their proper directions, viz., 1—2, 2—3, 3—4, and 4—1, closing on the point of beginning. Similarly, the vertical forces are laid off from 1 as a starting-point, at 1—5, 5—6, and 6—1. The line 1—0 is then drawn at 45° , representing a *perpendicular* to 2—3 and 1—5, and made of a convenient length, here 20,000 lbs.



BENDING MOMENTS ON PINS.



From the stress diagram 0 2 3 the equilibrium polygon CDF for horizontal forces is drawn, starting at the first horizontal force C with a line parallel to 0—2, and ending with one parallel to 0—1,

and hence parallel to AB . From the stress diagram 0 1 5 is obtained the equilibrium polygon for vertical forces, beginning at the first vertical force with a line parallel to 0—5 and ending with one parallel to 0—1. If one remembers that any side of an equilibrium polygon is parallel to the line in the stress diagram which runs to that point of division on the load line common to the two forces at the extremities of the side of the polygon, no difficulty need be experienced in the construction.

At any point of the pin, the vertical and the horizontal ordinate will have for their resultant a value equal to the hypotenuse of the right-angled triangle of which they are the legs. Hence it is easy to see, or to ascertain by scale, that GF will be the longest hypotenuse which can be drawn; and this hypotenuse multiplied by 0—1, or 20,000 lbs., gives, in this case, 60,050 inch lbs., as the maximum bending moment on the pin.

If one desires to try the effect of changing the order of the bars on the pin, it can readily be done. Suppose the diagonal tie to change places with the next chord bar. The horizontal stress diagram then becomes 1—2, 2—3, 3—4', and 4'—1. The equilibrium polygons will now be $CDEL$ and NIK ; and the maximum hypotenuse is KL , giving a moment of 80,000 inch lbs., showing that the change is for the worse, although the horizontal moment is diminished.

This method is useful also in cases where the pin should be treated as having three points of support, and in other problems of forces not parallel.

A Retaining Wall for Earth.—The principles underlying this method of designing a retaining wall may be briefly given if the explanation is limited to necessary points.

1. If, in the interior of a body, we select a small prismatic portion of cross-section ABC , Fig. 1, p. 182, and if we know that the stresses on the faces AC and CB , at right angles, are pressures perpendicular to those planes and equal in intensity per square inch or square foot, the pressure on the face AB must be perpendicular or normal to it, and of the same intensity as on the other faces. For, if we multiply the area of the face AC by the pressure, p , per square foot, and lay it off by scale in the proper direction, as the force OD , and likewise the area of face CB multiplied by p as the

force OE , the diagonal RO of the rectangle must give the direction and magnitude of the force on the third face, AB , which will balance OD and OE ; and as triangles ABC and ORD are similar, OR is perpendicular to the face AB , and the intensity of pressure on AB will be $OR \div AB = p$.

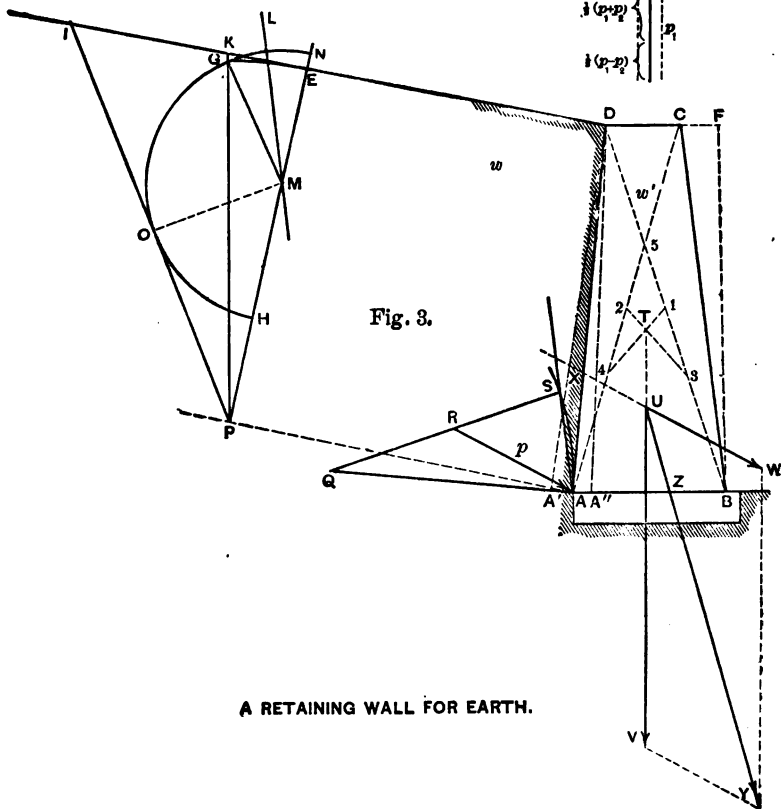
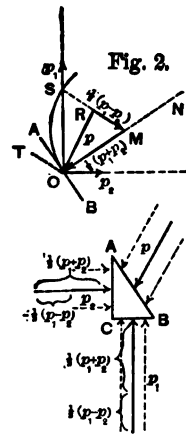
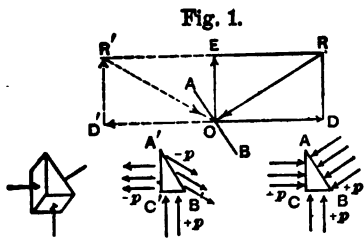
This case applies to fluid pressure, which is always normal to any plane, and of constant intensity for a given depth.

2. If, on the other hand, the force on the side $A'C'$ of a similar small prismatic body $A'B'C'$ is of the opposite kind, tension, but of the same intensity, p , as before, we must draw OD' in the opposite direction, and find $R'O$ as the force on the face $A'B'$. Then, since the triangles $A'B'C'$ and $R'D'O$ are again similar, the intensity of stress on plane $A'B'$ will still be $R'O \div A'B' = p$, but its kind, tension or pressure, will be that of the line, OD' or OE , to which it lies the nearer. In the sketch it is tension, as is evidently necessary to keep $A'B'C'$ in equilibrium. It will also be seen that the direction of this stress will make the same angle with the original stresses as in the case ABC , where they were pressures, but in the opposite direction; that is, angle $R'OE = \text{angle } ROE$.

3. Next, if the normal pressures on the two faces AC and CB , Fig. 2, are unequal, and we suppose that p_1 per square foot on CB is the greater and p_2 on AC is the less of the two, we may find the direction and intensity of the pressure on the face AB by the following construction:

Let the intensity of the pressure on face CB be divided into two parts, one equal to $\frac{1}{2}(p_1 + p_2)$ and the other to $\frac{1}{2}(p_1 - p_2)$, as indicated in the sketch. Similarly, the intensity of pressure on face AC may be divided, as shown, into $\frac{1}{2}(p_1 + p_2)$ and $-\frac{1}{2}(p_1 - p_2)$. The intensity of pressure on AB to balance the portion $\frac{1}{2}(p_1 + p_2)$ on CB and also on CA must then be, by § 1, the same, or MO , laid off perpendicularly to AB . The intensity of stress on AB to balance the remaining portions, a pressure of $\frac{1}{2}(p_1 - p_2)$ on CB and a stress of $-\frac{1}{2}(p_1 - p_2)$, or tension, on CA , must be, by § 2, a stress TO or RM of the same intensity, $\frac{1}{2}(p_1 - p_2)$, but making an angle equal to SOM with p_1 , but in the opposite direction. The angle NMR corresponds with ROR' of Fig. 1.

This angle can be readily found by taking M as a centre, MO as radius, and describing an arc OS to cut the line p_1 ; when MS ,



drawn through S , will make OMS an isosceles triangle, and the angle at S will equal that at O . Then lay off $MR = \frac{1}{2}(p_1 - p_2)$, to satisfy § 2. The resultant pressure on AB will be given in direction and intensity by $RO = p$, which is evidently the resultant of RM and MO , since these two components act on the same unit of area of plane AB . This resulting intensity of stress RO , or p , on plane AB , which balances p_1 on CB and p_2 on AC , makes an angle NOR with the normal NO to its plane.

4. If, for a given p_1 and p_2 , any number of planes AB are tried, at various angles with AC or CB , the triangle OMR , constructed on the normal to the plane AB , will change its angles and the length of RO , but not the lengths of RM and MO . Hence the greatest possible value of the angle MOR will occur when MR happens to be perpendicular to OR , and that value will give the greatest possible obliquity of the pressure on any plane, for given values of p_1 and p_2 .

5. If the material under consideration is *earth*, and it is assumed to be held in place by the friction alone of its particles, one on the other (the adhesion arising from moisture being neglected as always uncertain in amount and sometimes possibly absent), the greatest possible obliquity of pressure, consistent with equilibrium on any plane in the mass of earth, cannot exceed what is known as the angle of repose; for, if it did, sliding would take place along that plane.

6. Let a plane be passed through P , Fig. 3, parallel to the surface of the ground DI . The pressure on every square foot of this plane is vertical, and due to the earth above it, of depth KP . But the prism of earth resting on a square foot of this plane has a less horizontal cross-section than one square foot, and the ratio of the intensity of vertical pressure on the plane through P to the weight of a vertical column of earth one square foot in cross-section will be that of the normal PE , drawn from P to DI , to PK . Hence, revolve PE to PG , and GP will represent, in feet of earth, the pressure per square foot of the plane through P parallel to the surface of the ground.

If we knew what and where p_1 and p_2 were on such a prismatic body at P as we have previously considered, the plane previously denoted by AB being now the one that passes through P parallel to the surface of the ground, we could, by § 3, lay off on the normal

PN the distance $PM = \frac{1}{2}(p_1 + p_2)$, and MG would be $\frac{1}{2}(p_1 - p_2)$ to close on GP or p . But by § 5 the greatest obliquity of stress to the normal to any plane cannot exceed the angle of repose of the earth. Hence, if PI is drawn, making that angle with PN , the distance MG must, if applied at MO , make MOP a right angle. Therefore find by trial a centre M from which a semicircle NGQ can be drawn through G and tangent to the line PI . PM will then be $\frac{1}{2}(p_1 + p_2)$ and MG , $\frac{1}{2}(p_1 - p_2)$. By § 3, the direction of p_1 will be parallel to the line ML drawn bisecting the angle NMG .

7. It may be remarked in passing that $PH = p_1$ and $PN = p_2$, acting on the two right-angled faces of the prism at P , respectively parallel and perpendicular to the direction of the line LM ; that p_1 is the *least* pressure which is consistent with equilibrium, and that it is the one exerted by earth at rest under the action of its own weight only. Blows applied to the surface and vibration will probably increase the pressures.

8. To find the centre of pressure and the direction and intensity of pressure on any bed-joint of a retaining wall, weighing w' per cubic foot, pressed at the back by earth of a weight w per cubic foot and of a given angle of repose, proceed as follows :

The bed-joint is AB , Fig. 3, carrying the weight of masonry $ABCD$, whose centre of gravity is at T . To find T , draw diagonals AC and BD ; bisect each at 1 and 2 respectively; lay off $A-4 = C-5$ and $B-3 = D-5$; connect 1 with 4 and 2 with 3; these connecting lines will intersect at the centre of gravity, T .

9. At a point P , the same distance KP below the surface of the ground that A is, make the construction of § 6, that is, draw PN perpendicular to DI , and PI making with PN the given angle of repose. Revolve PE to PG , draw a semicircle, with centre on PN , through the point G and tangent to PI . Bisect the angle NMG by LM .

10. Now find the direction and intensity of pressure at A on the plane AD , by § 3, noting that A is similarly situated as P . To do so, draw AQ perpendicular or normal to AD ; lay off $AQ = PM = \frac{1}{2}(p_1 + p_2)$; draw AS parallel to LM , that being the direction of p_1 ; from Q as centre, with radius QA , draw an arc cutting AS at S ; draw QS and lay off on it, from Q , $MG = QR = \frac{1}{2}(p_1 - p_2)$; connect R with A , and RA will be the direction and intensity of

pressure per square foot at A on back of wall, in terms of cubic feet of earth, so that, if RA is measured by the scale of drawing and multiplied by the weight of a cubic foot of earth, the pressure on back of wall per square foot at A will be given.

As the pressure at back increases regularly with the depth below surface of ground, the centre of pressure will be at X , one-third of the slant height from A , and the total earth pressure against one foot in length of wall will be $\frac{1}{2} (AD \times AR)$.

11. Draw XUW through X , parallel to RA , and let fall TV vertically through T . Make $UV = (AB + CD) \frac{w'}{w}$ and $UW = AR \frac{AD}{BF}$ * Complete the parallelogram $UVYW$. UY will be the direction of the resultant pressure on the bed-joint AB , and the point Z where it cuts the joint will be the centre of resistance or pressure. The total pressure on the joint will be found by multiplying UY by one-half the height of the wall above AB and by the weight of a cubic foot of earth.

12. If one thinks that the centre of pressure is too near the front for safety or too near the middle of the joint for economy of masonry, change the section by drawing DA' or DA'' and try again, §§ 8, 10, and 11. A second trial will usually suffice.

13. If the distance BZ is more than one-third of BA , the maximum intensity per square foot, at B , is

$$p = \frac{2 \times \text{total pressure}}{AB} \left(2 - \frac{3BZ}{AB} \right).$$

If the distance BZ is less than one-third of BA , the maximum intensity, supposing the cement in the joint to offer no resistance to tension, is

$$p = \frac{2}{3} \cdot \frac{\text{total pressure}}{BZ}.$$

If this intensity is greater than the masonry can safely resist, make AB wider and try again.

* The ratio $\frac{AD}{BF}$ may be called unity without serious error, unless the wall has a strong batter at rear. By the use of the above factors, UV represents weight of wall, and UW the total earth pressure at back of same.

The wall will be satisfactory to many engineers if BZ is somewhat greater than $\frac{1}{3} AB$. A margin of safety is thus left for an increase of pressure beyond the *least* pressure here used.

The obliquity of UY to the perpendicular to AB will determine the tendency of the wall to slide forward. If such sliding seems likely to occur, the bed-joint AB may be inclined backwards.

The above constructions are simplified when the surface of the earth is horizontal, and also when it slopes at its angle of repose.

Extent of Uniform Load to Produce Maximum Stress in a Brace.—It may be interesting to offer an analytical investigation into the extent of a uniform load which shall produce the maximum stress on a brace of a truss of any form. The general steps of this treatment, substantially as here given, were contributed by S. W. Salmon to "The Analyst" for February, 1874.

Let the span AB of the truss, Fig. 36, $=l$, the distance AC to any joint $=a$, and the distance AD to the next joint $=b$; let the height CF of the truss at $C = h'$, and of DE at $D = h''$. Let H' = horizontal force at the section at C , and H'' = the same at D . The difference between H' and H'' is the horizontal component of the stress in DF . Let w' = moving load per unit of length extending from A a distance x . The upward reaction at $B = \frac{w' x^2}{2l}$. We may have three expressions for $H' - H''$; one when x is less than a , one when x is greater than b , and the last when the value of x lies between a and b . In the first two cases $H' - H''$ will increase as x approaches a and b respectively, and there is no absolute maximum. But the last case has a value of x which makes $H' - H''$ a maximum. At that time

$$H' = \frac{w' x^2}{2l} \cdot \frac{(l-b)}{h''}; \quad H' = \left\{ \frac{w' x^2}{2l} \cdot (l-a) - \frac{w' (x-a)^2}{2} \right\} \div h'$$

$$= \frac{w'}{h'} \left(a x - \frac{a x^2}{2l} - \frac{a^2}{2} \right);$$

$$H' - H'' = \frac{w'}{h'} \left(a x - \frac{a x^2}{2l} - \frac{a^2}{2} \right) - \frac{w' (l-b) x^2}{h'' \cdot 2l}, \text{ to be made a maximum.}$$

Differentiating relatively to x , and putting the first differential co-efficient equal to zero, we get

$$\frac{a}{h'} - \frac{a x}{h' l} - \frac{(l-b) x}{h'' l} = 0,$$

$$x = \frac{a l h''}{a h'' + (l-b) h'}.$$

Substitute in the value of $H' - H''$, which then becomes

$$H' - H'' (\text{max.}) = \frac{w' a^3}{2} \left\{ \frac{(l-a) h'' - (l-b) h'}{a h' h'' + (l-b) h'^2} \right\}, \text{ in the most general form.}$$

Applications.—If the truss has parallel chords, $h' = h'' = h$, and

$$H' - H'' (\text{max.}) = \frac{w' a^2}{2h} \left\{ \frac{b-a}{l-(b-a)} \right\}.$$

If $b-a = p$, a constant panel length or half-panel length, as the case may be, $l = Np$, and $a = np$:

$$H' - H'' = \frac{w' a^2 p}{2h(l-p)} = \frac{w' n^2 p^2}{2h(N-1)}.$$

The shear F , which is the other component of the stress in the brace, will be to $H' - H''$ in the ratio $\frac{h}{p}$: hence

$$F (\text{max.}) = \frac{w' n^2 p}{2(N-1)}.$$

This expression is the ordinate to a parabola which is convex to the horizontal axis, has its vertex at one abutment and its extreme ordinate, when $a = np = (N-1)p$, becomes $\frac{N-1}{2} w' p$, coincident with the ordinate for travelling load in the middle of the first panel which we have heretofore used. To ascertain how much of the truss must be successively loaded to cause the maximum shear in each panel, change the value of x by dividing out the common factor $h' = h''$ and making substitutions,

$$x = \frac{al}{l-(b-a)} = \frac{al}{l-p} = \frac{np l}{Np-p} = n \frac{l}{N-1}.$$

If, therefore, the span is divided into $N-1$ equal parts, the maximum shear in any panel will occur when the load extends up to the division in that panel, as shown in Fig. 37, and it will equal the ordinate to the parabola at that point. To the above expression for F must be added the shear for steady load. It is to be measured in the middle of the panel, as heretofore; and the two ordinates must, therefore, be measured separately.

If the truss is parabolic, the height h , at any point, will be, if k equals the rise at the middle,

$$h = \frac{4k}{l^2} x'(l-x');$$

and we obtain by substitution, making $x' = a$ or b , as required,

$$\begin{aligned} H' - H'' (\text{max.}) &= \frac{w' a^2}{2} \cdot \frac{l^2}{4k} \left\{ \frac{(l-a)b(l-b) - (l-b)a(l-a)}{a^2 b(l-b)(l-a) + a^2 (l-b)(l-a)^2} \right\} \\ &= \frac{w' l^2}{8k} \left\{ \frac{b-a}{b+l-a} \right\} = \frac{w' l^2}{8k} \cdot \frac{p}{l+p} = \frac{w' l^2}{8k} \cdot \frac{1}{N+1}, \end{aligned}$$

which is a constant quantity, as in the earlier investigation of the bowstring girder, w' in that case denoting a panel weight.

If the value of x is again reduced for this truss to obtain the extent of load to cause the maximum shear in each panel, it becomes

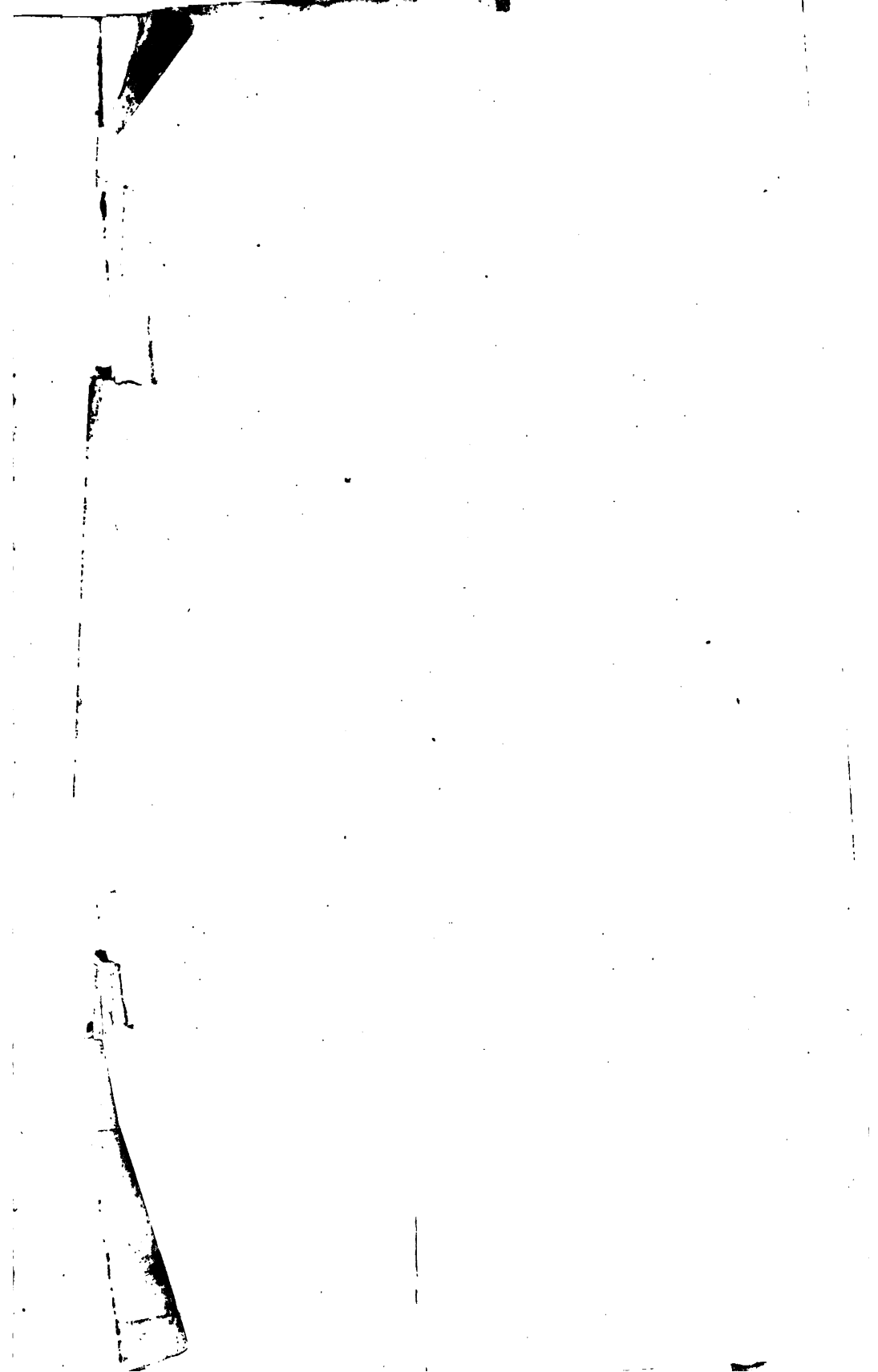
$$x = \frac{a l b (l - b)}{a b (l - b) + (l - b) a (l - a)} = \frac{l b}{b + l - a} = \frac{b l}{l + p} = (n + 1) \frac{l}{N + 1},$$

where the distance b exceeds a one panel length. If, then, the span of Fig. 88 is divided into $N + 1$ equal parts, the load, when extending up to the successive points of division, will give the maximum stress in the diagonal which crosses the head of the load. A diagram can therefore be constructed, as before, of the constant panel length $\frac{w' l^2}{8 (N + 1) k}$, by which to obtain the diagonal stresses.

INDEX.

	PAGE		PAGE
AREAS	119, 149	Formula for shear	31
Area moments, calculation of	103, 135	GRADE, truss on a	59
BALTIMORE Bridge Co.'s truss	51	HOWE truss	45
Beams, deflection of	105	INCLINED chords, effect of	78
Bending moment	18	" trusses	60
" " maximum	24, 29	JOINTS of continuous girder for special	
" " on pins	178	treatment	133, 145
Bollman truss	57	KELLOGE's truss	53
Bowstring girder	86, 98	Kentucky river bridge	154
CANTILEVER bridge	157	LATTICE girder	78
Centre of gravity of weights	23	Locomotive, effect of, on double web system	75
Clapeyron's formula	139	" stresses due to	42
Chord-stresses from polygon	36	MAXIMUM bending moment	24, 29
Concentrated moving loads	40, 61, 63	" shearing force	25, 29
Continuous girder, deflection of	149	Middle vertical	35
" " many spans	136	Moment diagram	22
" " three "	142	" " for partial moving load	40
" " two "	117, 124	" due to a force	23
" " piers not on level	141	NEGATIVE moments, continuous girder, 133, 145	
" " utility of	156	ODD number of panels	55
" load, extent of, to produce		Omission of locking bolt in draw	170
maximum web-stresses	186	PARABOLA for chord-stresses	38
Curve for maximum shears	30	Partially continuous trusses	150, 152, 157
DEFLECTION of beams	105	Piers not on a level	141, 150
" " continuous girder	149	Pier ordinates	120, 137
Diagonals, stresses in	33	Pins, bending moment on	178
" with inclined chords	79	Plate girder	53
Diagram of maximum shear	26	Points of contraflexure fixed by hinges	153
Double intersection (or quadrangular) truss	66	Pratt truss	46
" " inclined end posts	69	Pressure of earth	183
" " odd number of panels	69	REACTIONS	20
Draw spans	158	" of continuous girder, weigh-	
" of three spans	171	ing the	156
" " two spans, continuous	159, 166	Retaining wall	120
" " " independent	159	SETTLEMENT of piers, adjustment for	150, 153
" to rest on three points	132	Shear diagram	21, 28
" with tipper	174	" " for concentrated loads or	
EARTH pressure	180, 183	wheel-weights	61
Effect of movement of load	29, 40	Shear diagram for partial moving load	41
Equilibrium polygon	16, 27	" formula	31
FINK truss	56	" from moment diagram	97
Flexure of beams	100	Shearing force	17

	PAGE		PAGE
Shearing force, maximum	25, 29	Truss with inclined chord	78
" " curve for	30	VERTICALS, stresses in	36
Skew truss	59	" with inclined chords	80
THREE moment theorem	139, 140	WARREN girder	48
Trapezoidal truss	56	" " double	71
Triangle of forces	15	Weighing reactions	156-7
Triangular truss	48	Whipple truss	66
" " double	71	Wind bracing	57



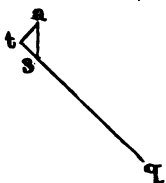
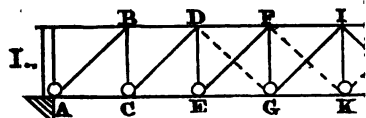
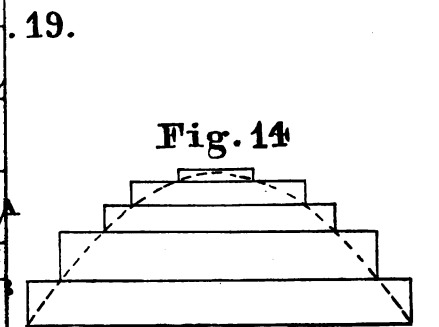
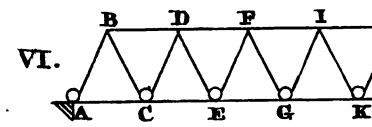
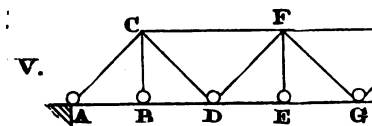
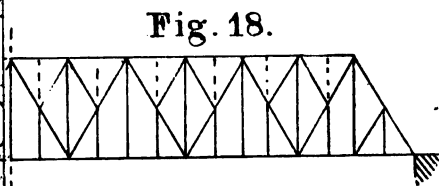
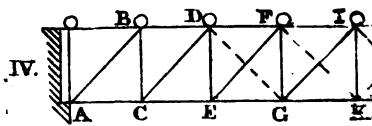
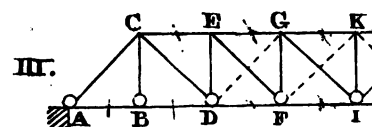
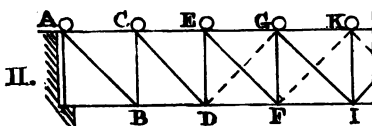
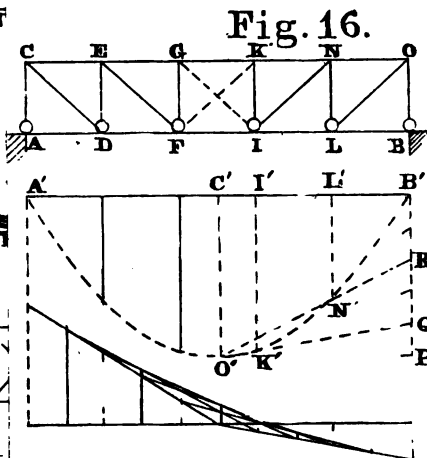
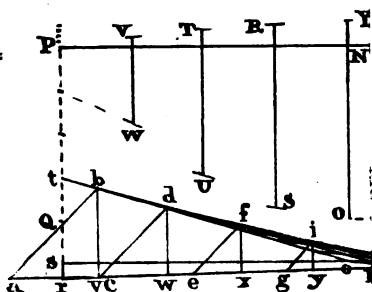


Fig. 13.



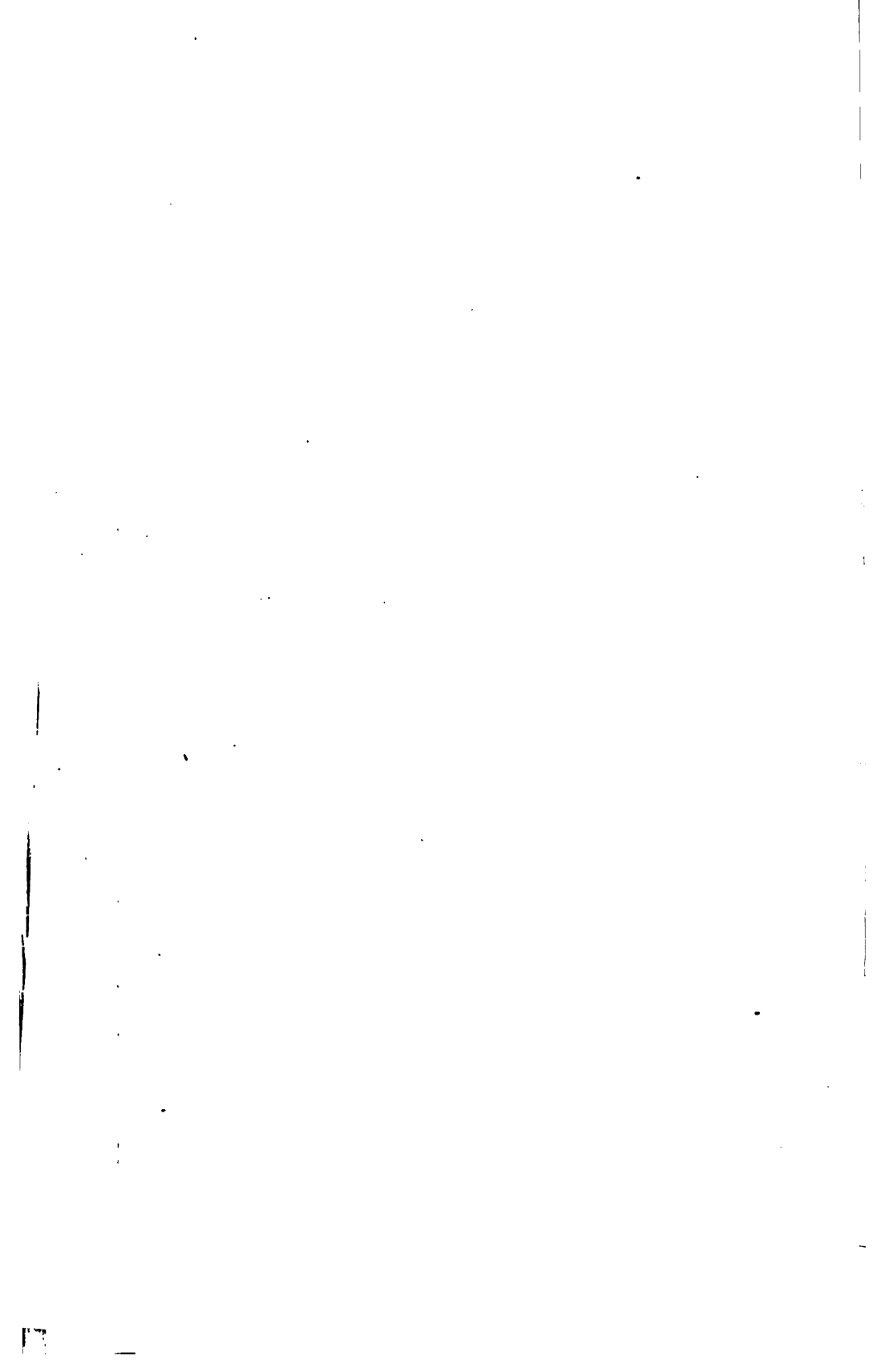


Fig. 2

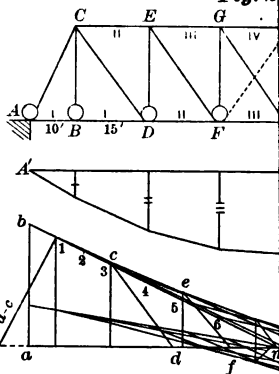


Fig. 22

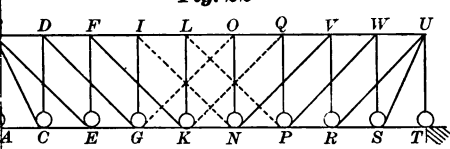


Fig. 25

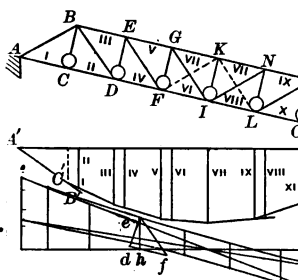
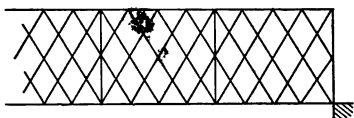


Fig. 29 A

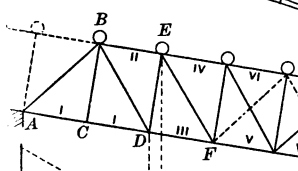
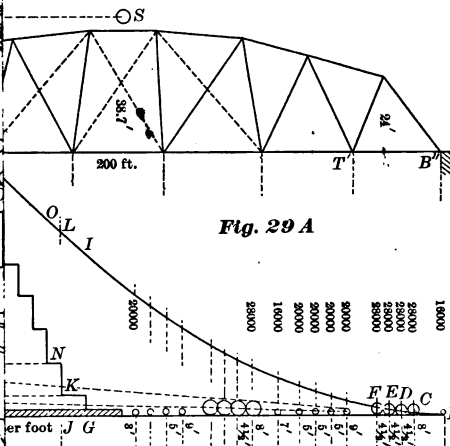


Fig. 26 B

Fig. 29 B

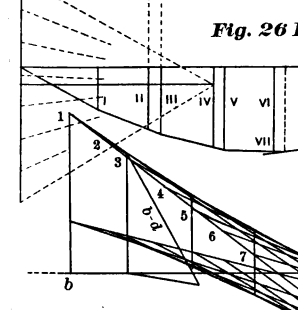
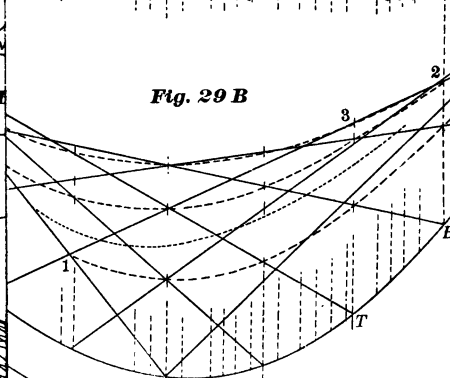
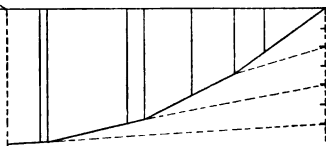
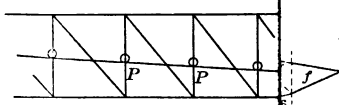
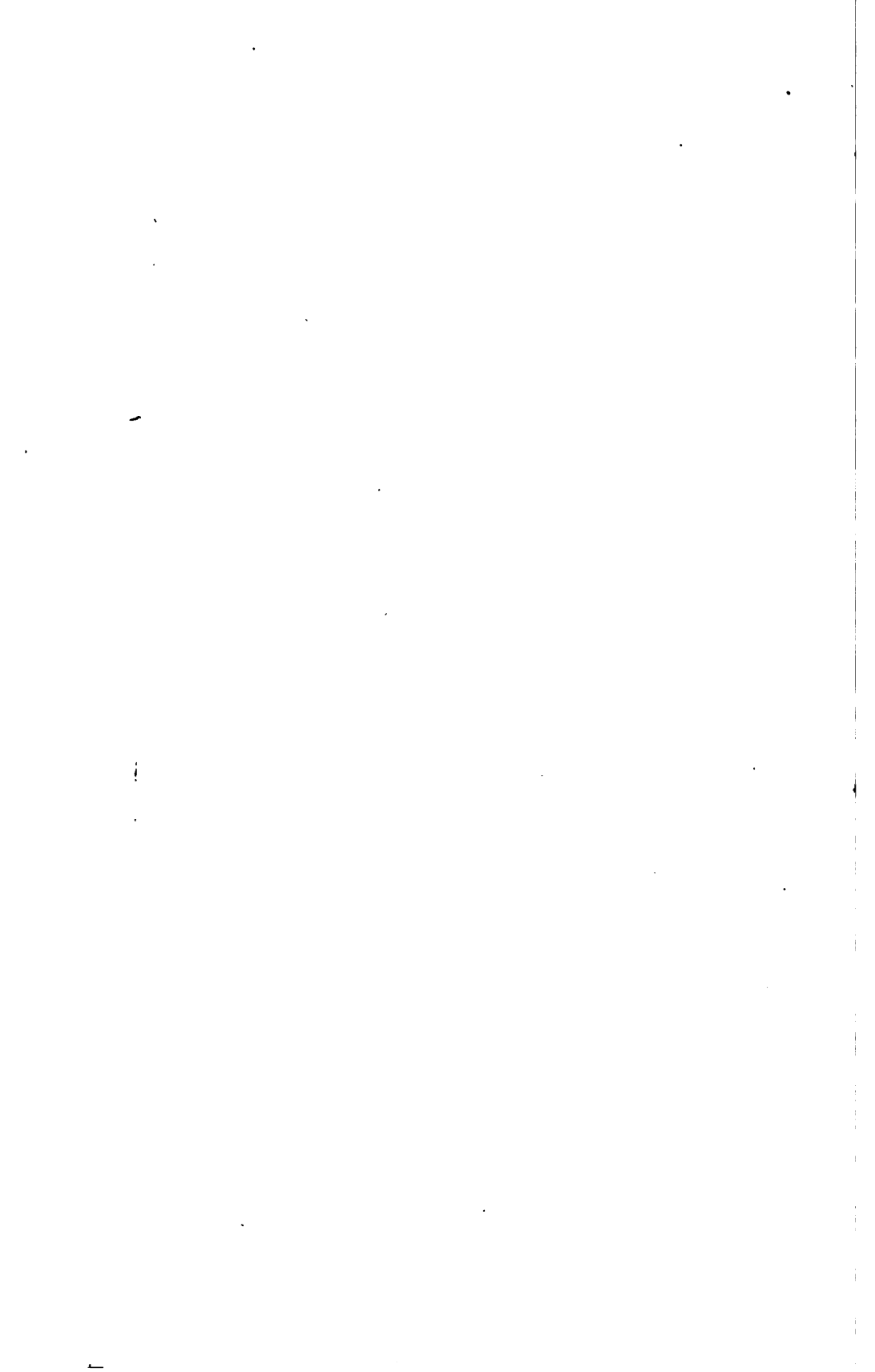


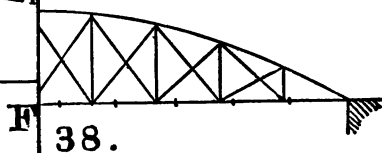
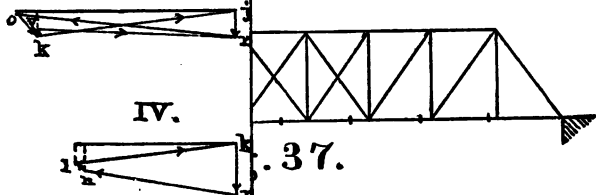
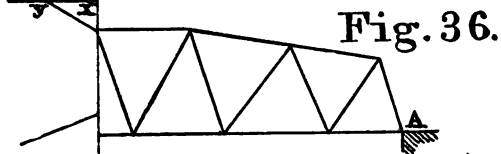
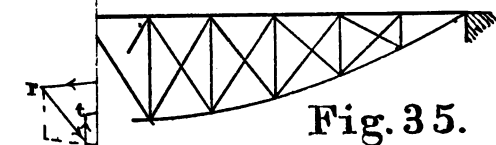
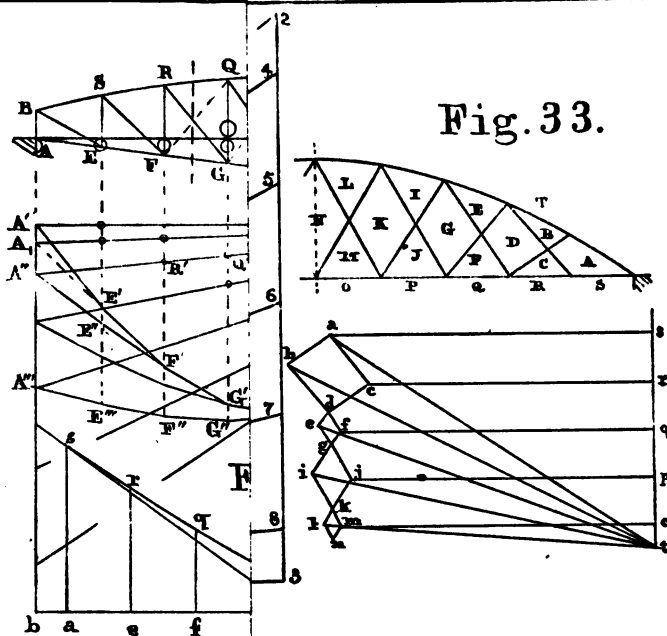
Fig. 26 C

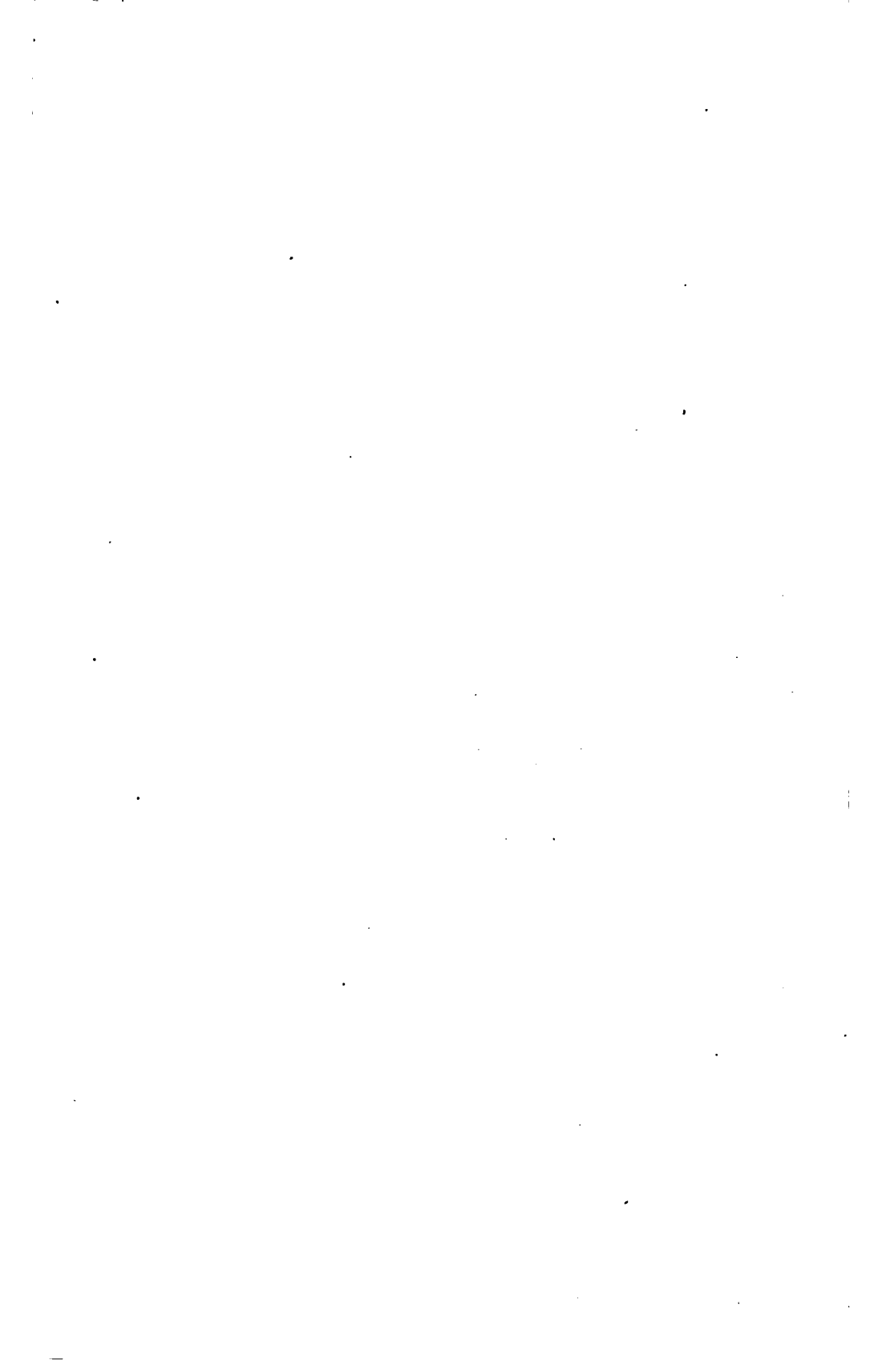


1 red/en & Pates, Eng'rs, N.Y.









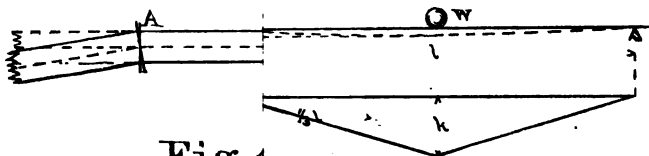


Fig. 44.

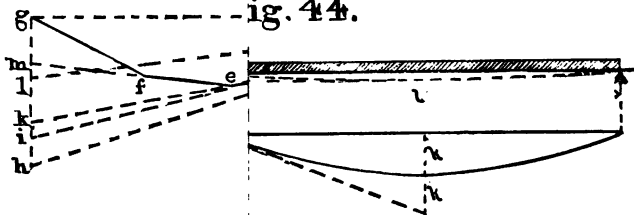


Fig. 45.

Fig. 47.

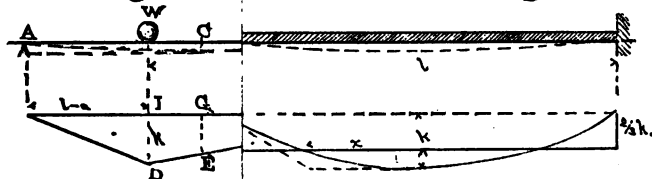


Fig. 50.

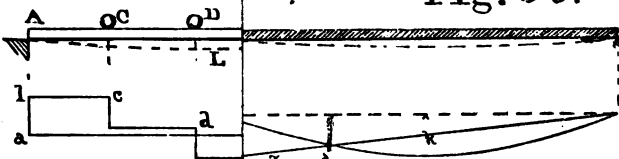
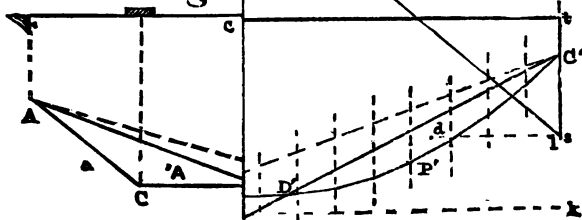


Fig. 5



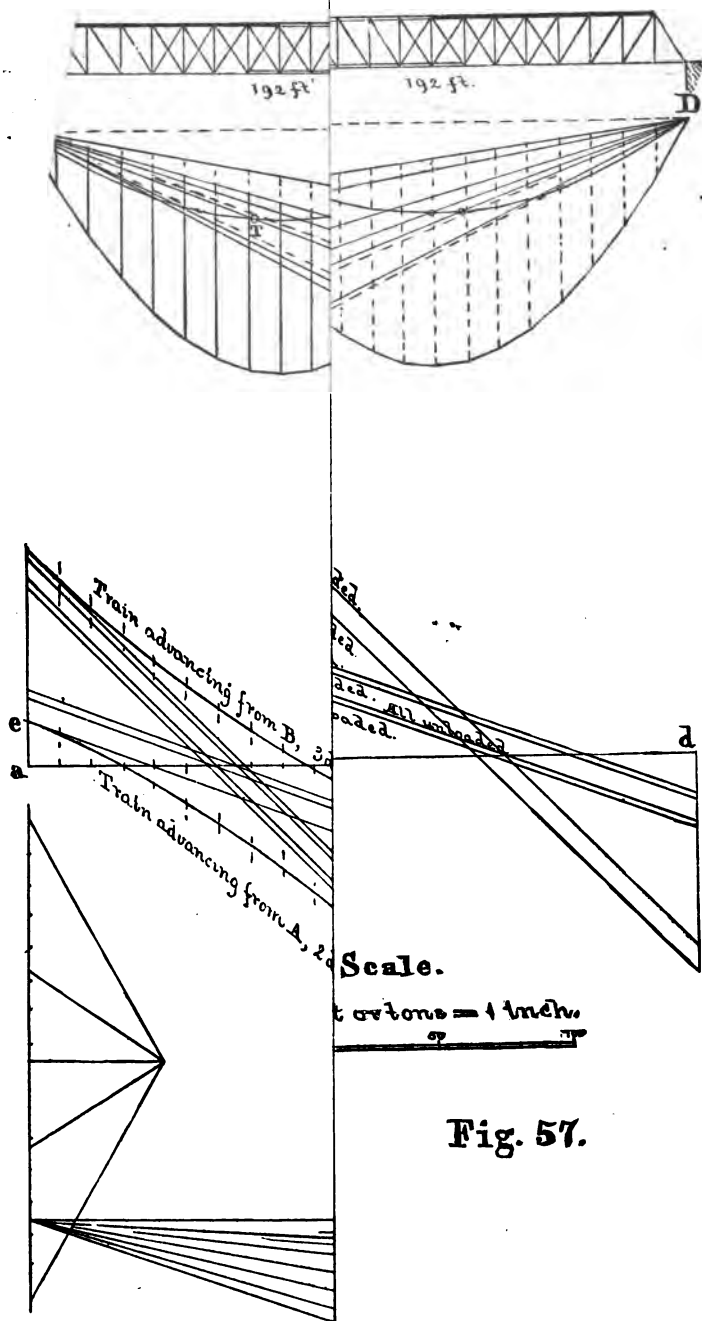
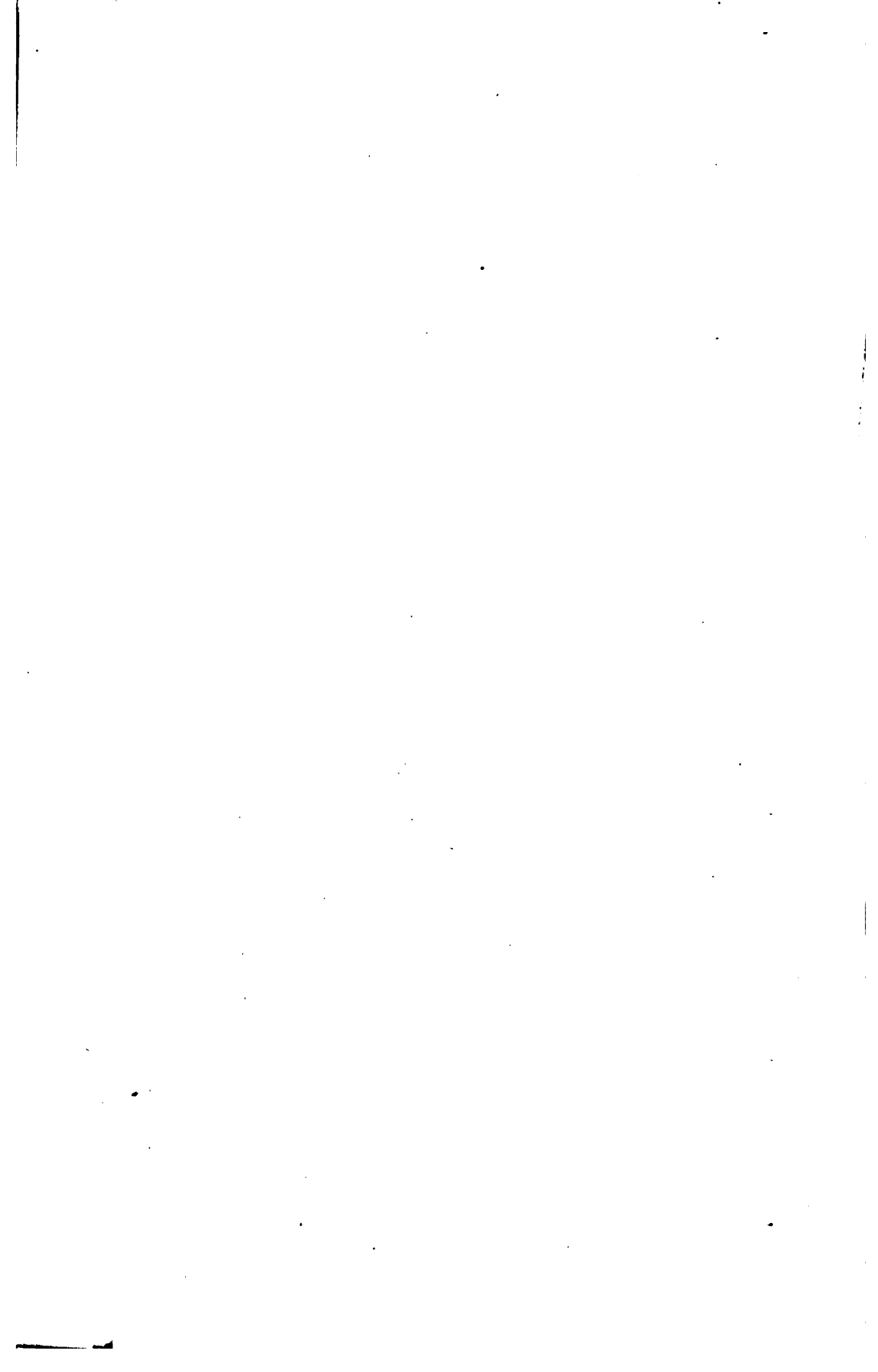


Fig. 57.



ages.

Plate VIII.

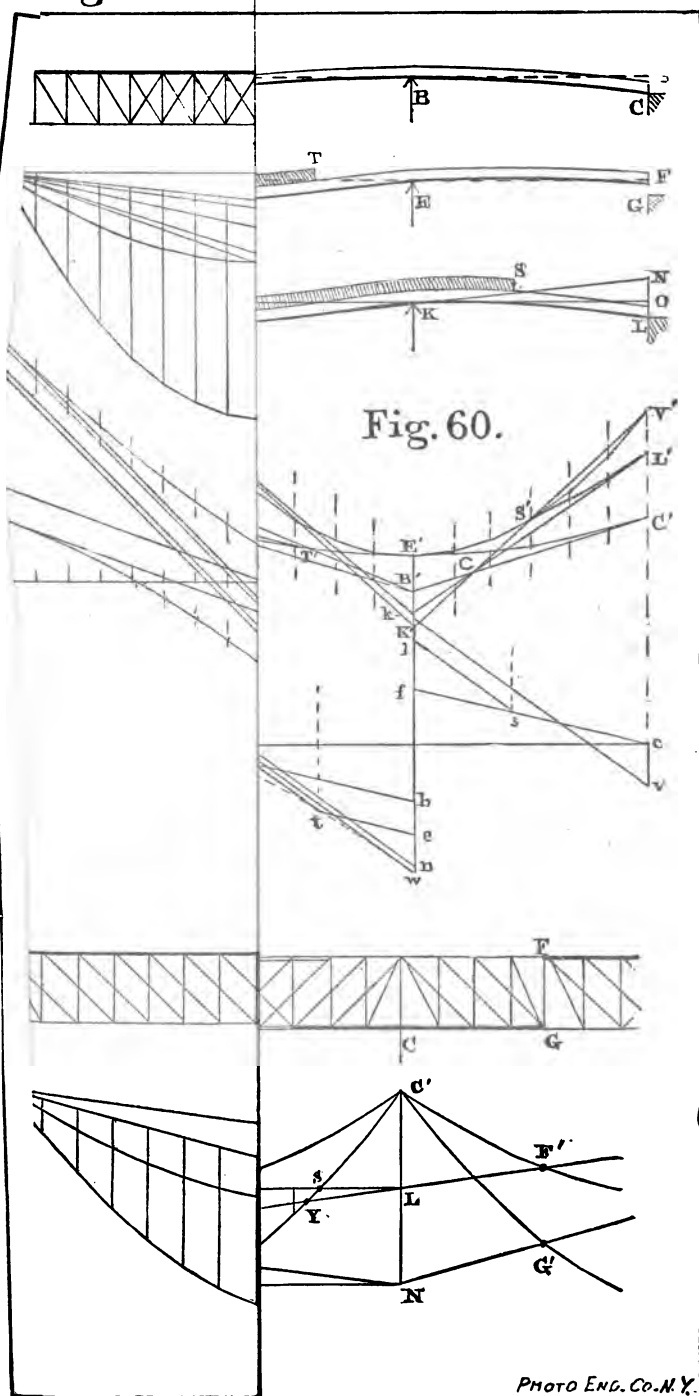




Plate IX.

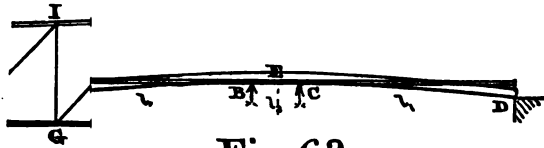


Fig. 62.

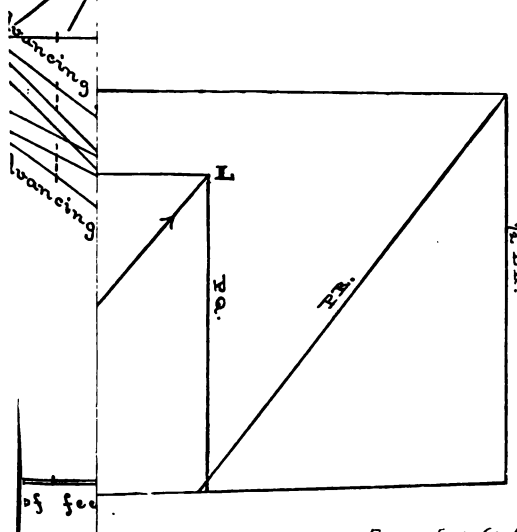
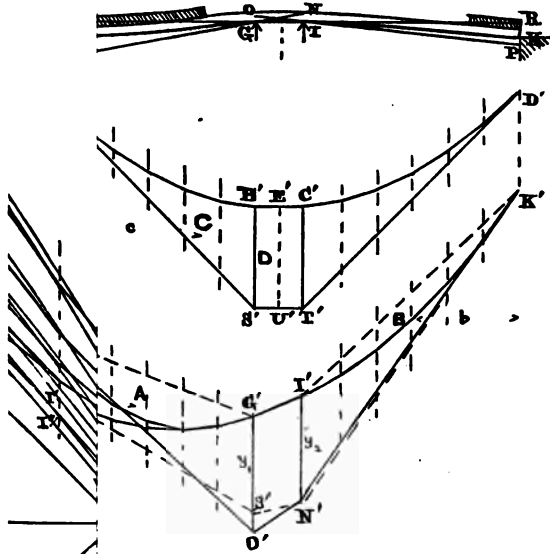
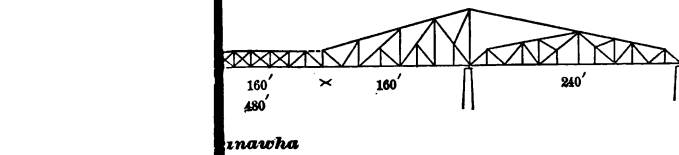
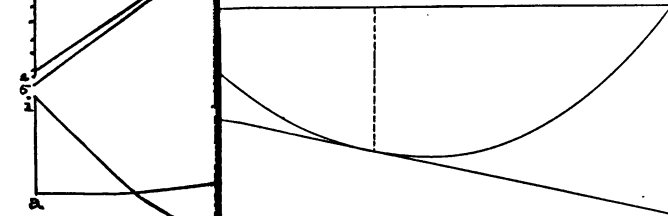
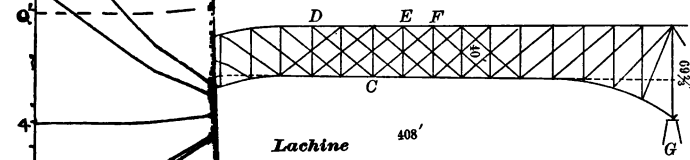
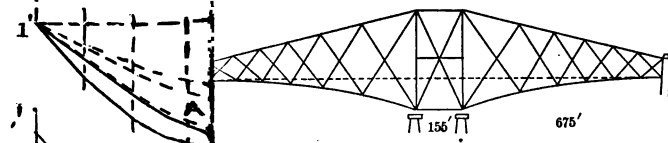
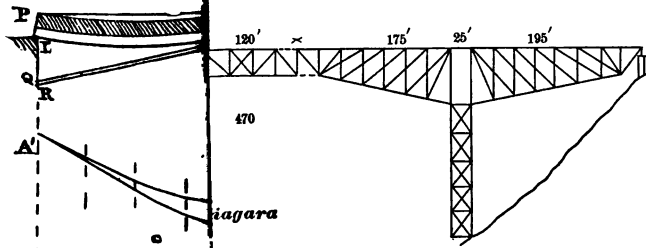
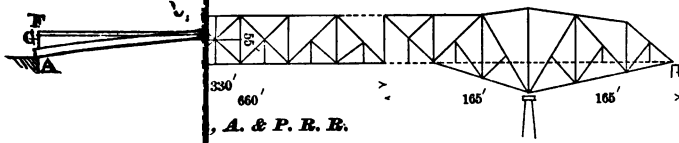


PHOTO ENG. CO. N. Y.







SHORT-TITLE CATALOGUE

OF THE
PUBLICATIONS

OF
JOHN WILEY & SONS,

NEW YORK.

LONDON: CHAPMAN & HALL, LIMITED.

ARRANGED UNDER SUBJECTS.

Descriptive circulars sent on application.

Books marked with an asterisk are sold at *net* prices only.

All books are bound in cloth unless otherwise stated.

AGRICULTURE.

CATTLE FEEDING—DAIRY PRACTICE—DISEASES OF ANIMALS—
GARDENING, ETC.

Armsby's Manual of Cattle Feeding.....	12mo,	\$1 75
Downing's Fruit and Fruit Trees.....	8vo,	5 00
Grotenfelt's The Principles of Modern Dairy Practice. (Woll.)		
	12mo,	2 00
Kemp's Landscape Gardening....	12mo,	2 50
Loudon's Gardening for Ladies. (Downing.).....	12mo,	1 50
Maynard's Landscape Gardening.....	12mo,	1 50
Steel's Treatise on the Diseases of the Dog.....	8vo,	3 50
" Treatise on the Diseases of the Ox.....	8vo,	6 00
Stockbridge's Rocks and Soils.....	8vo,	2 50
Woll's Handbook for Farmers and Dairymen.....	12mo,	1 50

ARCHITECTURE.

BUILDING—CARPENTRY—STAIRS—VENTILATION—LAW, ETC.

Berg's Buildings and Structures of American Railroads....	4to,	7 50
Birkmire's American Theatres—Planning and Construction.	8vo,	3 00
" Architectural Iron and Steel.....	8vo,	3 50
" Compound Riveted Girders.....	8vo,	2 00
" Skeleton Construction in Buildings.....	8vo,	3 00

Birkmire's Planning and Construction of High Office Buildings.

	8vo,	\$3 50
Briggs' Modern Am. School Building	8vo,	4 00
Carpenter's Heating and Ventilating of Buildings.....	8vo,	3 00
Freitag's Architectural Engineering.....	8vo,	2 50
“ The Fireproofing of Steel Buildings	8vo,	2 50
Gerhard's Sanitary House Inspection.....	16mo,	1 00
“ Theatre Fires and Panics.....	12mo,	1 50
Hatfield's American House Carpenter.....	8vo,	5 00
Holly's Carpenter and Joiner.. ..	18mo,	75
Kidder's Architect and Builder's Pocket-book...16mo, morocco,		4 00
Merrill's Stones for Building and Decoration.....	8vo,	5 00
Monckton's Stair Building—Wood, Iron, and Stone.....	4to,	4 00
Wait's Engineering and Architectural Jurisprudence.....	8vo,	6 00
	Sheep,	6 50
Worcester's Small Hospitals—Establishment and Maintenance, including Atkinson's Suggestions for Hospital Archi- tecture... ..	12mo,	1 25
World's Columbian Exposition of 1893.....	Large 4to,	2 50

ARMY, NAVY, Etc.

MILITARY ENGINEERING—ORDNANCE—LAW, ETC.

Bourne's Screw Propellers.....	4to,	5 00
* Bruff's Ordnance and Gunnery.....	8vo,	6 00
Chase's Screw Propellers.....	8vo,	3 00
Cooke's Naval Ordnance	8vo,	12 50
Cronkhite's Gunnery for Non-com. Officers.....	32mo, morocco,	2 00
* Davis's Treatise on Military Law.....	8vo,	7 00
	Sheep,	7 50
* “ Elements of Law.....	8vo,	2 50
De Brack's Cavalry Outpost Duties. (Carr.)....	32mo, morocco,	2 00
Dietz's Soldier's First Aid.....	16mo, morocco,	1 25
* Dredge's Modern French Artillery....	Large 4to, half morocco,	15 00
“ Record of the Transportation Exhibits Building, World's Columbian Exposition of 1893..	4to, half morocco,	10 00
Durand's Resistance and Propulsion of Ships.....	8vo,	5 00
Dyer's Light Artillery.....	12mo,	3 00
Hoff's Naval Tactics.....	8vo,	1 50

* Ingalls's Ballistic Tables.....	8vo,	\$1 50
" Handbook of Problems in Direct Fire.....	8vo,	4 00
Mahan's Permanent Fortifications. (Mercur.).....	8vo, half morocco,	7 50
* Mercur's Attack of Fortified Places.....	12mo,	2 00
* " Elements of the Art of War.....	8vo,	4 00
Metcalf's Ordnance and Gunnery.....	12mo, with Atlas,	5 00
Murray's A Manual for Courts-Martial.....	16mo, morocco,	1 50
" Infantry Drill Regulations adapted to the Springfield Rifle, Caliber .45.....	32mo, paper,	10
* Phelps's Practical Marine Surveying.....	8vo,	2 50
Powell's Army Officer's Examiner.....	12mo,	4 00
Sharpe's Subsisting Armies.....	32mo, morocco,	1 50
Very's Navies of the World.....	8vo, half morocco,	3 50
Wheeler's Siege Operations.....	8vo,	2 00
Winthrop's Abridgment of Military Law.....	12mo,	2 50
Woodhull's Notes on Military Hygiene.....	16mo,	1 50
Young's Simple Elements of Navigation.....	16mo, morocco,	2 00
" " " " " first edition.....		1 00

ASSAYING.

SMELTING—ORE DRESSING—ALLOYS, ETC.

Fletcher's Quant. Assaying with the Blowpipe.....	16mo, morocco,	1 50
Furman's Practical Assaying.....	8vo,	3 00
Kunhardt's Ore Dressing.....	8vo,	1 50
O'Driscoll's Treatment of Gold Ores.....	8vo,	2 00
Ricketts and Miller's Notes on Assaying.....	8vo,	3 00
Thurston's Alloys, Brasses, and Bronzes.....	8vo,	2 50
Wilson's Cyanide Processes.....	12mo,	1 50
" The Chlorination Process.....	12mo,	1 50

ASTRONOMY.

PRACTICAL, THEORETICAL, AND DESCRIPTIVE.

Craig's Azimuth.....	4to,	3 50
Doolittle's Practical Astronomy.....	8vo,	4 00
Gore's Elements of Geodesy.....	8vo,	2 50
Hayford's Text-book of Geodetic Astronomy.....	8vo,	3 00
* Michie and Harlow's Practical Astronomy.....	8vo,	3 00
* White's Theoretical and Descriptive Astronomy.....	12mo,	2 00

BOTANY.

GARDENING FOR LADIES, ETC.

Baldwin's Orchids of New England.....	Small 8vo,	\$1 50
Loudon's Gardening for Ladies. (Downing.).....	12mo,	1 50
Thomé's Structural Botany.....	16mo,	2 25
Westermaier's General Botany. (Schneider.).....	8vo,	2 00

BRIDGES, ROOFS, Etc.

CANTILEVER—DRAW—HIGHWAY—SUSPENSION.

(See also ENGINEERING, p. 8.)

Boller's Highway Bridges.....	8vo,	2 00
* " The Thames River Bridge.....	4to, paper,	5 00
Burr's Stresses in Bridges....	8vo,	3 50
Crehore's Mechanics of the Girder.....	8vo,	5 00
Dredge's Thames Bridges.....	7 parts, per part,	1 25
Du Bois's Stresses in Framed Structures.....	Small 4to,	10 00
Foster's Wooden Trestle Bridges.....	4to,	5 00
Greene's Arches in Wood, etc.....	8vo,	2 50
" Bridge Trusses.....	8vo,	2 50
" Roof Trusses.....	8vo,	1 25
Howe's Treatise on Arches	8vo,	4 00
Johnson's Modern Framed Structures.....	Small 4to,	10 00
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part I., Stresses.....	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part II., Graphic Statics.....	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part III., Bridge Design.....	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges.		
Part IV., Continuous, Draw, Cantilever, Suspension, and		
Arched Bridges.....	8vo,	2 50
* Morison's The Memphis Bridge.....	Oblong 4to,	10 00
Waddell's Iron Highway Bridges.....	8vo,	4 00
" De Pontibus (a Pocket-book for Bridge Engineers).		
16mo, morocco,		3 00
Wood's Construction of Bridges and Roofs.....	8vo,	2 00
Wright's Designing of Draw Spans. Parts I. and II..	8vo, each	2 50
" " " " " Complete.....	8vo,	3 50

CHEMISTRY—BIOLOGY—PHARMACY.

QUALITATIVE—QUANTITATIVE—ORGANIC—INORGANIC, ETC.

Adriance's Laboratory Calculations.....	12mo,	\$1 25
Allen's Tables for Iron Analysis.....	.8vo,	3 00
Austen's Notes for Chemical Students.....	12mo,	1 50
Bolton's Student's Guide in Quantitative Analysis.....	.8vo,	1 50
Boltwood's Elementary Electro Chemistry.....	(<i>In the press.</i>)	
Classen's Analysis by Electrolysis. (Herrick and Boltwood.)	.8vo,	3 00
Cohn's Indicators and Test-papers.....	12mo	2 00
Crafts's Qualitative Analysis. (Schaeffer.).....	12mo,	1 50
Davenport's Statistical Methods with Special Reference to Bio- logical Variations.....	12mo, morocco,	1 25
Drechsel's Chemical Reactions. (Merrill.).....	12mo,	1 25
Fresenius's Quantitative Chemical Analysis. (Allen.).....	.8vo,	6 00
“ Qualitative “ “ (Johnson.).....	.8vo,	3 00
“ “ “ “ (Wells.) Trans.		
16th German Edition.....	.8vo,	5 00
Fuertes's Water and Public Health.....	12mo,	1 50
Gill's Gas and Fuel Analysis.....	12mo,	1 25
Hammarsten's Physiological Chemistry. (Mandel.).....	.8vo,	4 00
Helm's Principles of Mathematical Chemistry. (Morgan).	12mo,	1 50
Kolbe's Inorganic Chemistry.....	12mo,	1 50
Ladd's Quantitative Chemical Analysis.....	12mo,	1 00
Landauer's Spectrum Analysis. (Tingle.).....	.8vo,	3 00
Löb's Electrolysis and Electrosynthesis of Organic Compounds. (Lorenz.).....	12mo,	1 00
Mandel's Bio-chemical Laboratory.....	12mo,	1 50
Mason's Water-supply.....	.8vo,	5 00
“ Examination of Water.....	12mo,	1 25
Meyer's Radicles in Carbon Compounds. (Tingle.) (<i>In the press.</i>)		
Miller's Chemical Physics.....	.8vo,	2 00
Mixter's Elementary Text-book of Chemistry.....	12mo,	1 50
Morgan's The Theory of Solutions and its Results.....	12mo,	1 00
“ Elements of Physical Chemistry.....	12mo,	2 00
Nichols's Water-supply (Chemical and Sanitary).....	.8vo,	2 50
O'Brine's Laboratory Guide to Chemical Analysis.....	.8vo,	2 00
Perkins's Qualitative Analysis.....	12mo,	1 00
Pinner's Organic Chemistry. (Austen.).....	12mo,	1 50

Poole's Calorific Power of Fuels.....	8vo,	\$3 00
Ricketts and Russell's Notes on Inorganic Chemistry (Non-metallic).....	Oblong 8vo, morocco,	75
Ruddiman's Incompatibilities in Prescriptions.....	8vo,	2 00
Schimpf's Volumetric Analysis.....	12mo,	2 50
Spencer's Sugar Manufacturer's Handbook.....	16mo, morocco,	2 00
" Handbook for Chemists of Beet Sugar Houses.	16mo, morocco,	3 00
Stockbridge's Rocks and Soils.....	8vo,	2 50
* Tillman's Descriptive General Chemistry.....	8vo,	3 00
Van Deventer's Physical Chemistry for Beginners. (Boltwood.)	12mo,	1 50
Wells's Inorganic Qualitative Analysis.....	12mo,	1 50
" Laboratory Guide in Qualitative Chemical Analysis.	8vo,	1 50
Whipple's Microscopy of Drinking-water.....	8vo,	3 50
Wiechmann's Chemical Lecture Notes.....	12mo,	3 00
" Sugar Analysis.....	Small 8vo,	2 50
Wulling's Inorganic Phar. and Med. Chemistry.....	12mo,	2 00

DRAWING.

ELEMENTARY—GEOMETRICAL—MECHANICAL—TOPOGRAPHICAL.

Hill's Shades and Shadows and Perspective.....	8vo,	2 00
MacCord's Descriptive Geometry.....	8vo,	3 00
" Kinematics.....	8vo,	5 00
" Mechanical Drawing.....	8vo,	4 00
Mahan's Industrial Drawing. (Thompson.).....	2 vols., 8vo,	3 50
Reed's Topographical Drawing. (H. A.).....	4to,	5 00
Reid's A Course in Mechanical Drawing.....	8vo.	2 00
" Mechanical Drawing and Elementary Machine Design.	8vo. (<i>In the press.</i>)	
Smith's Topographical Drawing. (Macmillan.).....	8vo,	2 50
Warren's Descriptive Geometry.....	2 vols., 8vo,	3 50
" Drafting Instruments.....	12mo,	1 25
" Free-hand Drawing.....	12mo,	1 00
" Linear Perspective.....	12mo,	1 00
" Machine Construction.....	2 vols., 8vo,	7 50

Warren's Plane Problems.....	12mo,	\$1 25
“ Primary Geometry.....	12mo,	75
“ Problems and Theorems.....	8vo,	2 50
“ Projection Drawing	12mo,	1 50
Warren's Shades and Shadows.....	8vo,	3 00
“ Stereotomy—Stone-cutting.....	8vo,	2 50
Whelpley's Letter Engraving	12mo,	2 00

ELECTRICITY AND MAGNETISM.

ILLUMINATION—BATTERIES—PHYSICS—RAILWAYS. •

Anthony and Brackett's Text-book of Physics. (Magie.) Small		
	8vo,	3 00
Anthony's Theory of Electrical Measurements.....	12mo,	1 00
Barker's Deep-sea Soundings.....	8vo,	2 00
Benjamin's Voltaic Cell.....	8vo,	3 00
“ History of Electricity.....	8vo,	3 00
Classen's Analysis by Electrolysis. (Herrick and Boltwood.)	8vo,	3 00
Cosmic Law of Thermal Repulsion.....	12mo,	75
Crehore and Squier's Experiments with a New Polarizing Photo- Chronograph.....	8vo,	3 00
Dawson's Electric Railways and Tramways. Small, 4to, half		
	morocco,	12 50
* Dredge's Electric Illuminations... 2 vols., 4to, half morocco,		25 00
“ “ “ Vol. II.....	4to,	7 50
Gilbert's De magnete. (Mottelay.).....	8vo,	2 50
Holman's Precision of Measurements.....	8vo,	2 00
“ Telescope-mirror-scale Method.....	Large 8vo,	75
Löb's Electrolysis and Electrosynthesis of Organic Compounds. (Lorenz.).....	12mo,	1 00
* Michie's Wave Motion Relating to Sound and Light.....	8vo,	4 00
Morgan's The Theory of Solutions and its Results.....	12mo,	1 00
Niaudet's Electric Batteries. (Fishback.).....	12mo,	2 50
Pratt and Alden's Street-railway Road-beds.....	8vo,	2 00
Reagan's Steam and Electric Locomotives.....	12mo,	2 00
Thurston's Stationary Steam Engines for Electric Lighting Pur- poses.....	8vo,	2 50
* Tillman's Heat.....	8vo,	1 50

ENGINEERING.

CIVIL—MECHANICAL—SANITARY, ETC.

(See also BRIDGES, p. 4; HYDRAULICS, p. 9; MATERIALS OF ENGINEERING, p. 10; MECHANICS AND MACHINERY, p. 12; STEAM ENGINES AND BOILERS, p. 14.)

Baker's Masonry Construction.....	8vo,	\$5 00
“ Surveying Instruments.....	12mo,	3 00
Black's U. S. Public Works.....	Oblong 4to,	5 00
Brooks's Street-railway Location.....	16mo, morocco,	1 50
Butts's Civil Engineers' Field Book.....	16mo, morocco,	2 50
Byrne's Highway Construction.....	8vo,	5 00
“ Inspection of Materials and Workmanship.....	16mo,	3 00
Carpenter's Experimental Engineering	8vo,	6 00
Church's Mechanics of Engineering—Solids and Fluids....	8vo,	6 00
“ Notes and Examples in Mechanics.....	8vo,	2 00
Crandall's Earthwork Tables.....	8vo,	1 50
“ The Transition Curve.....	16mo, morocco,	1 50
* Dredge's Penn. Railroad Construction, etc. Large 4to,		
	half morocco,	20 00
* Drinker's Tunnelling.....	4to, half morocco,	25 00
Eissler's Explosives—Nitroglycerine and Dynamite.....	8vo,	4 00
Folwell's Sewerage.....	8vo,	3 00
Fowler's Coffe-dam Process for Piers.....	8vo,	2 50
Gerhard's Sanitary House Inspection.....	12mo,	1 00
Godwin's Railroad Engineer's Field-book.....	16mo, morocco,	2 50
Gore's Elements of Geodesy.....	8vo,	2 50
Howard's Transition Curve Field-book.....	16mo, morocco,	1 50
Howe's Retaining Walls (New Edition.).....	12mo,	1 25
Hudson's Excavation Tables. Vol. II.....	8vo,	1 00
Hutton's Mechanical Engineering of Power Plants.....	8vo,	5 00
“ Heat and Heat Engines.....	8vo,	5 00
Johnson's Materials of Construction.....	Large 8vo,	6 00
“ Stadia Reduction Diagram. Sheet, 22½ × 28½ inches,		50
“ Theory and Practice of Surveying.....	Small 8vo,	4 00
Kent's Mechanical Engineer's Pocket-book....	16mo, morocco,	5 00
Kiersted's Sewage Disposal.....	12mo,	1 25
Mahan's Civil Engineering. (Wood.).....	8vo,	5 00
Merriman and Brook's Handbook for Surveyors....	16mo, mor.,	2 00
Merriman's Precise Surveying and Geodesy	8vo,	2 50
“ Retaining Walls and Masonry Dams.....	8vo,	2 00
“ Sanitary Engineering.....	8vo,	2 00
Nagle's Manual for Railroad Engineers.....	16mo, morocco,	3 00
Ogden's Sewer Design.....	12mo,	2 00
Patton's Civil Engineering.....	8vo, half morocco,	7 50

Patton's Foundations.....	8vo,	\$5 00
Pratt and Alden's Street-railway Road-beds.....	8vo,	2 00
Rockwell's Roads and Pavements in France.....	12mo,	1 25
Searles's Field Engineering	16mo, morocco,	3 00
" Railroad Spiral.....	16mo, morocco,	1 50
Siebert and Biggin's Modern Stone Cutting and Masonry...	8vo,	1 50
Smart's Engineering Laboratory Practice.....	12mo,	2 50
Smith's Wire Manufacture and Uses.....	Small 4to,	3 00
Spalding's Roads and Pavements.....	12mo,	2 00
" Hydraulic Cement.....	12mo,	2 00
Taylor's Prismoidal Formulas and Earthwork.....	8vo,	1 50
Thurston's Materials of Construction	8vo,	5 00
* Trautwine's Civil Engineer's Pocket-book ...	16mo, morocco,	5 00
* " Cross-section.....	Sheet,	25
* " Excavations and Embankments.....	8vo,	2 00
* " Laying Out Curves.....	12mo, morocco,	2 50
Waddell's De Pontibus (A Pocket-book for Bridge Engineers).	16mo, morocco,	3 00
Wait's Engineering and Architectural Jurisprudence.....	8vo,	6 00
" Law of Field Operation in Engineering, etc.....	8vo,	6 50
Warren's Stereotomy—Stone-cutting.....	8vo,	2 50
Webb's Engineering Instruments. New Edition.	16mo, morocco,	1 25
Wegmann's Construction of Masonry Dams.....	4to,	5 00
Wellington's Location of Railways... ..	Small 8vo,	5 00
Wheeler's Civil Engineering.....	8vo,	4 00
Wolf's Windmill as a Prime Mover.....	8vo,	3 00

HYDRAULICS.

WATER-WHEELS—WINDMILLS—SERVICE PIPE—DRAINAGE, ETC.

(See also ENGINEERING, p. 8.)

Bazin's Experiments upon the Contraction of the Liquid Vein. (Trautwine.).....	8vo,	2 00
Bovey's Treatise on Hydraulics.....	8vo,	4 00
Coffin's Graphical Solution of Hydraulic Problems.....	12mo,	2 50
Ferrel's Treatise on the Winds, Cyclones, and Tornadoes...	8vo,	4 00
Fuertes's Water and Public Health.....	12mo,	1 50
Ganguillet & Kutter's Flow of Water. (Hering & Trautwine.)	8vo,	4 00
Hazen's Filtration of Public Water Supply.....	8vo,	2 00
Herschel's 115 Experiments	8vo,	2 00
Kiersted's Sewage Disposal.....	12mo,	1 25

Mason's Water Supply.....	8vo,	\$5 00
" Examination of Water.....	12mo,	1 25
Merriman's Treatise on Hydraulics.....	8vo,	4 00
Nichols's Water Supply (Chemical and Sanitary).....	8vo,	2 50
Wegmann's Water Supply of the City of New York.....	4to,	10 00
Weisbach's Hydraulics. (Du Bois.).....	8vo,	5 00
Whipple's Microscopy of Drinking Water.....	8vo,	3 50
Wilson's Irrigation Engineering.....	8vo,	4 00
" Hydraulic and Placer Mining.....	12mo,	2 00
Wolff's Windmill as a Prime Mover.....	8vo,	3 00
Wood's Theory of Turbines.....	8vo,	2 50

MANUFACTURES.

BOILERS—EXPLOSIVES—IRON—STEEL—SUGAR—WOOLLENS, ETC.

Allen's Tables for Iron Analysis.....	8vo,	3 00
Beaumont's Woollen and Worsted Manufacture.....	12mo,	1 50
Belland's Encyclopædia of Founding Terms.....	12mo,	3 00
" The Iron Founder.....	12mo,	2 50
" " " " Supplement.....	12mo,	2 50
Bouvier's Handbook on Oil Painting.....	12mo,	2 00
Eissler's Explosives, Nitroglycerine and Dynamite.....	8vo,	4 00
Ford's Boiler Making for Boiler Makers.....	18mo,	1 00
Metcalf's Cost of Manufactures.....	8vo,	5 00
Metcalf's Steel—A Manual for Steel Users.....	12mo,	2 00
* Reisig's Guide to Piece Dyeing.....	8vo,	25 00
Spencer's Sugar Manufacturer's Handbook....	16mo, morocco,	2 00
" Handbook for Chemists of Beet Sugar Houses.	16mo, morocco,	3 00
Thurston's Manual of Steam Boilers.....	8vo,	5 00
Walke's Lectures on Explosives.....	8vo,	4 00
West's American Foundry Practice.....	12mo,	2 50
" Moulder's Text-book.....	12mo,	2 50
Wiehmann's Sugar Analysis.....	Small 8vo,	2 50
Woodbury's Fire Protection of Mills.....	8vo,	2 50

MATERIALS OF ENGINEERING.

STRENGTH—ELASTICITY—RESISTANCE, ETC.

(See also ENGINEERING, p. 8.)

Baker's Masonry Construction.....	8vo,	5 00
Beardslee and Kent's Strength of Wrought Iron.....	8vo,	1 50
Bovey's Strength of Materials.....	8vo,	7 50
Burr's Elasticity and Resistance of Materials.....	8vo,	5 00
Byrne's Highway Construction.....	8vo,	5 00

Church's Mechanics of Engineering—Solids and Fluids.....	8vo,	\$6 00
Du Bois's Stresses in Framed Structures.....	Small 4to,	10 00
Johnson's Materials of Construction.....	8vo,	6 00
Lanza's Applied Mechanics.....	8vo,	7 50
Martens's Testing Materials. (Henning.).....	2 vols., 8vo,	7 50
Merrill's Stones for Building and Decoration.....	8vo,	5 00
Merriman's Mechanics of Materials.....	8vo,	4 00
" Strength of Materials.....	12mo,	1 00
Patton's Treatise on Foundations.....	8vo,	5 00
Rockwell's Roads and Pavements in France.....	12mo,	1 25
Spalding's Roads and Pavements.....	12mo,	2 00
Thurston's Materials of Construction.....	8vo,	5 00
" Materials of Engineering.....	3 vols., 8vo,	8 00
Vol. I., Non-metallic.....	8vo,	2 00
Vol. II., Iron and Steel.....	8vo,	3 50
Vol. III., Alloys, Brasses, and Bronzes.....	8vo,	2 50
Wood's Resistance of Materials.....	8vo,	2 00

MATHEMATICS.

CALCULUS—GEOMETRY—TRIGONOMETRY, ETC.

Baker's Elliptic Functions.....	8vo,	1 50
Ballard's Pyramid Problem.....	8vo,	1 50
Barnard's Pyramid Problem.....	8vo,	1 50
*Bass's Differential Calculus.....	12mo,	4 00
Briggs's Plane Analytical Geometry.....	12mo,	1 00
Chapman's Theory of Equations.....	12mo,	1 50
Compton's Logarithmic Computations.....	12mo,	1 50
Davis's Introduction to the Logic of Algebra.....	8vo,	1 50
Halsted's Elements of Geometry.....	8vo,	1 75
" Synthetic Geometry.....	8vo,	1 50
Johnson's Curve Tracing.....	12mo,	1 00
" Differential Equations—Ordinary and Partial.		
Small 8vo,		3 50
" Integral Calculus.....	12mo,	1 50
" " Unabridged. Small 8vo.		
(In the press.)		
" Least Squares.....	12mo,	1 50
*Ludlow's Logarithmic and Other Tables. (Bass.).....	8vo,	2 00
* " Trigonometry with Tables. (Bass.).....	8vo,	3 00
*Mahan's Descriptive Geometry (Stone Cutting).....	8vo,	1 50
Merriman and Woodward's Higher Mathematics.....	8vo,	5 00
Merriman's Method of Least Squares.....	8vo,	2 00
Parker's Quadrature of the Circle.....	8vo,	2 50
Rice and Johnson's Differential and Integral Calculus,		
2 vols. in 1, small 8vo,		2 50

Rice and Johnson's Differential Calculus.....	Small 8vo,	\$3 00
" Abridgment of Differential Calculus.....		
	Small 8vo,	1 50
Totten's Metrology.....	8vo,	2 50
Warren's Descriptive Geometry.....	2 vols., 8vo,	3 50
" Drafting Instruments.....	12mo,	1 25
" Free-hand Drawing.....	12mo,	1 00
" Linear Perspective.....	12mo,	1 00
" Primary Geometry.....	12mo,	75
" Plane Problems.....	12mo,	1 25
" Problems and Theorems.....	8vo,	2 50
" Projection Drawing.....	12mo,	1 50
Wood's Co-ordinate Geometry.....	8vo,	2 00
" Trigonometry.....	12mo,	1 00
Woolf's Descriptive Geometry.....	Large 8vo,	3 00

MECHANICS—MACHINERY.

TEXT-BOOKS AND PRACTICAL WORKS.

(See also ENGINEERING, p. 8.)

Baldwin's Steam Heating for Buildings.....	12mo,	2 50
Barr's Kinematics of Machinery.....	8vo,	
Benjamin's Wrinkles and Recipes.....	12mo,	2 00
Chordal's Letters to Mechanics.....	12mo,	2 00
Church's Mechanics of Engineering....	8vo,	6 00
" Notes and Examples in Mechanics.....	8vo,	2 00
Crehore's Mechanics of the Girder.....	8vo,	5 00
Cromwell's Belts and Pulleys.....	12mo,	1 50
" Toothed Gearing.....	12mo,	1 50
Compton's First Lessons in Metal Working.....	12mo,	1 50
Compton and De Groodt's Speed Lathe.....	12mo,	1 50
Dana's Elementary Mechanics.....	12mo,	1 50
Dingey's Machinery Pattern Making.....	12mo,	2 00
Dredge's Trans. Exhibits Building, World Exposition.		
	Large 4to, half morocco,	10 00
Du Bois's Mechanics. Vol. I., Kinematics.....	8vo,	3 50
" " Vol. II., Statics.....	8vo,	4 00
" " Vol. III., Kinetics.....	8vo,	3 50
Fitzgerald's Boston Machinist.....	18mo,	1 00
Flather's Dynamometers.....	12mo,	2 00
" Rope Driving.....	12mo,	2 00
Hall's Car Lubrication.....	12mo,	1 00
Holly's Saw Filing.....	18mo,	75
Johnson's Theoretical Mechanics. An Elementary Treatise.		
(In the press.)		
Jones's Machine Design. Part I., Kinematics.....	8vo,	1 50

Jones's Machine Design. Part II., Strength and Proportion of Machine Parts.....	8vo,	\$3 00
Lanza's Applied Mechanics.....	8vo,	7 50
MacCord's Kinematics.....	8vo,	5 00
Merriman's Mechanics of Materials.....	8vo,	4 00
Metcalf's Cost of Manufactures.....	8vo,	5 00
*Michie's Analytical Mechanics.....	8vo,	4 00
Richards's Compressed Air.....	12mo,	1 50
Robinson's Principles of Mechanism.....	8vo,	3 00
Smith's Press-working of Metals.....	8vo,	3 00
Thurston's Friction and Lost Work.....	8vo,	3 00
" The Animal as a Machine.....	12mo,	1 00
Warren's Machine Construction.....	2 vols., 8vo,	7 50
Weisbach's Hydraulics and Hydraulic Motors. (Du Bois.).....	8vo,	5 00
" Mechanics of Engineering. Vol. III, Part I., Sec. I. (Klein.).....	8vo,	5 00
Weisbach's Mechanics of Engineering. Vol. III, Part I., Sec. II. (Klein.).....	8vo,	5 00
Weisbach's Steam Engines. (Du Bois.).....	8vo,	5 00
Wood's Analytical Mechanics.....	8vo,	3 00
" Elementary Mechanics.....	12mo,	1 25
" " " Supplement and Key.....	12mo,	1 25

METALLURGY.

IRON—GOLD—SILVER—ALLOYS, ETC.

Allen's Tables for Iron Analysis.....	8vo,	3 00
Egleston's Gold and Mercury.....	Large 8vo,	7 50
" Metallurgy of Silver.....	Large 8vo,	7 50
* Kerl's Metallurgy—Copper and Iron.....	8vo,	15 00
" " Steel, Fuel, etc.....	8vo,	15 00
Kunhardt's Ore Dressing in Europe.....	8vo,	1 50
Metcalf's Steel—A Manual for Steel Users.....	12mo,	2 00
O'Driscoll's Treatment of Gold Ores.....	8vo,	2 00
Thurston's Iron and Steel.....	8vo,	3 50
" Alloys.....	8vo,	2 50
Wilson's Cyanide Processes.....	12mo,	1 50

MINERALOGY AND MINING.

MINE ACCIDENTS—VENTILATION—ORE DRESSING, ETC.

Barringer's Minerals of Commercial Value....	Oblong morocco,	2 50
Beard's Ventilation of Mines.....	12mo,	2 50
Boyd's Resources of South Western Virginia.....	8vo,	3 00
" Map of South Western Virginia.....	Pocket-book form,	2 00
Brush and Penfield's Determinative Mineralogy. New Ed.	8vo,	4 00

Chester's Catalogue of Minerals.....	8vo,	\$1 25
“ “ “ “	Paper,	50
“ Dictionary of the Names of Minerals.....	8vo,	3 00
Dana's American Localities of Minerals.....	Large 8vo,	1 00
“ Descriptive Mineralogy (E. S.) Large 8vo. half morocco,		12 50
“ First Appendix to System of Mineralogy. ...	Large 8vo,	1 00
“ Mineralogy and Petrography. (J. D.).....	12mo,	2 00
“ Minerals and How to Study Them. (E. S.).	12mo,	1 50
“ Text-book of Mineralogy. (E. S.)...New Edition.	8vo,	4 00
* Drinker's Tunnelling, Explosives, Compounds, and Rock Drills.		
	4to, half morocco,	25 00
Egleston's Catalogue of Minerals and Synonyms.....	8vo,	2 50
Eissler's Explosives—Nitroglycerine and Dynamite.....	8vo,	4 00
Hussak's Rock-forming Minerals. (Smith.).....	Small 8vo,	2 00
Ihlseng's Manual of Mining.. ..	8vo,	4 00
Kunhardt's Ore Dressing in Europe.....	8vo,	1 50
O'Driscoll's Treatment of Gold Ores.....	8vo,	2 00
* Penfield's Record of Mineral Tests.....	Paper, 8vo,	50
Rosenbusch's Microscopical Physlography of Minerals and		
Rocks. (Iddings.).....	8vo,	5 00
Sawyer's Accidents in Mines.....	Large 8vo,	7 00
Stockbridge's Rocks and Soils.....	8vo,	2 50
Walke's Lectures on Explosives.....	8vo,	4 00
Williams's Lithology.....	8vo,	3 00
Wilson's Mine Ventilation.....	12mo,	1 25
“ Hydraulic and Placer Mining.....	12mo,	2 50

STEAM AND ELECTRICAL ENGINES, BOILERS, Etc.

STATIONARY—MARINE—LOCOMOTIVE—GAS ENGINES, ETC.

(See also ENGINEERING, p. 8.)

Baldwin's Steam Heating for Buildings.....	12mo,	2 50
Clerk's Gas Engine.....	Small 8vo,	4 00
Ford's Boiler Making for Boiler Makers.....	18mo,	1 00
Hemenway's Indicator Practice.....	12mo,	2 00
Hoadley's Warm-blast Furnace.....	8vo,	1 50
Kneass's Practice and Theory of the Injector	8vo,	1 50
MacCord's Slide Valve.....	8vo,	2 00
Meyer's Modern Locomotive Construction.....	4to,	10 00
Peabody and Miller's Steam-boilers.....	8vo,	4 00
Peabody's Tables of Saturated Steam.....	8vo,	1 00
“ Thermodynamics of the Steam Engine.....	8vo,	5 00
“ Valve Gears for the Steam Engine.....	8vo,	2 50
Pray's Twenty Years with the Indicator.....	Large 8vo,	2 50
Pupin and Osterberg's Thermodynamics.....	12mo,	1 25

Reagan's Steam and Electric Locomotives.....	12mo,	\$2 00
Röntgen's Thermodynamics. (Du Bois.).....	8vo,	5 00
Sinclair's Locomotive Running.....	12mo,	2 00
Snow's Steam-boiler Practice.....	8vo. (<i>In the press.</i>)	
Thurston's Boiler Explosions.....	12mo,	1 50
" Engine and Boiler Trials.....	8vo,	5 00
" Manual of the Steam Engine. Part I., Structure and Theory.....	8vo,	6 00
" Manual of the Steam Engine. Part II., Design, Construction, and Operation.....	8vo,	6 00
	2 parts,	10 00
Thurston's Philosophy of the Steam Engine.....	12mo,	75
" Reflection on the Motive Power of Heat. (Carnot.)	12mo,	1 50
" Stationary Steam Engines.....	8vo,	2 50
" Steam-boiler Construction and Operation.....	8vo,	5 00
Spangler's Valve Gears.....	8vo,	2 50
Weisbach's Steam Engine. (Du Bois.).....	8vo,	5 00
Whitham's Constructive Steam Engineering.....	8vo,	6 00
" Steam-engine Design.....	8vo,	5 00
Wilson's Steam Boilers. (Flather.).....	12mo,	2 50
Wood's Thermodynamics, Heat Motors, etc.....	8vo,	4 00

TABLES, WEIGHTS, AND MEASURES.

FOR ACTUARIES, CHEMISTS, ENGINEERS, MECHANICS—METRIC TABLES, ETC.

Adriance's Laboratory Calculations.....	12mo,	1 25
Allen's Tables for Iron Analysis.....	8vo,	3 00
Bixby's Graphical Computing Tables.....	Sheet,	25
Compton's Logarithms.....	12mo,	1 50
Crandall's Railway and Earthwork Tables.....	8vo,	1 50
Egleston's Weights and Measures.....	18mo,	75
Fisher's Table of Cubic Yards.....	Cardboard,	25
Hudson's Excavation Tables. Vol. II.....	8vo,	1 00
Johnson's Stadia and Earthwork Tables.....	8vo,	1 25
Ludlow's Logarithmic and Other Tables. (Bass.).....	12mo,	2 00
Totten's Metrology.....	8vo,	2 50

VENTILATION.

STEAM HEATING—HOUSE INSPECTION—MINE VENTILATION.

Baldwin's Steam Heating.....	12mo,	2 50
Beard's Ventilation of Mines.....	12mo,	2 50
Carpenter's Heating and Ventilating of Buildings.....	8vo,	3 00
Gerhard's Sanitary House Inspection.....	12mo,	1 00
Reid's Ventilation of American Dwellings.....	12mo,	1 50
Wilson's Mine Ventilation.....	12mo,	1 25

MISCELLANEOUS PUBLICATIONS.

Alcott's Gems, Sentiment, Language.....	Gilt edges,	\$5 00
Bailey's The New Tale of a Tub.....	8vo,	75
Ballard's Solution of the Pyramid Problem.....	8vo,	1 50
Barnard's The Metrological System of the Great Pyramid..	8vo,	1 50
Davis's Elements of Law.....	8vo,	2 00
Emmon's Geological Guide-book of the Rocky Mountains..	8vo,	1 50
Ferrel's Treatise on the Winds.....	8vo,	4 00
Hannes's Addresses Delivered before the Am. Ry. Assn....	12mo,	2 50
Mott's The Fallacy of the Present Theory of Sound..	Sq. 16mo,	1 00
Ricketts's History of Rensselaer Polytechnic Institute....	8vo,	3 00
Rotherham's The New Testament Critically Emphasized.	12mo,	1 50
" The Emphasized New Test. A new translation.	Large 8vo,	2 00
Totten's An Important Question in Metrology.....	8vo,	2 50
Whitehouse's Lake Mœris.....	Paper,	25
* Wiley's Yosemite, Alaska, and Yellowstone	4to,	3 00

HEBREW AND CHALDEE TEXT-BOOKS.

FOR SCHOOLS AND THEOLOGICAL SEMINARIES.

Gesenius's Hebrew and Chaldee Lexicon to Old Testament. (Tregelles.).....	Small 4to, half morocco,	5 00
Green's Elementary Hebrew Grammar.....	12mo,	1 25
" Grammar of the Hebrew Language (New Edition).	8vo,	3 00
" Hebrew Chrestomathy.....	8vo,	2 00
Letteris's Hebrew Bible (Massoretic Notes in English).	8vo, arabesque,	2 25

MEDICAL.

Bull's Maternal Management in Health and Disease.....	12mo,	1 00
Hammarsten's Physiological Chemistry. (Mandel.).....	8vo,	4 00
Mott's Composition, Digestibility, and Nutritive Value of Food.	Large mounted chart,	1 25
Ruddiman's Incompatibilities in Prescriptions.....	8vo,	2 00
Steel's Treatise on the Diseases of the Ox.....	8vo,	6 00
" Treatise on the Diseases of the Dog.....	8vo,	3 50
Woodhull's Military Hygiene.....	16mo,	1 50
Worcester's Small Hospitals—Establishment and Maintenance, including Atkinson's Suggestions for Hospital Archi- tecture.....	12mo,	1 25



